Weak Gravitational Lensing
cycle 2

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Overview

[Day 1] Reminders from last year
- The lens equation
- Convergence, shear, and ellipticity
- Basic equation of weak lensing

[Day 1] Measurement of weak lensing
- Galaxy shape measurement
- Shear biases and calibration
- PSF correction & diagnostics
- PSF systematics

[Day 2] Measurement of weak lensing continued
- From pixels to cosmology
- Shear calibration revisited

[Day 2] Galaxy-galaxy lensing theory
- Tangential shear, and surface mass excess
- Galaxy – dark-matter connection I

[Day 3] Galaxy-galaxy lensing measurements
- Galaxy – dark-matter connection II
- Testing GR

[Day 3]: More lensing theory
- Back to the aperture mass: Filter function relations
- Spherical-sky lensing projections
- E-/B-mode estimators
- Measurements & systematics

Books, Reviews and Lecture Notes

- Kochanek, Schneider & Wambsganss 2004, book (Saas Fee) Gravitational lensing: Strong, weak & micro. Download Part I (Introduction) and Part III (Weak lensing) from my homepage `http://www.cosmostat.org/people/kilbinger`.
- Kilbinger 2015, review Cosmology from cosmic shear observations Reports on Progress in Physics, 78, 086901, arXiv:1411.0155
- Bartelmann & Maturi 2017, review Weak gravitational lensing, Scholarpedia 12(1):32440, arXiv:1612.06535
- Mandelbaum 2018, review Weak lensing for precision cosmology, ARAA submitted, arXiv:1710.03235
- Henk Hoekstra 2013, lecture notes (Varenna) arXiv:1312.5981
- Sarah Bridle 2014, lecture videos (Saas Fee) `http://archiveweb.epfl.ch/saasfee2014.epfl.ch/page-110036-en.html`
- Alan Heavens, 2015, lecture notes (Rio de Janeiro) `www.on.br/cce/2015/br/arq/Heavens_Lecture_4.pdf`

Day 1: Reminders from last year

The lens equation

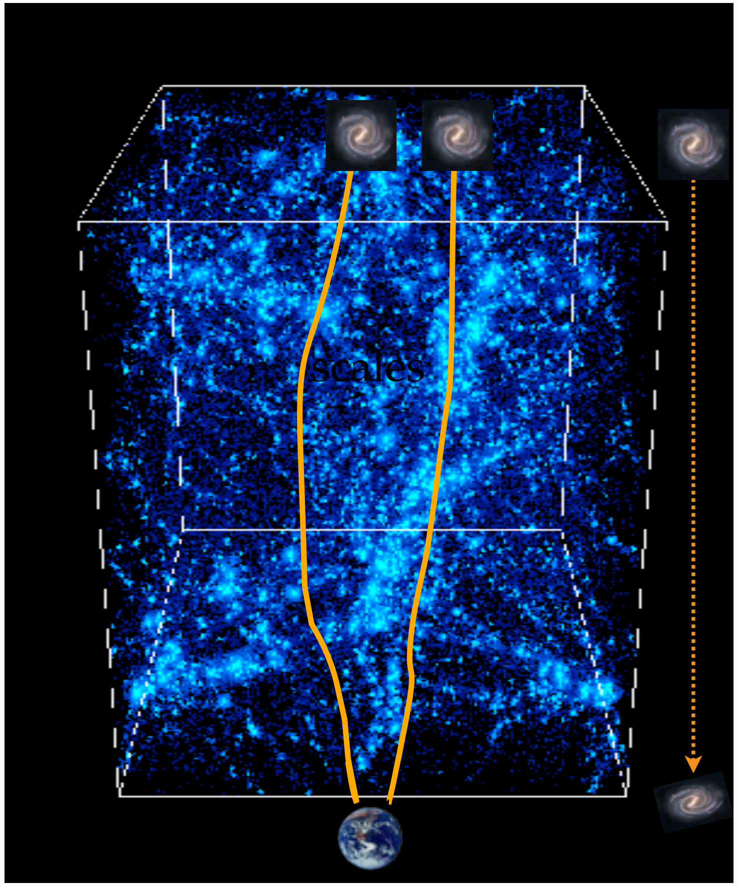
Convergence, shear, and ellipticity

Basic equation of weak lensing

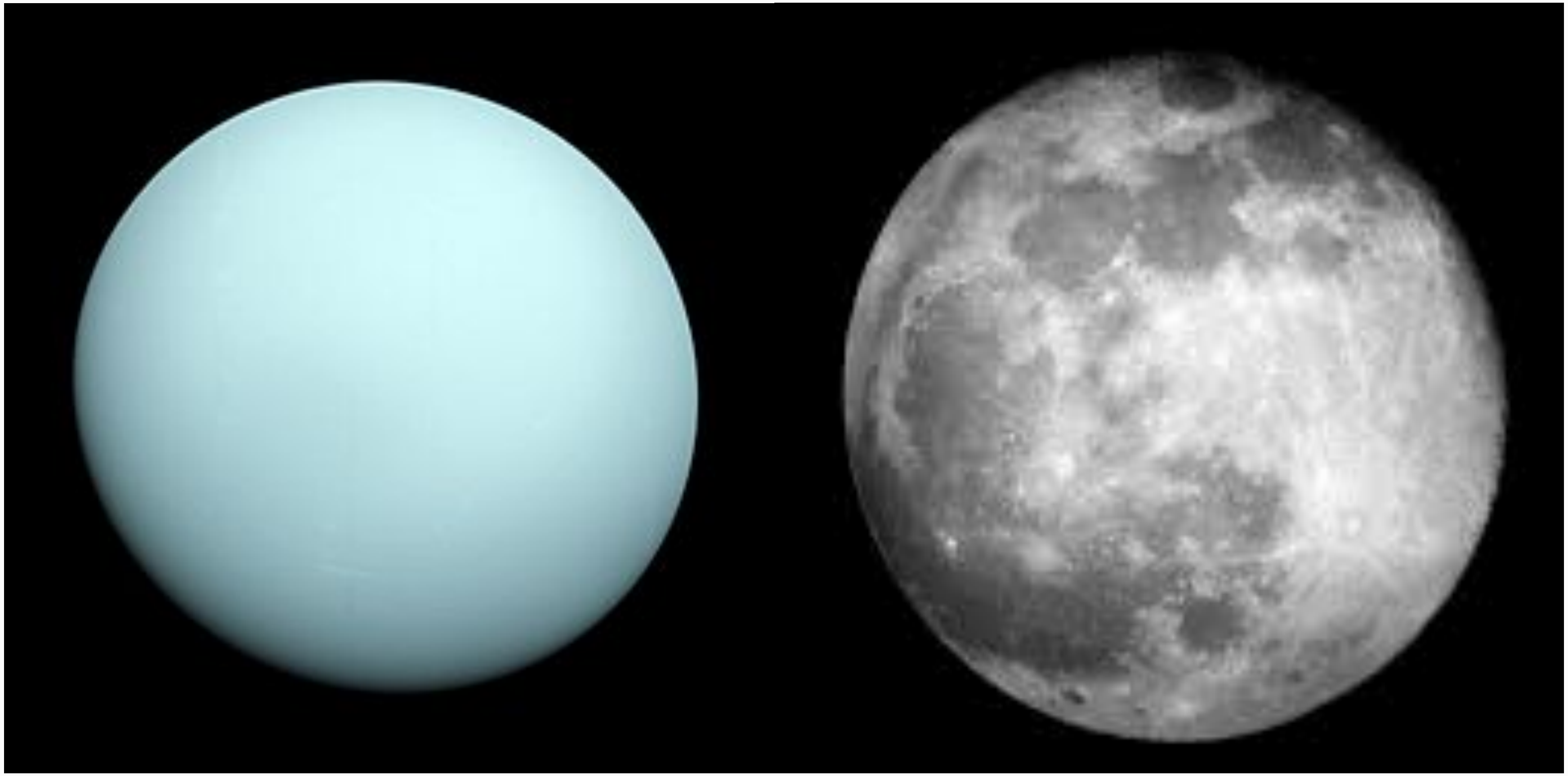
Cosmic shear, or weak cosmological lensing

Light of distant galaxies is deflected while travelling through inhomogeneous Universe. Information about mass distribution is imprinted on observed galaxy images.

- Continuous deflection: sensitive to projected 2D mass distribution.
- Differential deflection: magnification, distortions of images.
- Small distortions, few percent change of images: need statistical measurement.
- Coherent distortions: measure correlations, scales few Mpc to few 100 Mpc.

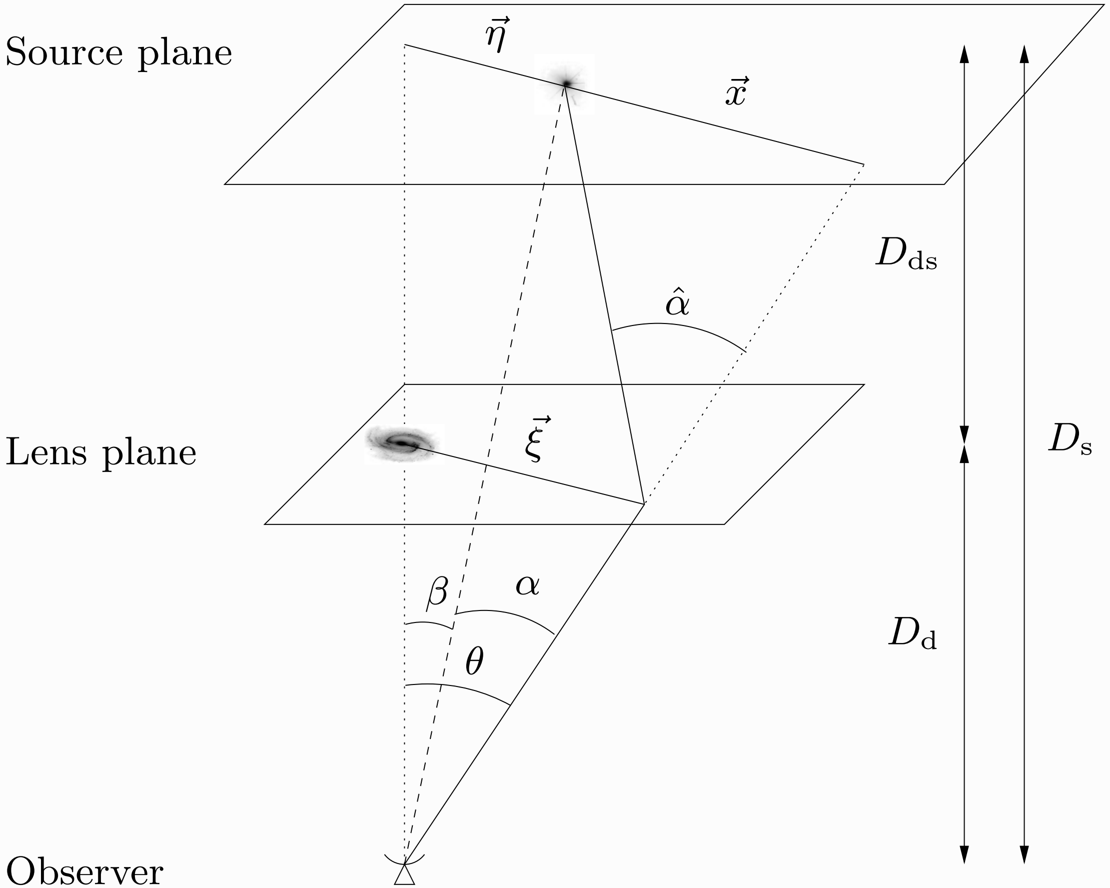


Measuring cosmic shear



Typical shear of a few percent equivalent to difference in ellipticity between Uranus and the Moon.

The lens equation



The lens equation is

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}.$$

This is a mapping from lens coordinates $\boldsymbol{\theta}$ to source coordinates $\boldsymbol{\beta}$.

(Q: why not the other way round?)

The deflection angle $\boldsymbol{\alpha}(\boldsymbol{\theta})$ depends on the mass distribution of the lens. It is the gradient of the 2D lensing potential,

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \boldsymbol{\nabla}\psi(\boldsymbol{\theta}).$$

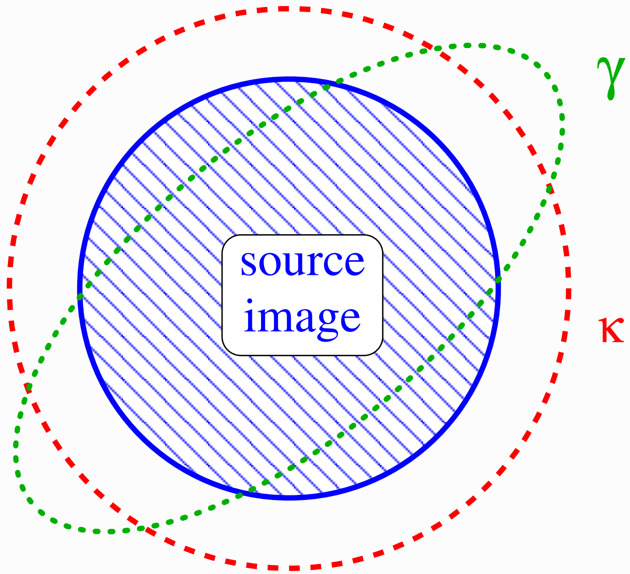
Linearized lensing quantities I

Linearizing lens equation

We talked about differential deflection last year. To first order, this involves the derivative of the deflection angle.

Or the lens mapping:

$$\frac{\partial \beta_i}{\partial \theta_j} \equiv A_{ij} = \delta_{ij} - \partial_i \partial_j \psi.$$



Jacobi (symmetric) matrix

$$A = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}.$$

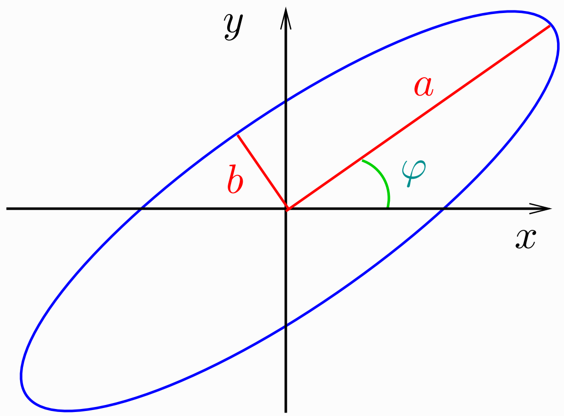
- convergence κ: isotropic magnification
- shear γ: anisotropic stretching

Convergence and shear are second derivatives of the 2D lensing potential.

Convergence and shear I

Define complex shear

$$\gamma = \gamma_1 + \mathrm{i}\gamma_2 = |\gamma| \mathrm{e}^{2\mathrm{i}\varphi};$$



The relation between convergence, shear, and the axis ratio of elliptical isophotes is then

$$|\gamma| = |1 - \kappa| \frac{1 - b/a}{1 + b/a}$$

Summary:

- Convergence and shear describe linearised lensing transformations
- They encompass information about projected mass distribution (lensing potential ψ).
- They quantify how lensed images are magnified, enlarged, and stretched.
- These are the main quantities in (weak) lensing.
- Shear is easier to measure (see below), convergence more intuitive to interpret and plot ("mass" maps). One can be transformed into the other, with caveats

Basic equation of weak lensing

Weak lensing regime

$\kappa \ll 1, |\gamma| \ll 1.$

The observed ellipticity of a galaxy is the sum of the intrinsic ellipticity and the shear:

$$\boxed{\varepsilon^{\mathrm{obs}} \approx \varepsilon^{\mathrm{s}} + \gamma}$$

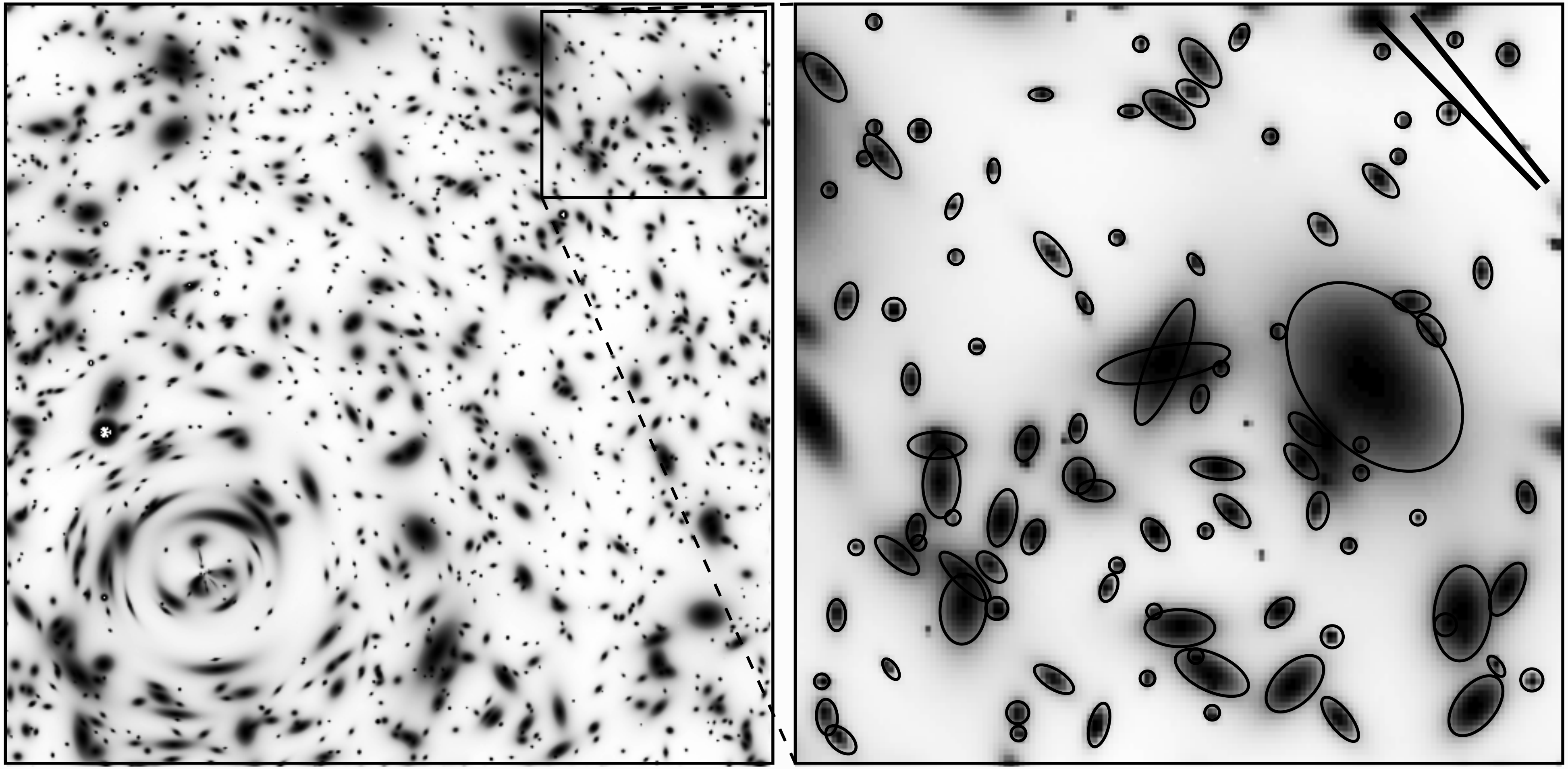
Random intrinsic orientation of galaxies

$$\langle \varepsilon^{\mathrm{s}} \rangle = 0 \quad \longrightarrow \quad \boxed{\langle \varepsilon^{\mathrm{obs}} \rangle = \gamma}$$

The observed ellipticity is an unbiased estimator of the shear. Very noisy though! $\sigma_\varepsilon = \langle |\varepsilon^{\mathrm{s}}|^2 \rangle^{1/2} \approx 0.4 \gg \gamma \sim 0.03$. Increase S/N and beat down noise by averaging over large number of galaxies.

Question: Why is the equivalent estimation of the convergence and/or magnification more difficult?

Ellipticity and local shear



[from Y. Mellier]

Galaxy ellipticities are an estimator of the local shear.

Day 1: Measurement of weak lensing

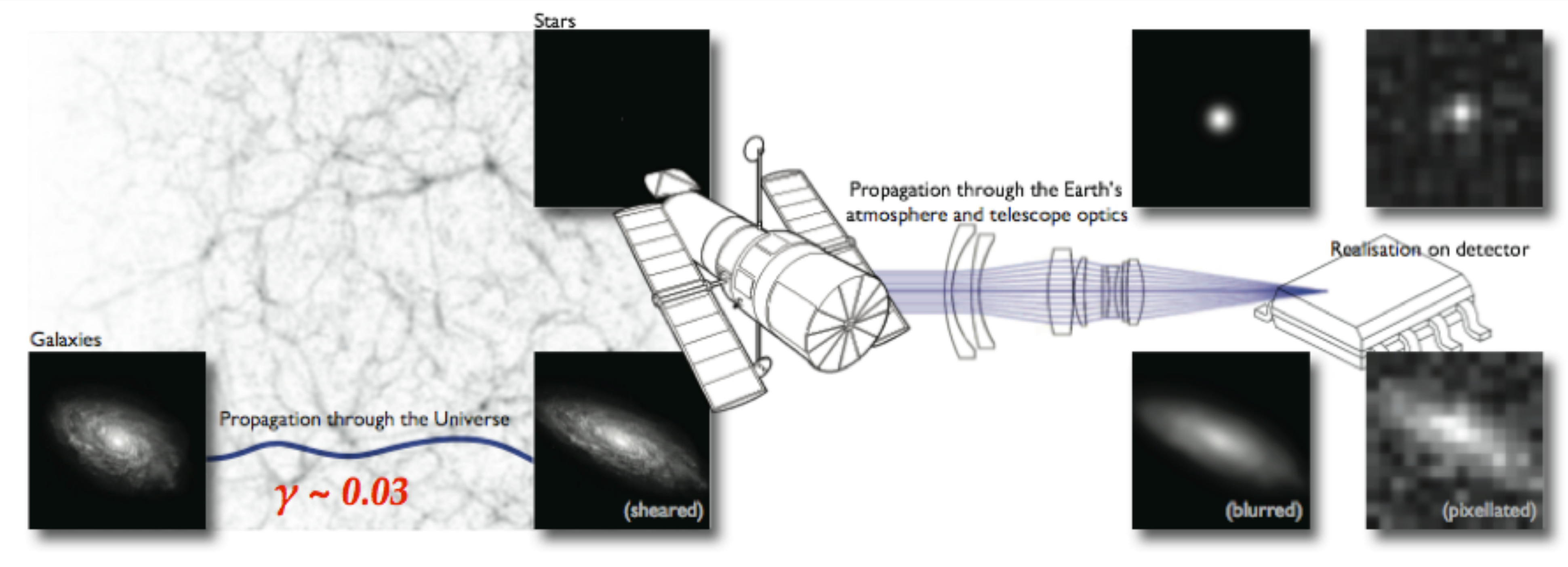
Galaxy shape measurement

Shear biases and calibration

PSF correction & diagnostics

PSF systematics

The shape measurement challenge

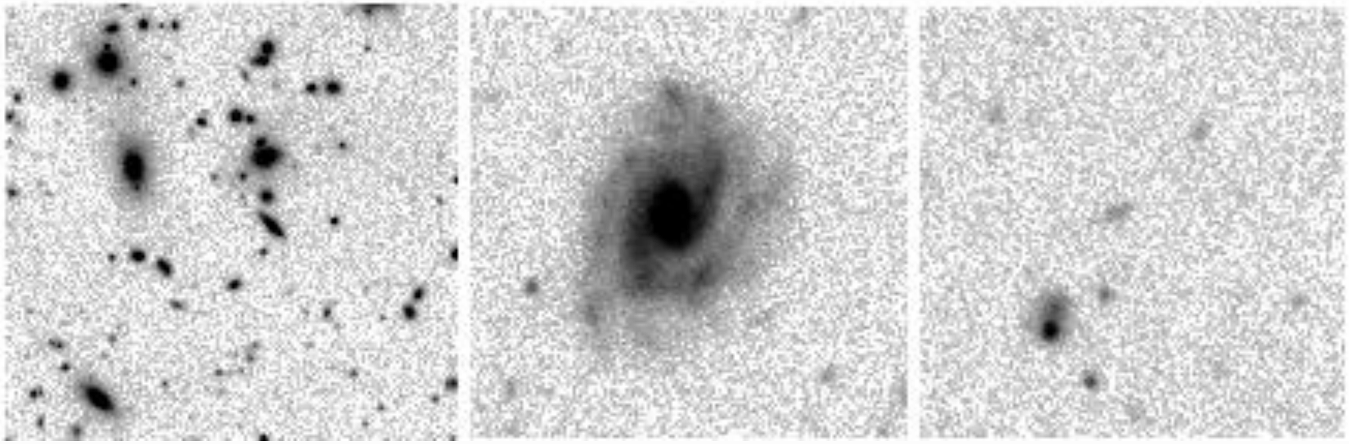


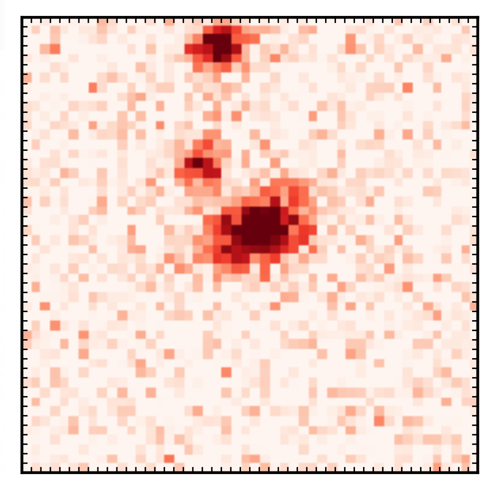
Bridle et al. 2008, great08 handbook

- Cosmological shear $|\gamma| \ll |\varepsilon|$ intrinsic ellipticity
- Galaxy images corrupted by PSF
- Measured shapes are biased

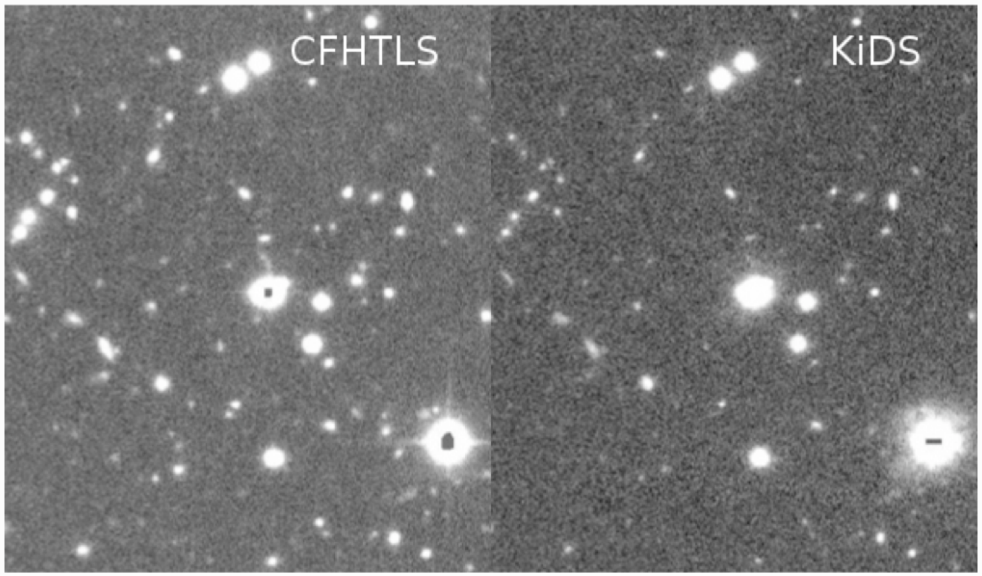
The shape measurement challenge

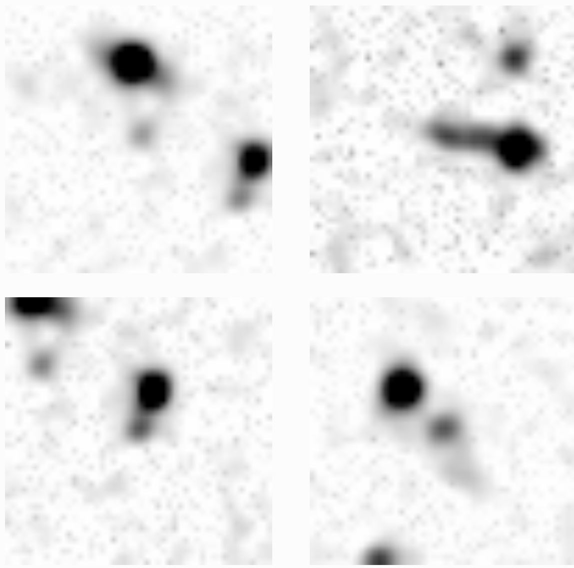
How do we measure "ellipticity" for irregular, faint, noisy objects?





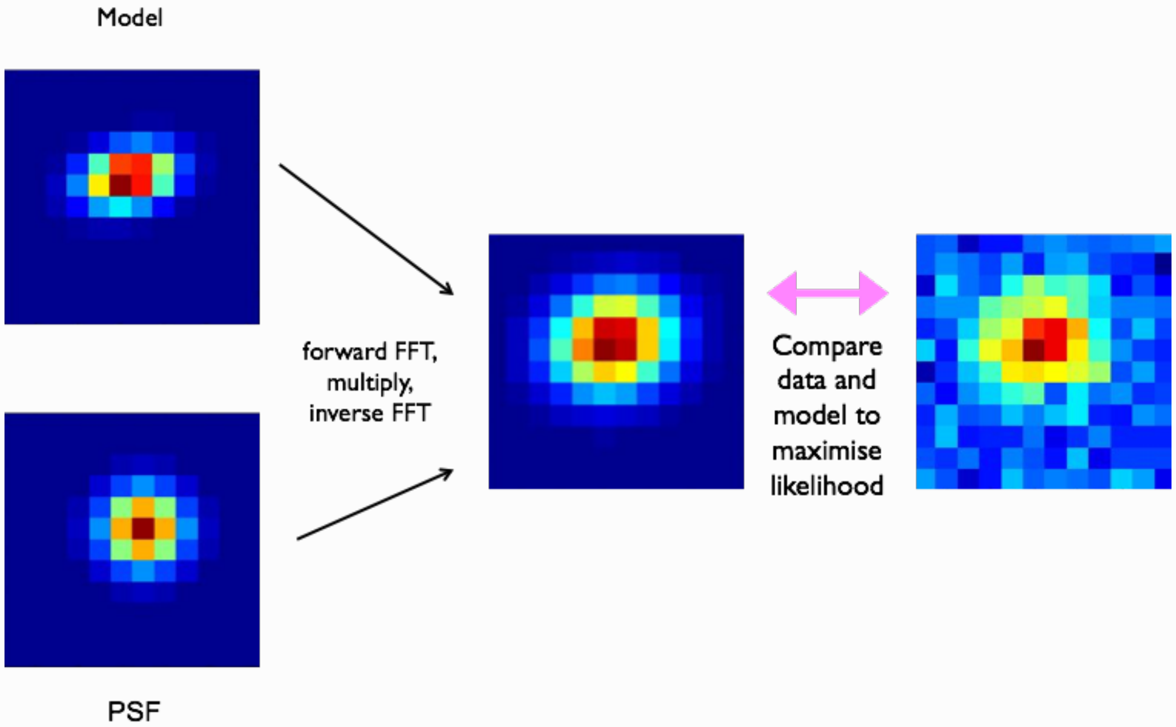
[Y. Mellier/CFHT(?)] — (Jarvis et al. 2016)





[CFHTLenS/KiDS image — CFHTLenS postage stamps]

Model fitting methods



Forward model-fitting (example *lens*fit)

- Convolution of model with PSF instead of devonvolution of image
- Combine multiple exposures avoiding co-adding of (dithered) images.
  - Bayesian: fit each exposure independently, multiply posterior density
  - Freqeuentist: fit joint model to each exposure

Moment-based methods

Moments and ellipticity

Simple case: qualitatively, what are the $0^{\mathrm{th}}, 1^{\mathrm{st}}, 2^{\mathrm{nd}}$ moments of a 1D distribution? Of a 2D distribution?

Quadrupole moment of weighted light distribution $I(\boldsymbol{\theta})$:

$$Q_{ij} = \frac{\int \mathrm{d}^2\theta \, q[I(\boldsymbol{\theta})] \, (\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int \mathrm{d}^2 \, \theta \, q[I(\boldsymbol{\theta})]}, \quad i, j = 1, 2$$

q : weight function

$$\bar{\boldsymbol{\theta}} = \frac{\int \mathrm{d}^2\theta \, q_I[I(\boldsymbol{\theta})] \, \boldsymbol{\theta}}{\int \mathrm{d}^2\theta \, q_I[I(\boldsymbol{\theta})]} : \quad \text{barycenter (first moment!)}$$

Ellipticity

$$\varepsilon = \frac{Q_{11} - Q_{22} + 2\mathrm{i}Q_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$$

Circular object $Q_{11} = Q_{22}, Q_{12} = Q_{21} = 0$

Shear measurement biases I

For ~~basically~~ all shape measurement methods: observed shear $\neq$ true shear. This is called shear bias.

Origins

- **Noise bias**
  In general, ellipticity is non-linear in pixel data (e.g. normalization by flux). Pixel noise $\rightarrow$ biased estimators.
- **Model bias**
  Assumption about galaxy light distribution is in general wrong.
  - Model-fitting method: wrong model
  - Perturbative methods (*KSB*, *DEIMOS*, *HOLICS*): weight function not appropriate
  - Non-perturbative methods (*shapelets*): truncated expansion, bad eigenfunction representation
  - Color gradients
  - Non-elliptical isophotes

Shear measurement biases II

- **Other**
  - Imperfect PSF correction
  - Detector effects (CTI — charge transfer inefficiency)
  - Selection effects (probab. of detection/sucessful ε measurement depends on ε and PSF)

Characterisation

Bias can be multiplicative ($\boldsymbol{m}$) and additive ($\boldsymbol{c}$):

$$\left\langle \varepsilon_i^{\rm obs} \right\rangle = \gamma_i^{\rm obs} = (1 + m_i)\gamma_i^{\rm true} + c_i; \quad i = 1, 2.$$

Biases $\boldsymbol{m}$, $\boldsymbol{c}$ are typically complicated functions of galaxy properties (e.g. size, magnitude, ellipticity), redshift, PSF, ... They can be scale-dependent.

Current methods: $|m| =$ a few to a few 10

Challenges such as STEP1, STEP2, great08, great10, great3 quantified these biases with blind simulationes.

Shear measurement biases III

Calibration

Usually biases are calibrated using simulated or emulated data, or self-calibration using the observed data themselves.

Many surveys produce their own image simulations with properties of galaxy sample and PSF matching to data.

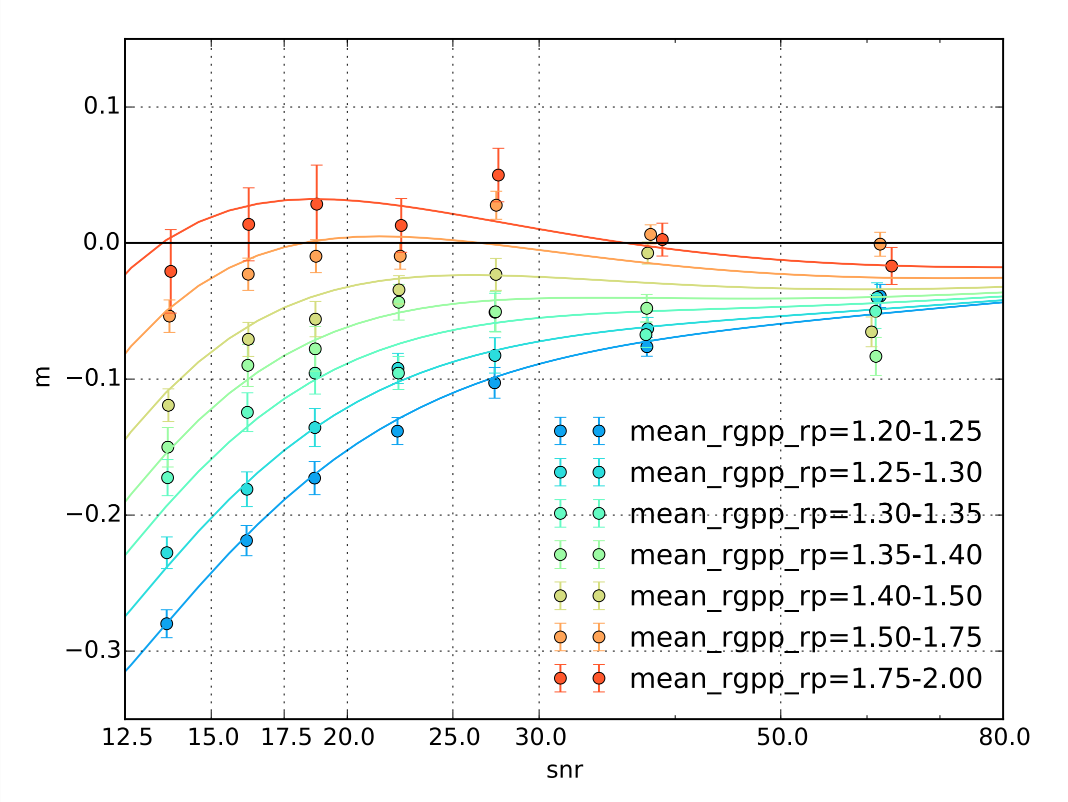
Calibration using the observed data has been developped in the last 5 years (mainly by DES people), this is called *Metacalibration*.

However, image simulations are still required to

- Check and validate the metacalibrated shear measurements
- Quantify other biasa, e.g. detection bias

Shear measurement biases IV

Functional dependence of m on observables must not be too complicated (e.g. not smooth, many variables, large parameter space), or else measurement is *not calibratable*!



(Jarvis et al. 2016)

Requirements

Normalisation $\sigma_8 \propto m$!

Necessary knowledge of residual biases $|\Delta m|, |\Delta c|$ (after calibration):

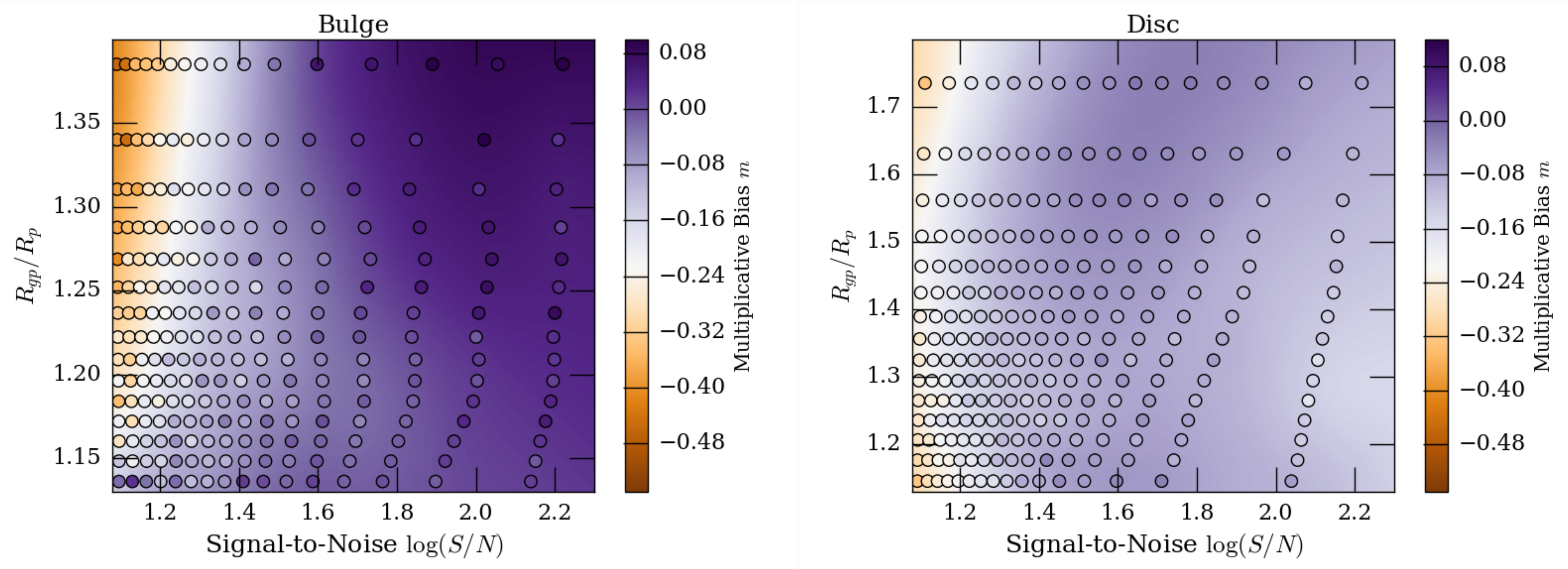
Current surveys 1-5%.

Future large missions (Euclid, LSST, ...) $10^{-4} = 0.1\%$!

Shear measurement biases V

Complex bias dependencies

Need to account for bias as function of more than one galaxy property. E.g. size and SNR. Also need to know bulge and disc fraction of observed population.



(Zuntz et al. 2018)

Metacalibration I

Going back to the definition of multipliative and additive shear bias:

$$\left\langle \varepsilon_i^{\rm obs} \right\rangle = \gamma_i^{\rm obs} = (1 + m_i)\gamma_i^{\rm true} + c_i; \quad i = 1, 2.$$

This ensemble estimator was derived from the equation for a single galaxy:

$$\varepsilon_i^{\rm obs} = \varepsilon_i^{\rm s} + \gamma_i^{\rm obs} = \varepsilon_i^{\rm s} + (1 + m_i)\gamma_i^{\rm true} + c_i; \quad i = 1, 2.$$

Interpreting the l.h.s. as a function of the shear, we can write the multiplicative bias as first derivative of that function:

$$\frac{\partial \varepsilon_i^{\rm obs}}{\partial \gamma_i^{\rm true}} = 1 + m_i.$$

Since both ellipticity and shear are two-component quantities, we can generalise this expression and write it as matrix equation. This introduces the shear response matrix **R**.

$$\frac{\partial \varepsilon_i^{\rm obs}}{\partial \gamma_j} = R_{ij}.$$

Metacalibration II

On the diagonal we find the original scalars $1 + m_i$. On the off-diagonal there are cross-terms of multiplicative bias,

$$\mathbf{R} = \begin{pmatrix} 1 + m_1 & R_{12} \\ R_{21} & 1 + m_2 \end{pmatrix}$$

We have to go back to an ensemble of galaxies, to estimate shear in a sensible way. For that we compute the ensemble average of the shear response, $\langle \mathbf{R} \rangle$ as the average shear bias of the sample, and get

$$\left\langle \boldsymbol{\varepsilon}^{\mathrm{obs}} \right\rangle = \boldsymbol{\gamma}^{\mathrm{obs}} = \langle \mathbf{R} \rangle \, \boldsymbol{\gamma}^{\mathrm{true}} + \boldsymbol{c}.$$

To calibrate the ensemble, we subtract the additive bias $\boldsymbol{c}$ and multiply with the inverse response matrix $\langle \mathbf{R} \rangle^{-1}$.
Therefore, to calibrate, we can do this for each individual galaxy. The calibrated shape of a galaxy is then

$$\boldsymbol{\varepsilon}^{\mathrm{cal}} = \langle \mathbf{R} \rangle^{-1} \left(\boldsymbol{\varepsilon}^{\mathrm{obs}} - \boldsymbol{c} \right),$$

Metacalibration III

and we see, by forming the ensemble average, that this is indeed unbiased:

$$\left\langle \boldsymbol{\varepsilon}^{\mathrm{cal}} \right\rangle = \boldsymbol{\gamma}.$$

Note: Calibrating each galaxy by its own **R** is generally a bad idea, since:

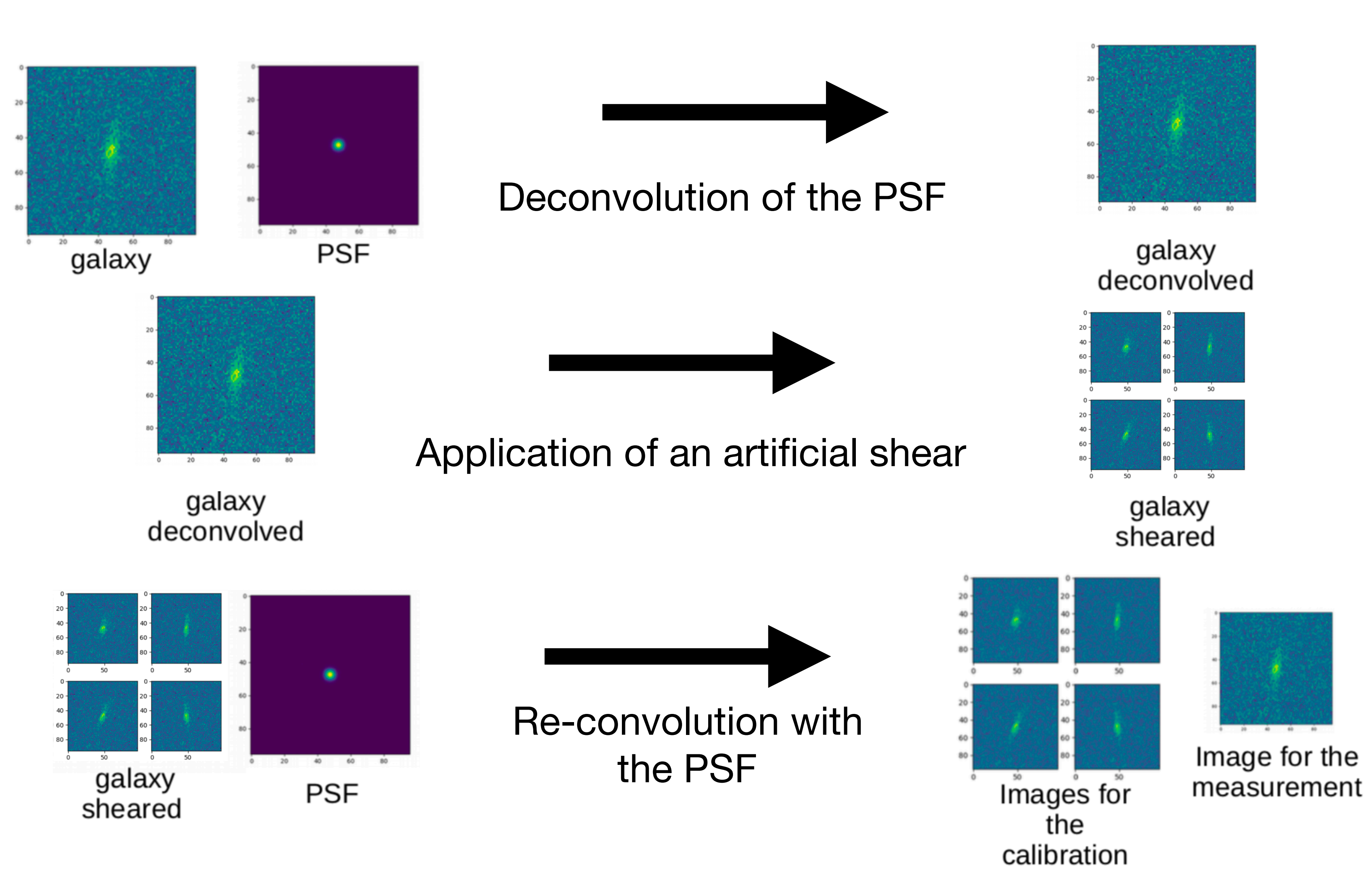
- The estimate of a single **R** is extremely noisy (see TDs!), the matrix might not be invertible.
- Correlations between **R** and γ might be amplified.

In practise, the derivative **R** is computed with finite differences. For that, we add some small shear $\pm\Delta\gamma_{1,2} \approx 0.02$ to each observed galaxy image, and re-measure the ellipticity $\boldsymbol{\varepsilon}^{\pm}$. Then

$$R_{ij} \approx \frac{\varepsilon_i^+ - \varepsilon_i^-}{2\Delta\gamma_j}.$$

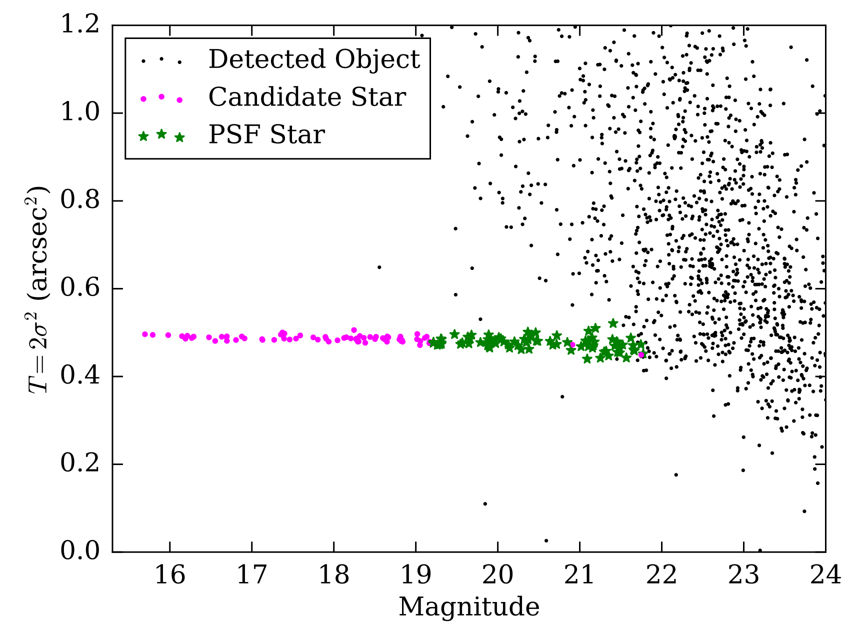
Main difficulty: Need to deconvolve with the PSF first.

Metacalibration IV

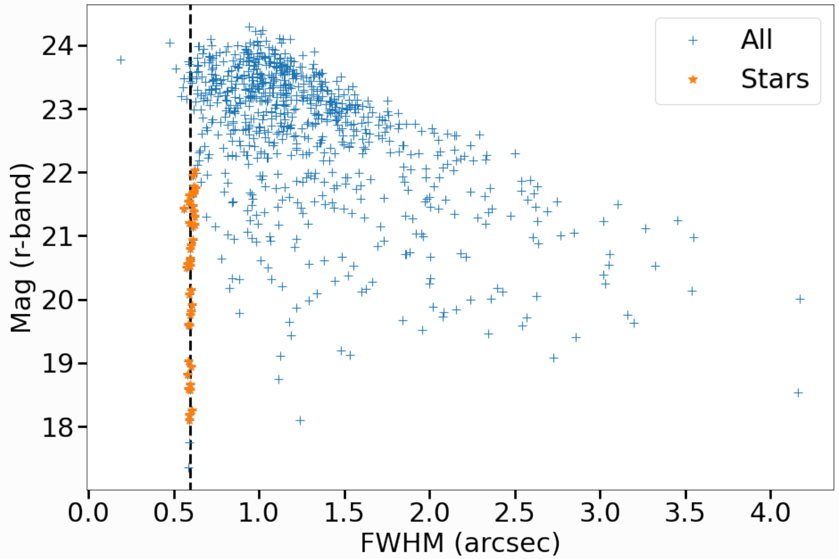


(Sheldon & Huff 2017, Huff & Mandelbaum 2017) — (Slide from A. Guinot.)

PSF correction



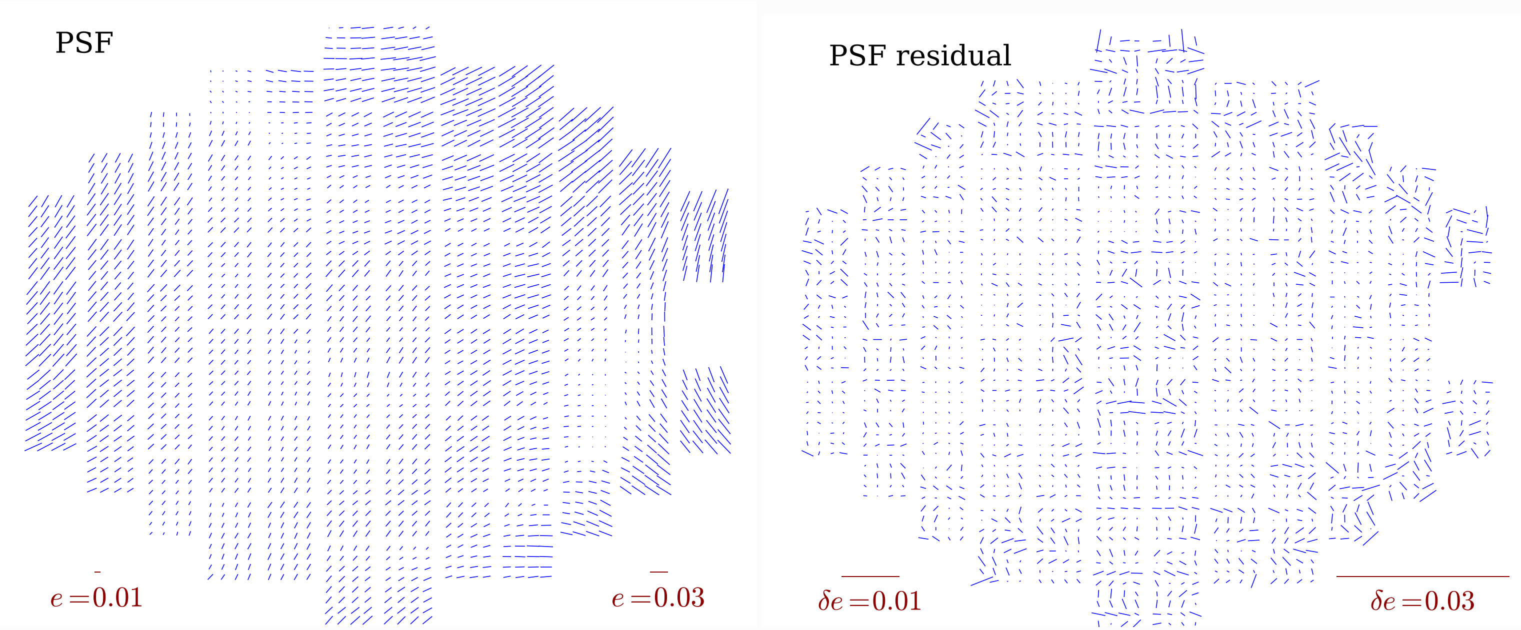
(Zuntz et al. 2018)



(Guinot et al. 2021)

- Select clean sample of stars
- Measure star shapes
- Create PSF model and interpolate (pixel values, ellipticity, PCA coefficients, ...) to galaxy positions. Space-based observations: global PSF model from many exposures possible
- Correct for PSF: galaxy image devonvolution or other (e.g. linearized) correction, or convolve model

PSF correction



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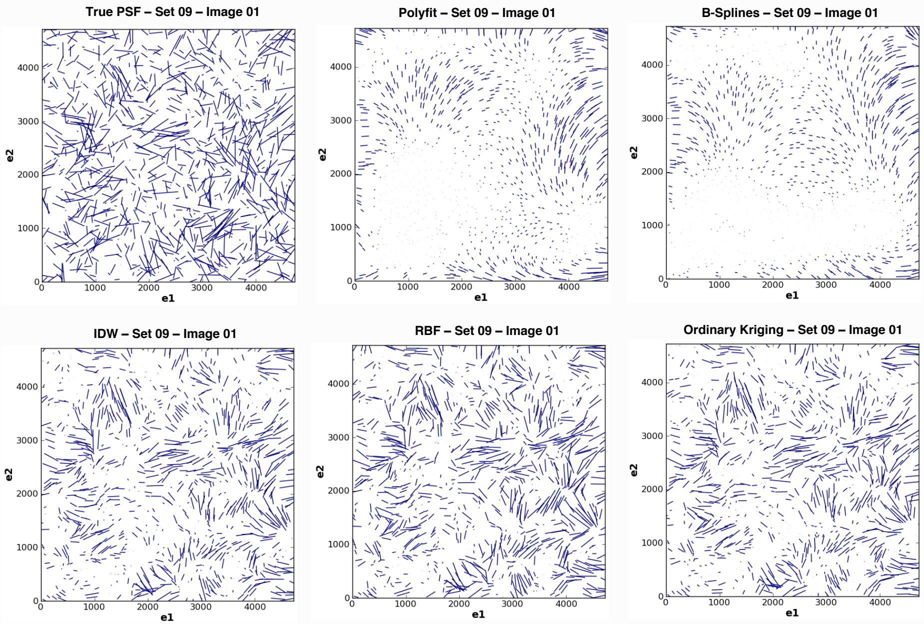

(Gentile et al. 2013)

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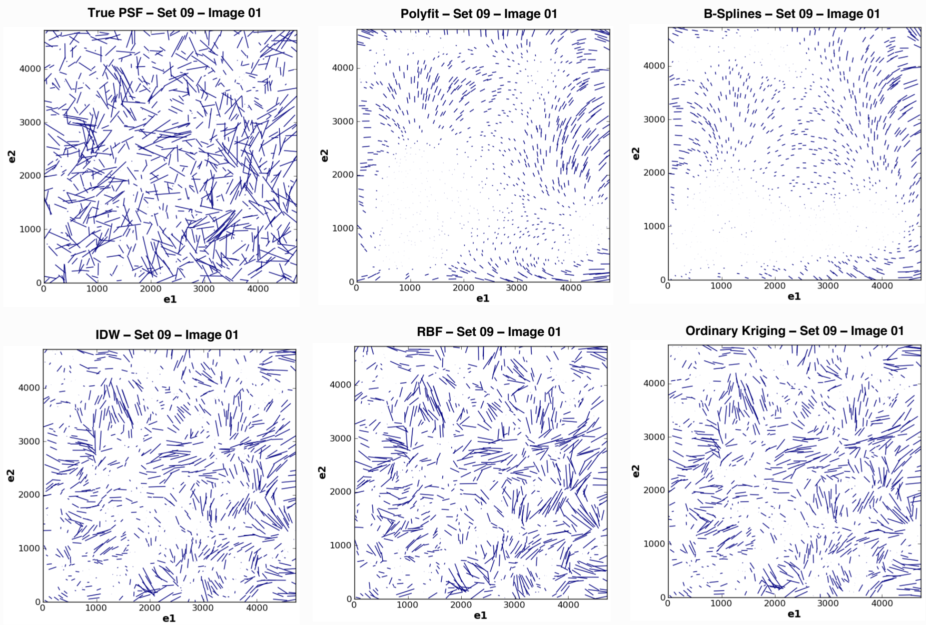

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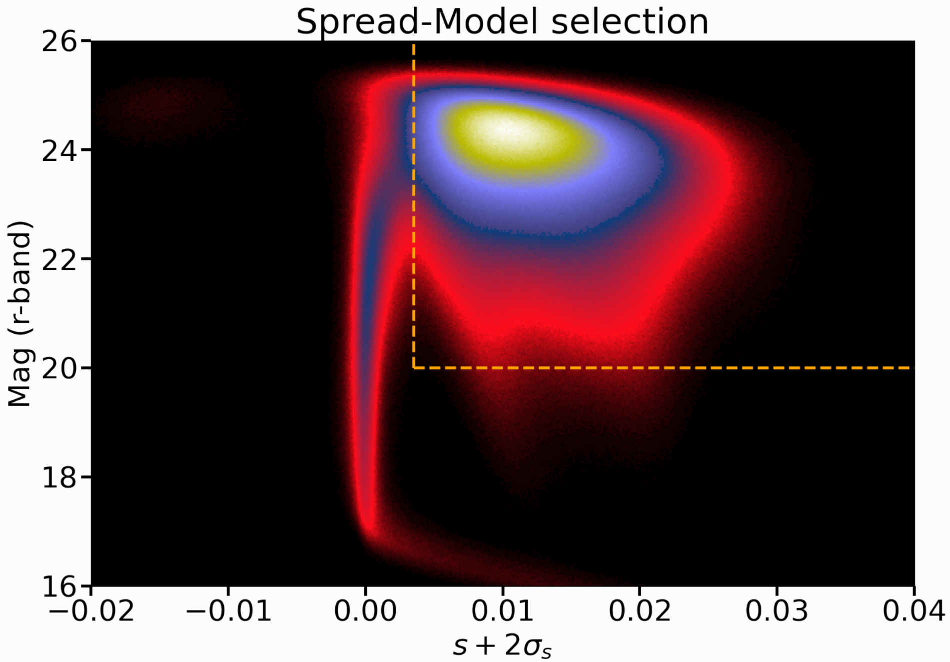
Galaxy selection

Galaxy = extended objects, larger than the PSF. The *spread model* uses the PSF model, to account for spatially varying PSF.
Compare image to extended source, and PSF.

$$s = \frac{\mathbf{G}^{\mathrm{T}}\,\mathbf{W}\,\mathbf{I}}{\mathbf{P}^{\mathrm{T}}\,\mathbf{W}\,\mathbf{I}} - \frac{\mathbf{G}^{\mathrm{T}}\,\mathbf{W}\,\mathbf{P}}{\mathbf{I}^{\mathrm{T}}\,\mathbf{W}\,\mathbf{P}}$$

- **P** PSF
- **G** model of extended source $* \mathbf{P}$
- **I** observed image
- **W** weight



Quantifying PSF systematics: leakage I

PSF leakage

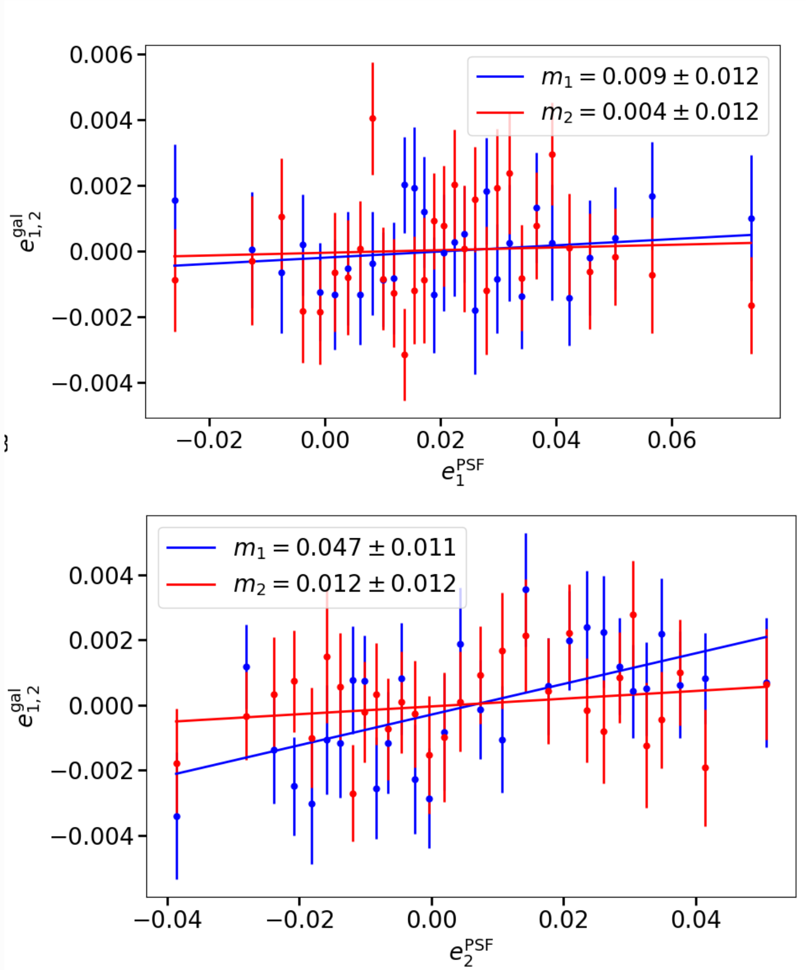
Define PSF leakage via additional term with amplitude α,

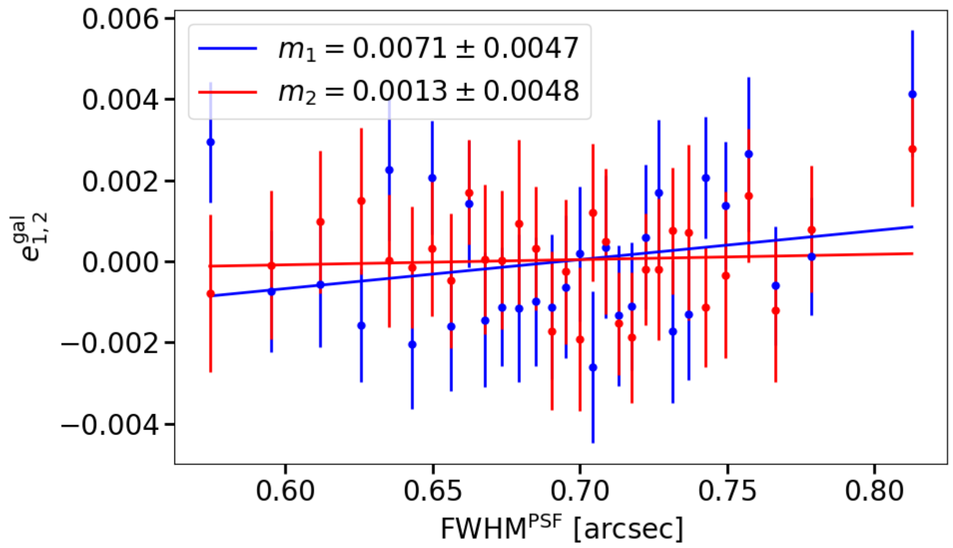
$$\gamma_i^{\rm obs} = (1 + m_i)\gamma_i^{\rm true} + c_i + \alpha\, \varepsilon_i^{\rm PSF}; \quad i = 1, 2.$$

There are two methods to determine α.

1. Via linear regression. Fit $\boldsymbol{\varepsilon}^{\rm obs}$ (remember: $\langle \varepsilon^{\rm obs} \rangle = \gamma$) as function of $\boldsymbol{\varepsilon}^{\rm PSF}$. E.g. in bins of PSF ellipticity.
   We can also look at galaxy ellipticity as function of PSF size, as cross check.

Quantifying PSF systematics: leakage II





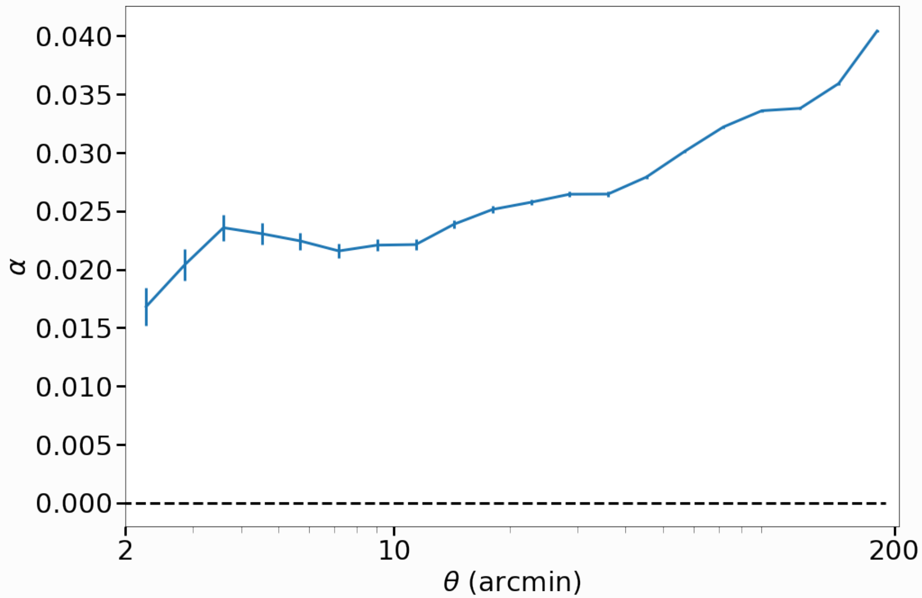
CFIS W3 field.

Quantifying PSF systematics: leakage III

2. Via correlation functions.

$$\alpha(\theta) = \frac{\xi_+^{\mathrm{gp}}(\theta) - \langle e_{\mathrm{gal}} \rangle^* \langle e_{\mathrm{PSF}} \rangle}{\xi_+^{\mathrm{pp}}(\theta) - |\langle e_{\mathrm{PSF}} \rangle|^2},$$

This results in a scale-dependent estimate.



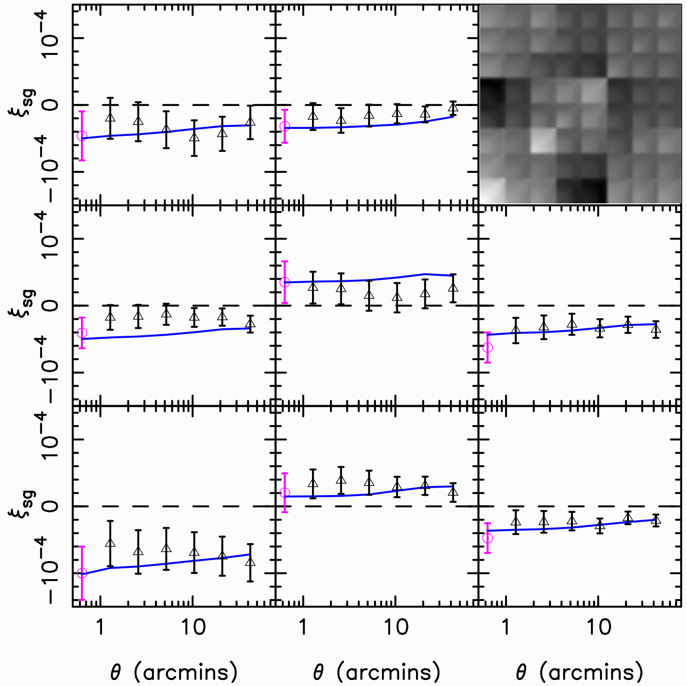
CFIS, (Guinot et al. 2021).

Quantifying PSF systematics: Cross-correlation function. I

Null test: $\xi_{\rm sys}$ correlation between star and galaxy shapes expected to vanish, unless PSF correction (using stars to correct galaxy shapes) is not perfect.

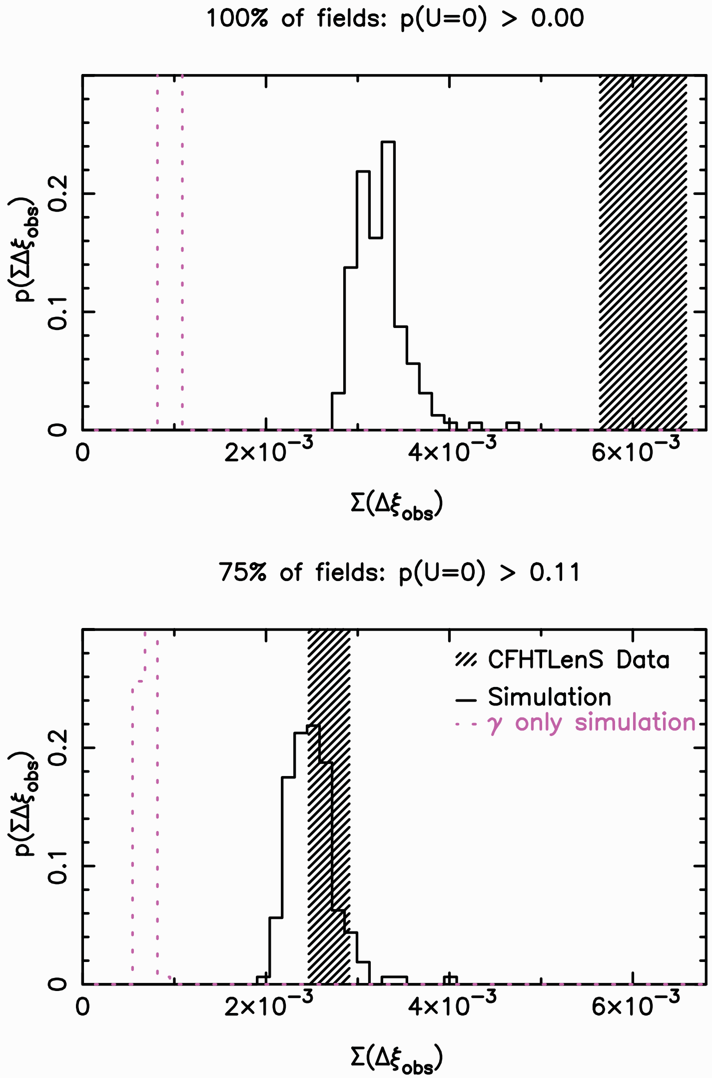
$$\xi_{\rm sys} = \langle \varepsilon^* \varepsilon \rangle$$

This measures residual PSF pattern leakage onto galaxy field.
Caveat: LSS can show chance alignments with PSF pattern. Sample or *cosmic variance* has to be accunted for → N-body simulations!



(Heymans et al. 2012)

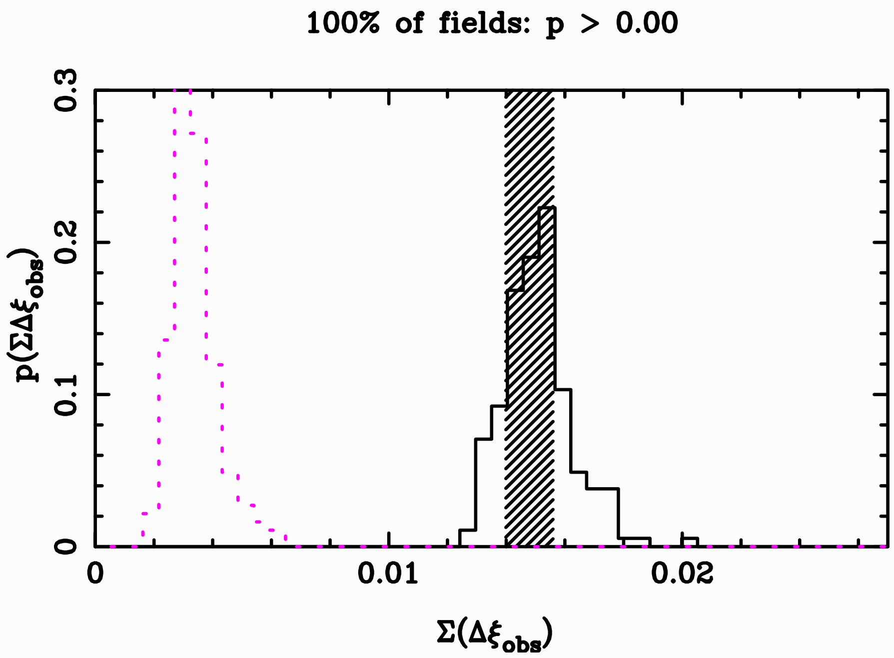
Quantifying PSF systematics: Cross-correlation function. II



Histogram of probability p that $\Sigma\xi_{\rm obs} \sim \Sigma|\xi_{\rm sys}|$ is not zero (sum over all pointings), from simulations.

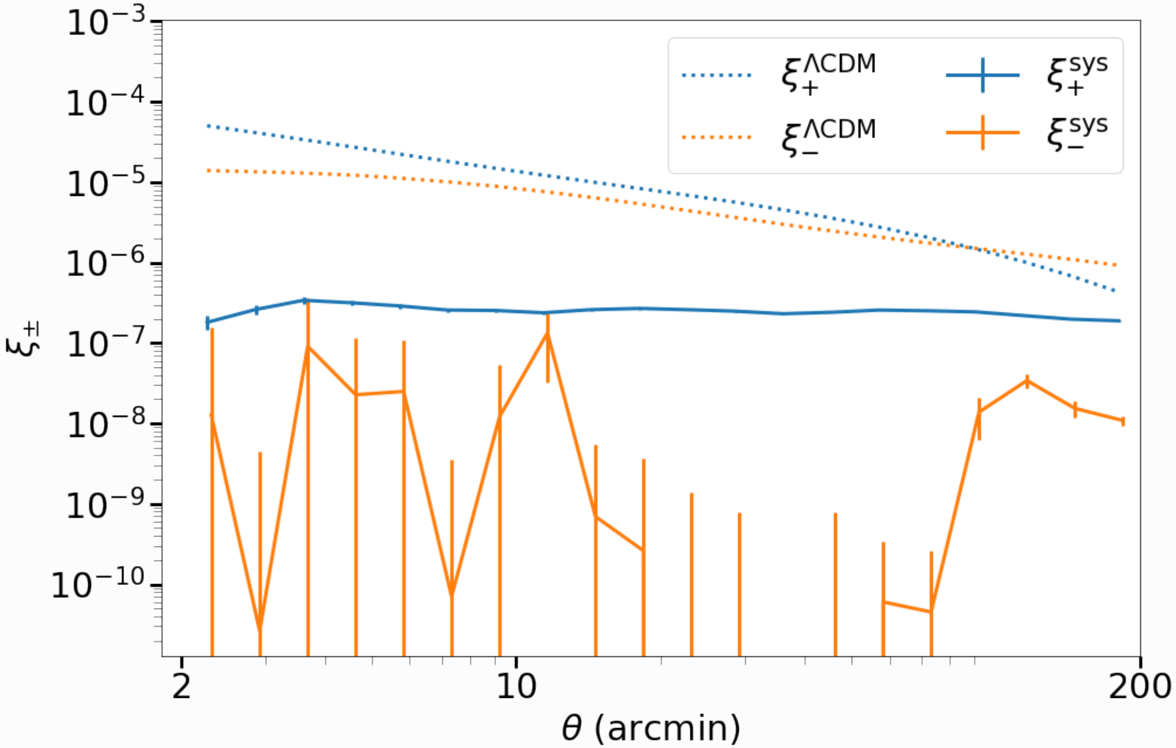
Shaded region = data.

Magenta: simulations without LSS.



[Hildebrandt et al. 2016, KiDS-450]

Quantifying PSF systematics: Cross-correlation function. III



CFIS, (Guinot et al. 2021).