Weak Lensing and Cosmology

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Overview

Lensing by the large-scale structure

- (Weak) gravitational lensing in a nutshell
- Deflection in an inhomogeneous Universe
- Shear and convergence
- Projected power spectrum and cosmological parameters

Weak lensing and cosmology

- Second-order cosmic shear statistics
- Shear tomography (2 1/2 D lensing)
- Third-order cosmic shear statistics
- 3D lensing
- Peak statistics
- Shear-ratio geometry test

Observational aspects of weak lensing

- Shape measurement
- Photometric redshifts
- Intrinsic alignment
- Non-linear structure formation
- Non-Gaussian errors

Books & Reviews

- Kochanek, Schneider & Wambsganss, Gravitational lensing: Strong, weak & micro, proceedings of the 33rd Saas-Fee Advanced Course, 2004, Springer (http://www.astro.uni-bonn.de/~peter/SaasFee.html)
- Bartelmann & Schneider, Weak gravitational lensing, 2001, Phys. Rep. , 340, 297 (astro-ph/9912508)
- Refregier, Weak gravitational lensing by large-scale structure, 2003, ARA&A, 41, 645 (astro-ph/0307212)
- van Waerbeke & Mellier, Gravitational lensing by large scale structures: A review, Aussois winter school, astro-ph/0305089)
- Munshi et al. 2007, Cosmology with weak lensing surveys, submitted to Phys.Rep., astro-ph/0612667

Lensing by the large-scale structure

Overview

- (Weak) gravitational lensing in a nutshell
- Deflection of light in an inhomogeneous Universe
- Shear γ and convergence κ
- Projected power spectrum and cosmological parameters

(Weak) gravitational lensing in a nutshell

Gravitational lensing theory

Phenomenon of gravitational light deflection in the limits of weak, stationary fields and small deflection angles

Basis is General Theory of Relativity

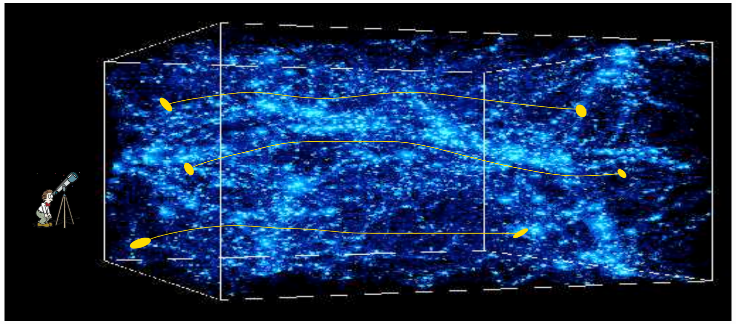
Photons travel on null geodesics of space-time metric. Simplified mathematical treatment of GL.

Achievements of weak lensing

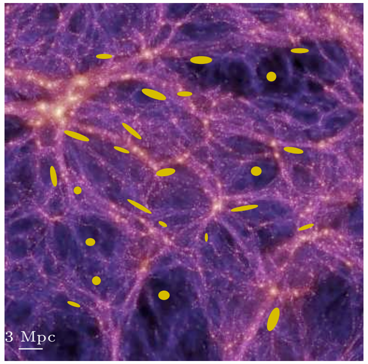
Cluster masses, mass profiles, M/L-relation, SI cross-section of dark matter, galaxy halos at large scales, power spectrum normalization σ_8, $\Omega_{\rm m}$, structure growth

Probing matter distribution using distant galaxies

- Light from distant galaxies is continuously deflected on its way through an inhomogeneous Universe
- Light bundles are differentially distorted due to gravitational lensing by tidal field of large-scale structure (LSS)

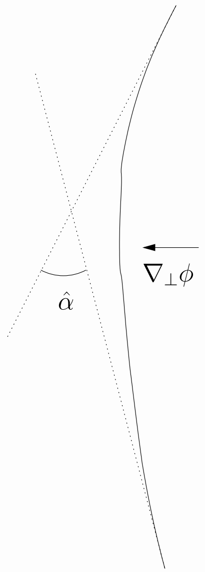


- Images of galaxies are coherently distorted leading to shape correlations which depend on statistical properties of LSS
- Probes total (dark+luminous) matter, no tracer for dark matter needed
- Distortions are very small (weak lensing regime), can be detected only statistically using large number of galaxies



"Cosmic shear"

Deflection angle



- Perturbed Minkowski metric, weak field $\phi \ll c^2$

$$\mathrm{d}s^2 = \left(1 + 2\phi/c^2\right) c^2 \mathrm{d}t^2 - \left(1 - 2\phi/c^2\right) \mathrm{d}\ell^2$$

- Fermat's principle: light travel time stationary

$$t = \frac{1}{c} \int_{\text{path}} \left(1 - 2\phi/c^2\right) \mathrm{d}\ell$$

- Deflection angle

$$\boldsymbol{\alpha} = -\frac{2}{c^2} \int_{\mathrm{S}}^{\mathrm{O}} \boldsymbol{\nabla}_\perp \phi \, \mathrm{d}\ell$$

Propagation of light bundles

- Comoving separation $\boldsymbol{x}$ between two light rays from geodesic deviation equation, relating neighboring geodesics via Riemann tensor

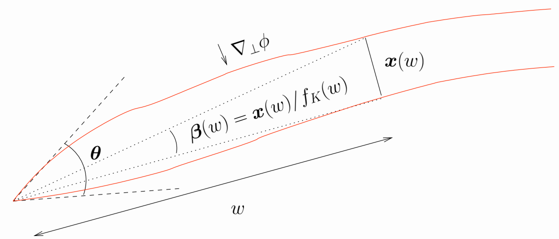
$$\longrightarrow \frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}w^2} + K\boldsymbol{x} = -\frac{2}{c^2}\Delta\Big(\nabla_\perp\phi(\boldsymbol{x}, w)\Big).$$

  (w = comoving distance, K = spatial curvature)

- Solution is integral equation

$$\boldsymbol{x}(\boldsymbol{\theta}, w) = f_K(w)\boldsymbol{\theta} - \frac{2}{c^2}\int_0^w dw' f_K(w-w')\Delta\Big(\nabla_\perp\phi(\boldsymbol{x}(\boldsymbol{\theta}, w'), w')\Big).$$

  ($f_K(w)$ = comoving angular diameter distance)



Deflection angle

Solving differential equation

- Born approximation: replace $\boldsymbol{x}$ on r.h.s. with $\boldsymbol{x}_0(\boldsymbol{\theta}, w) = f_K(w)\,\boldsymbol{\theta}$ (integrate along unperturbed ray)
- Deflection angle = difference between angular separation of two light rays in unperturbed and perturbed Universe, at comoving distance w

$$\boldsymbol{\alpha}(\boldsymbol{\theta}, w) \equiv \boldsymbol{\theta} - \boldsymbol{\beta}(\boldsymbol{\theta}, w) = \frac{f_K(w)\boldsymbol{\theta} - \boldsymbol{x}(\boldsymbol{\theta}, w)}{f_K(w)}$$
$$= \frac{2}{c^2} \int_0^w \mathrm{d}w' \, \frac{f_K(w - w')}{f_K(w)} \nabla_\perp \phi[f_K(w')\boldsymbol{\theta}, w'), w']$$

- Lensing potential, $\boldsymbol{\alpha} = \nabla\psi$

$$\psi(\boldsymbol{\theta}, w) = \frac{2}{c^2} \int_0^w \mathrm{d}w' \, \frac{f_K(w - w')}{f_K(w') f_K(w)} \phi(f_K(w')\boldsymbol{\theta}, w')$$

Linearizing the lens mapping

- $\boldsymbol{\beta}(\boldsymbol{\theta}) = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$ is mapping from unperturbed ($\boldsymbol{\theta}$) to unperturbed ($\boldsymbol{\beta}$) coordinates (lens equation)
- Linearize mapping, defining Jacobian

$$\mathcal{A}_{ij} = \frac{\partial \beta_i}{\partial \theta_j} == \delta_{ij} - \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}$$

defining convergence κ and shear γ as second-order derivatives of lensing potential

$$\kappa = \frac{1}{2}(\partial_1 \partial_1 + \partial_2 \partial_2)\psi$$

$$\gamma_1 = \frac{1}{2}(\partial_1 \partial_1 - \partial_2 \partial_2)\psi; \qquad \gamma_2 = \partial_1 \partial_2 \psi$$

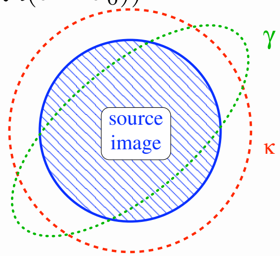
- Reduced shear $g_i = \gamma_i / (1 - \kappa)$

Shear and convergence

Liouville's theorem: Surface brightness is conserved

$$I(\boldsymbol{\theta}) = I^{\mathrm{s}}(\boldsymbol{\beta}(\boldsymbol{\theta})) \approx I^{\mathrm{s}}(\boldsymbol{\beta}(\boldsymbol{\theta}_0) + \mathcal{A}(\boldsymbol{\theta} - \boldsymbol{\theta}_0))$$

Effect of lensing

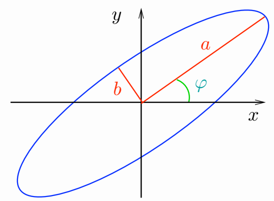


- isotropic magnification (convergence κ)
- anisotropic stretching (shear γ)

Shear transforms a circle into an ellipse.
Define complex ellipticity

$$\gamma = \gamma_1 + \mathrm{i}\gamma_2 = |\gamma|\mathrm{e}^{2\mathrm{i}\varphi};$$

$$|\gamma| = |1-\kappa|\frac{1-b/a}{1+b/a}$$



Basic equation of weak lensing

Weak lensing regime

$\kappa \ll 1, |\gamma| \ll 1.$
The observed ellipticity of a galaxy is the sum of the intrinsic ellipticity and the shear:

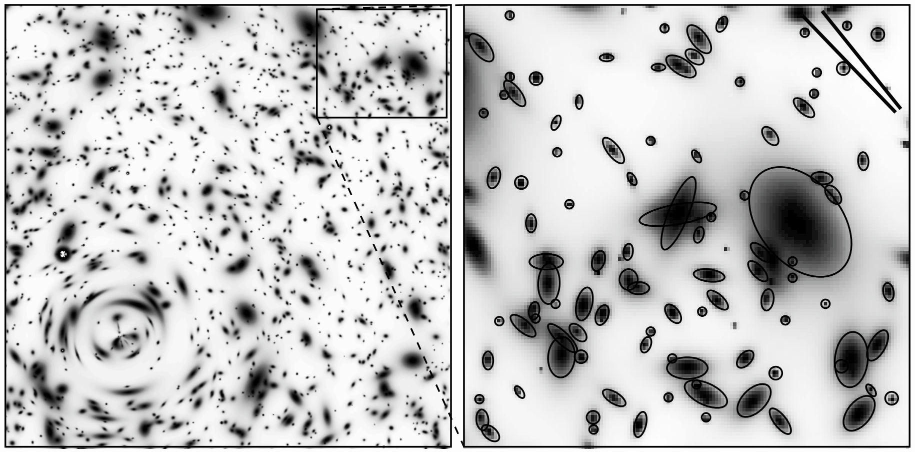
$$\boxed{\varepsilon \approx \varepsilon^{\mathrm{s}} + \gamma}$$

Random intrinsic orientation of galaxies

$$\langle \varepsilon^{\mathrm{s}} \rangle = 0 \quad \longrightarrow \quad \boxed{\langle \varepsilon \rangle = \gamma}$$

The observed ellipticity is an unbiased estimator of the shear. Very noisy though! $\sigma_\varepsilon = \langle |\varepsilon^{\mathrm{s}}|^2 \rangle^{1/2} \approx 0.3 - 0.4 \gg \gamma$. Beat down noise by averaging over large number of galaxies.

Ellipticity and local shear



[from Y. Mellier]

Galaxy ellipticities are an estimator of the local shear.

Typical numbers

Regime	γ	$\gamma/\sigma_\varepsilon$	$N_{\rm gal}$ for S/N~ 1
weak lensing by clusters	0.03	0.1	10^2
galaxy-galaxy lensing	0.003	0.01	10^4
cosmic shear	0.001	0.003	10^5

Much more galaxies for precision measurements needed.

Cosmic shear galaxy surveys

$n_{\rm gal}$ [arcmin^{-2}]	10 – 30	(from ground)
	60 – 100	(from space)

Area:
- past: from < 1 deg^2 to ≈ 100 deg^2.
- ongoing: Subaru (33 deg^2), DLS (36 deg^2), CFHTLS-Wide (170 deg^2)
- future: DES, KIDS, SNAP (1000–5000 deg^2), Pan-STARRS-4, LSST, DUNE (20 000 deg^2)

Relation to density contrast

Back to the propagation equation

- Since $\kappa = \frac{1}{2}\Delta\psi$:

$$\kappa(\boldsymbol{\theta}, w) = \frac{1}{c^2} \int_0^w \mathrm{d}w' \, \frac{f_K(w-w')f_K(w')}{f_K(w)} \Delta_{\boldsymbol{\theta}} \Phi(f_K(w')\boldsymbol{\theta}, w')$$

- Terms $\Delta_{w'w'}\Phi$ average out when integrating along line of sight, can be added to yield 3d Laplacian (error $\mathcal{O}(\Phi) \sim 10^{-5}$).
- Poisson equation

$$\Delta\Phi = \frac{3H_0^2\Omega_\mathrm{m}}{2a} \, \delta$$

$$\rightarrow \kappa(\boldsymbol{\theta}, w) = \frac{3}{2}\Omega_\mathrm{m} \left(\frac{H_0}{c}\right)^2 \int_0^w \mathrm{d}w' \, \frac{f_K(w-w')f_K(w')}{f_K(w)a(w')} \, \delta\left(f_K(w')\boldsymbol{\theta}, w'\right) .$$

Amplitude of the cosmic shear signal

Order-of magnitude estimate

$$\kappa(\boldsymbol{\theta}, w) = \frac{3}{2}\Omega_{\rm m}\left(\frac{H_0}{c}\right)^2 \int_0^w {\rm d}w' \frac{f_K(w-w')f_K(w')}{f_K(w)a(w')}\,\delta\left(f_K(w')\boldsymbol{\theta}, w'\right).$$

for simple case: single lens at at redshift $z_{\rm L} = 0.4$ with size R, source at $z_{\rm S} = 0.8$.

$$\kappa \approx \frac{3}{2}\Omega_{\rm m}\left(\frac{H_0}{c}\right)^2 \frac{D_{\rm LS}D_{\rm L}}{D_{\rm S}} \frac{R}{a^2(z_{\rm L})} \frac{\delta\rho}{\rho}$$

Add signal from $N \approx D_{\rm S}/R$ crossings:

$$\begin{aligned}\langle\kappa^2\rangle^{1/2} \approx & \frac{3}{2}\Omega_{\rm m}\frac{D_{\rm LS}D_{\rm L}}{R_{\rm H}^2}\sqrt{\frac{R}{D_{\rm S}}}a^{-1.5}(z_{\rm L})\left\langle\left(\frac{\delta\rho}{\rho}\right)^2\right\rangle \\ \approx & \frac{3}{2}\,0.3\times 0.1 \quad\times\ 0.1 \quad\times 2 \qquad\times 1 \qquad\approx 0.01\end{aligned}$$

- Convergence signal from a distribution of source galaxies with pdf $p(w)\mathrm{d}w$

$$\kappa(\boldsymbol{\theta}) = \int_0^{w_{\mathrm{lim}}} \mathrm{d}w \, p(w) \, \kappa(\boldsymbol{\theta}, w) = \int_0^{w_{\mathrm{lim}}} \mathrm{d}w \, G(w) \, f_K(w) \, \delta \left(f_K(w)\boldsymbol{\theta}, w \right)$$

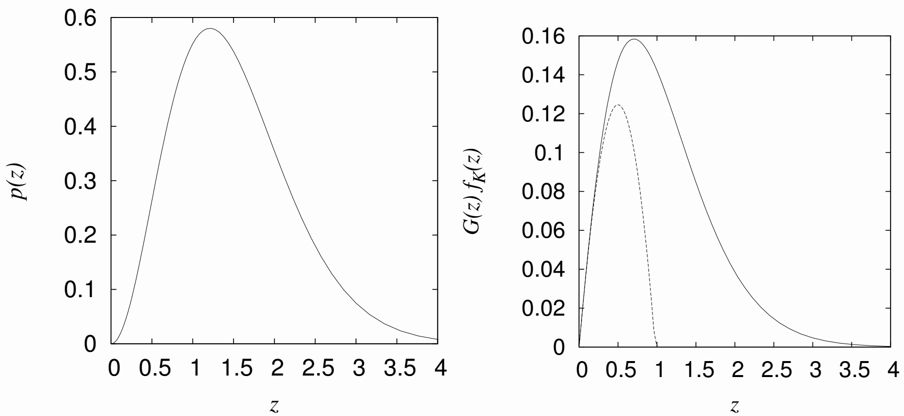
with lens efficiency

$$G(w) = \frac{3}{2} \left(\frac{H_0}{c} \right)^2 \frac{\Omega_{\mathrm{m}}}{a(w)} \int_w^{w_{\mathrm{lim}}} \mathrm{d}w' \, p(w') \frac{f_K(w' - w)}{f_K(w')}.$$

The convergence is a projection of the matter-density contrast, weighted by the source galaxy distribution and angular distances.

Parametrization of redshift distribution, e.g.

$$p(w)\mathrm{d}w = p(z)\mathrm{d}z \propto (z/z0)^{\alpha} \exp[-(z/z0)^{\beta}]$$



$\alpha = 2, \beta = 1.5, z_0 = 1$
dashed line: all sources at redshift 1

The convergence power spectrum

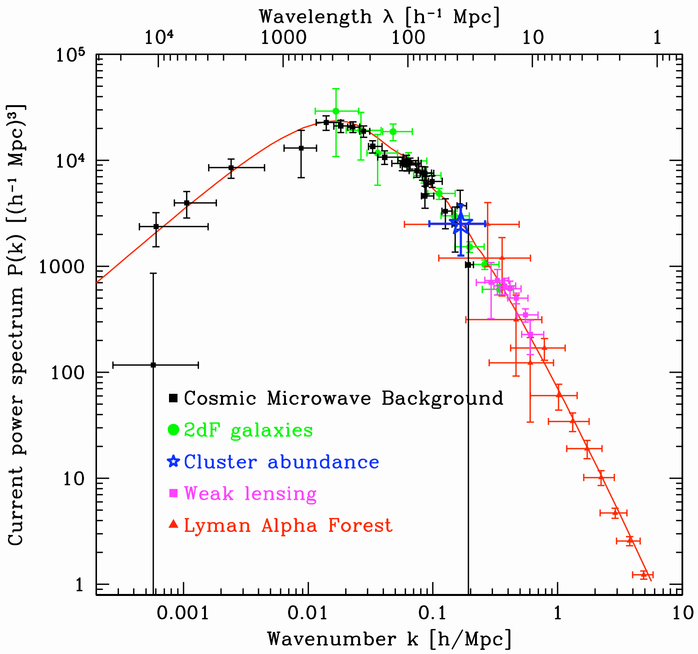
- Variance of convergence $\langle \kappa(\boldsymbol{\vartheta} + \boldsymbol{\theta}) \kappa(\boldsymbol{\vartheta}) \rangle = \langle \kappa\kappa \rangle(\theta)$ depends on variance of the density contrast $\langle \delta\delta \rangle$
- In Fourier space:

$$\left\langle \hat{\kappa}(\boldsymbol{\ell}) \hat{\kappa}^*(\boldsymbol{\ell}') \right\rangle = (2\pi)^2 \delta_{\mathrm{D}}(\boldsymbol{\ell} - \boldsymbol{\ell}') P_\kappa(\ell)$$
$$\left\langle \hat{\delta}(\boldsymbol{k}) \hat{\delta}^*(\boldsymbol{k}') \right\rangle = (2\pi)^3 \delta_{\mathrm{D}}(\boldsymbol{k} - \boldsymbol{k}') P_\delta(k)$$

- Limber's equation

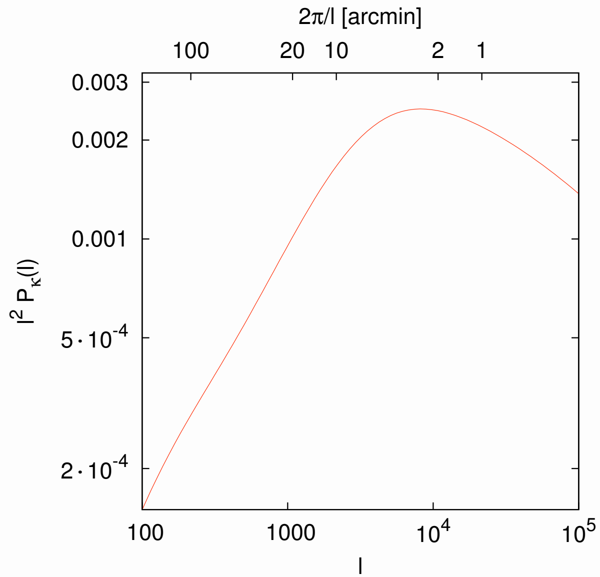
$$P_\kappa(\ell) = \int \mathrm{d}w \, G^2(w) P_\delta\left(\frac{\ell}{f_K(w)}\right)$$

  using small-angle approximation, $P_\delta(k) \approx P_\delta(k_\perp)$, contribution only from Fourier modes $\perp$ to line of sight
- Relations between κ and γ $\longrightarrow$ $\boxed{P_\kappa = P_\gamma}$


Wavelength λ [h⁻¹ Mpc]
10⁴
1000
100
10
1
10⁵
10⁴
1000
100
10
1
Current power spectrum P(k) [(h⁻¹ Mpc)³]
Cosmic Microwave Background
2dF galaxies
Cluster abundance
Weak lensing
Lyman Alpha Forest
0.001
0.01
0.1
1
10
Wavenumber k [h/Mpc]

[M. Tegmark]

Convergence power spectrum

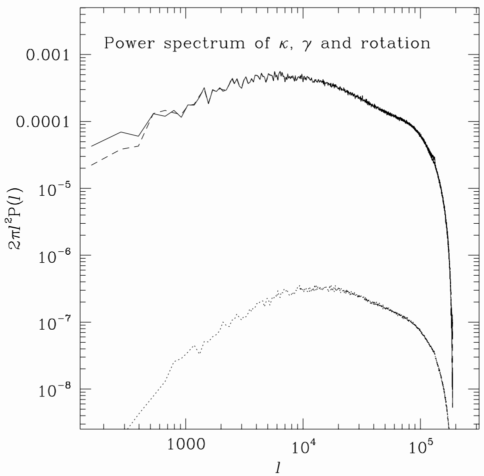


Example

A simple toy model: single lens plane at redshift z_0, $P_\delta(k) \propto \sigma_8^2 k^n$, CDM, no Λ, linear growth:

$$\langle \kappa^2(\theta) \rangle^{1/2} = \langle \gamma^2(\theta) \rangle^{1/2} \approx 0.01\, \sigma_8\, \Omega_{\rm m}^{0.8} \left(\frac{\theta}{1 \deg} \right)^{-(n+2)/2} z_0^{0.75}$$

Born-approximation tested with numerical (ray-tracing) simulations.



Asymmetry of Jacobi-matrix $\mathcal{A}$ due to lens-lens coupling negligible
[Jain, Seljak & White 2000]