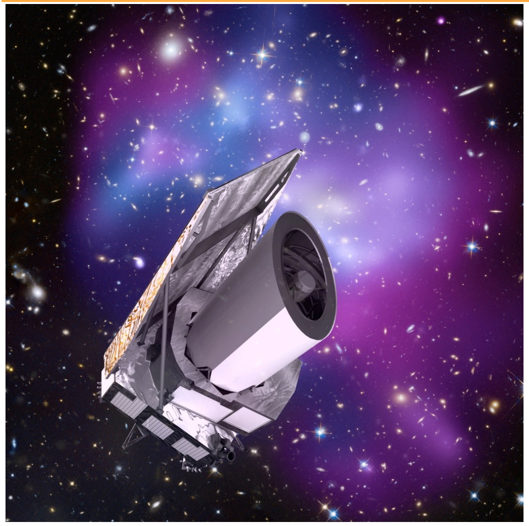


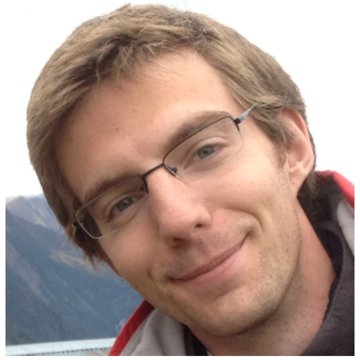
Weak Lensing Mass Mapping

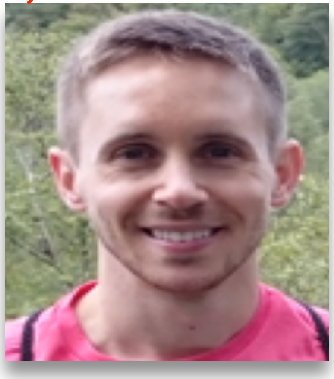
Jean-Luc Starck

http://jstarck.cosmostat.org

Collaborators:

Francois Lanusse, Adrienne Leonard, Austin Peel



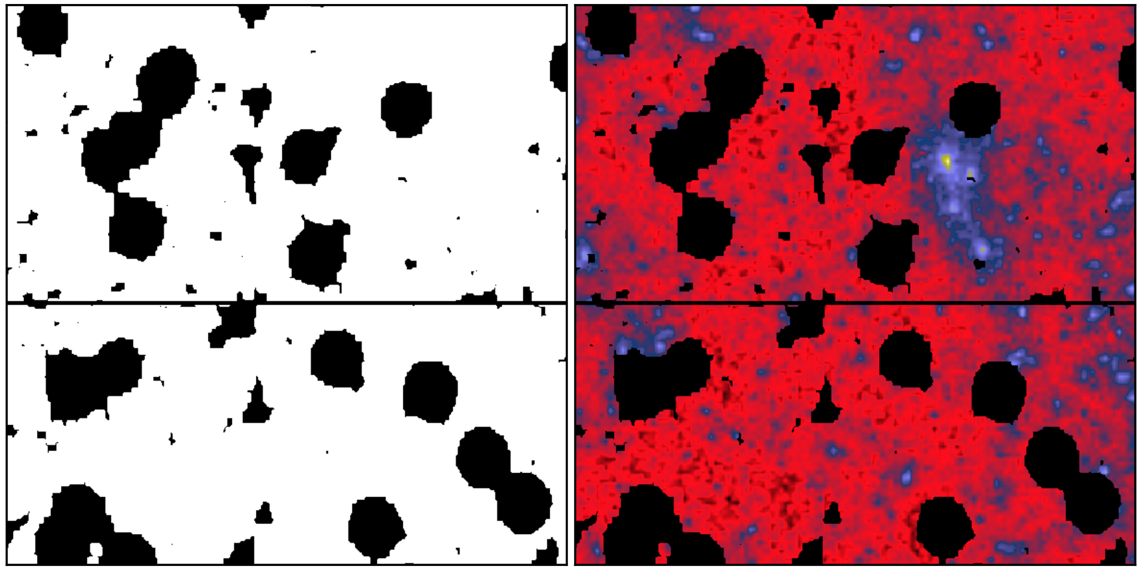


2D Mass Mapping Problems

$$\boxed{\hat{\kappa} = P_1 \hat{\gamma}_1 + P_2 \hat{\gamma}_2}$$

$$P_1(k) = \frac{k_1^2 - k_2^2}{k^2}$$
$$P_2(k) = \frac{2k_1 k_2}{k^2}$$

- **Irregular sampling**
- **Missing data** (mask and limited number densities):



- **Shape noise:**
- **Reduced shear:**

$$g = \frac{\gamma}{1-\kappa}$$

the observed reduced shear is invariant under the transformation: $\kappa^{'} = \lambda\kappa(1-\lambda)$

=> Mass-sheet degeneracy problem

State of the Art

Linear Methods:

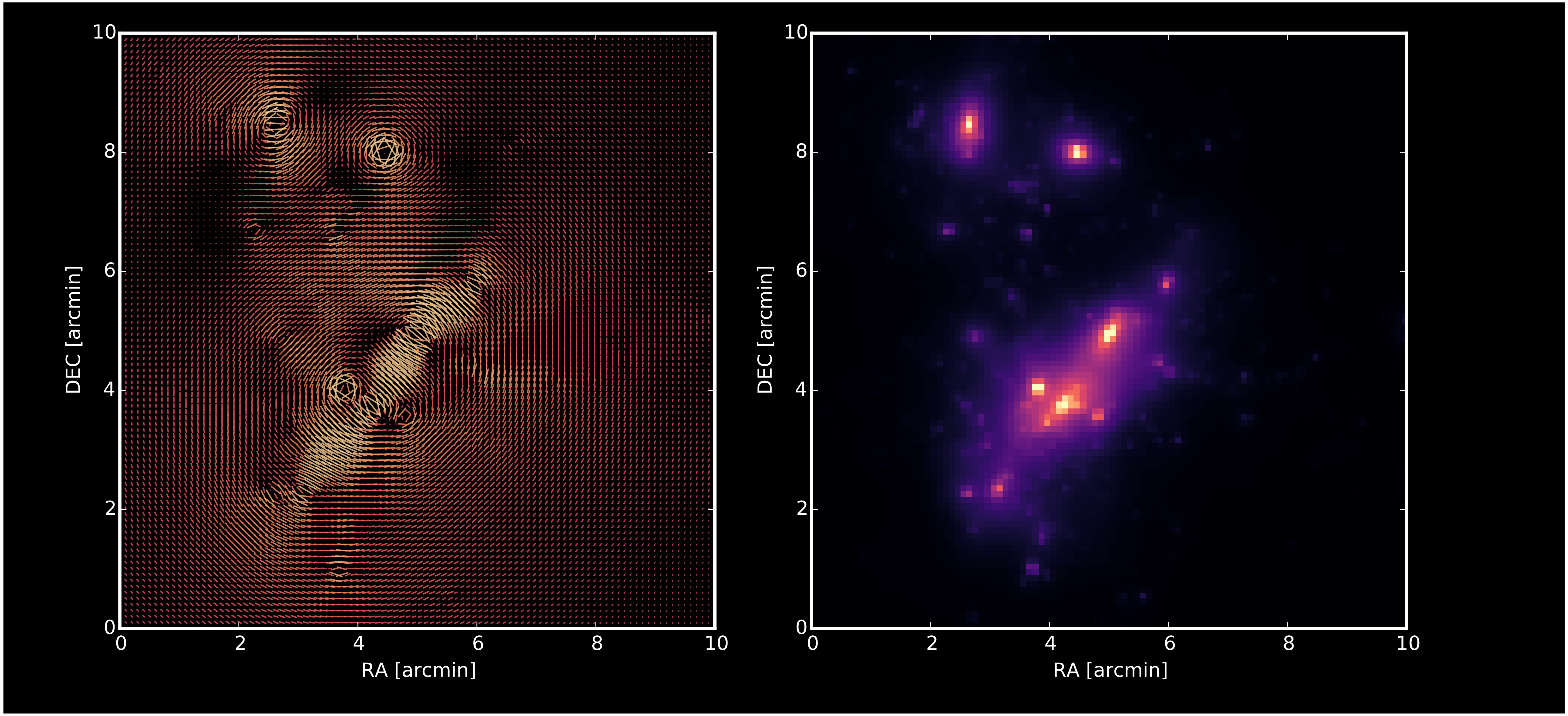
- Kaiser-Squires (1993) + Gaussian smoothing : HSC (2018), DES (2017)
- 3D SVD Inversion (Simon et al, 2009) -> HSC (2018)

Non Linear methods:

- **For clusters:**
  - Model fitting algorithms (Bartelmann et al, 1996; Bradac et al, 2005;   Jullo and Kneib, 2009).
  - Aperture Mass (Seitz and Schneider, 1996; 2002).
- **For larger fields:**
  - Maximum Likehood (Bartelmann et al, 1996).
  - MemLens (Bridle et al, 1998; Marshal and Hobson, 2002).
  - FastLens + MR-Lens (Starck et al 2006; Pires et al, 2009).
  - Bayesian approches (Heavens et al, **2016**, Alsing et al , **2017**, Schneider et al, **2017**).
  - **Glimpse2D** (Lanusse et al, **2016**).
- **3D Mass Mapping:**
  - Bayesian approches (Bohm et al, **2017**).
  - **Glimpse3D** (Leonard et al, 2012, Leonard et al, **2014**).

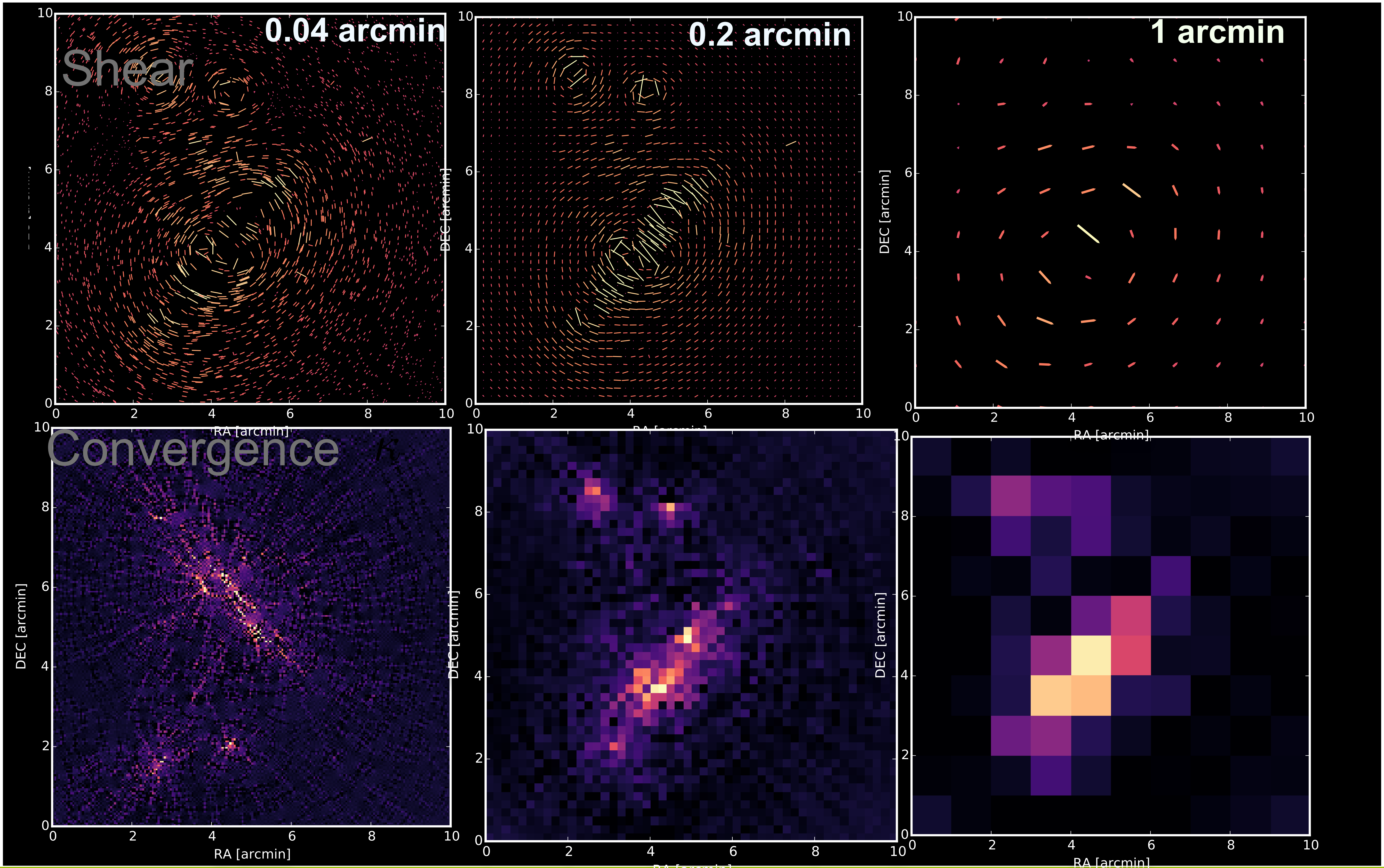
Regular shear sampling

Shear γ Convergence κ



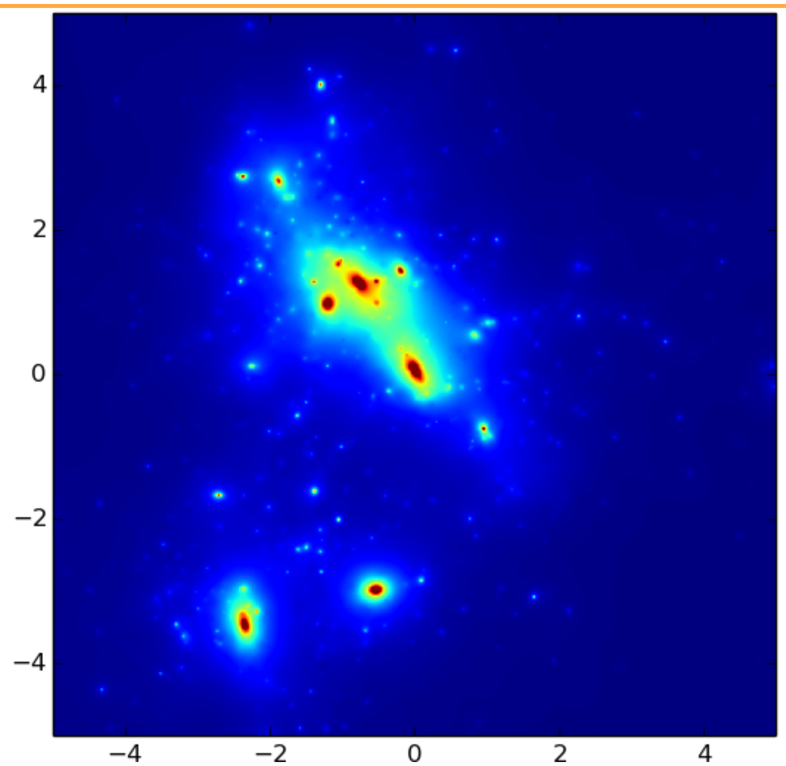
$$\kappa = F^* P F \gamma$$

Irregular shear sampling

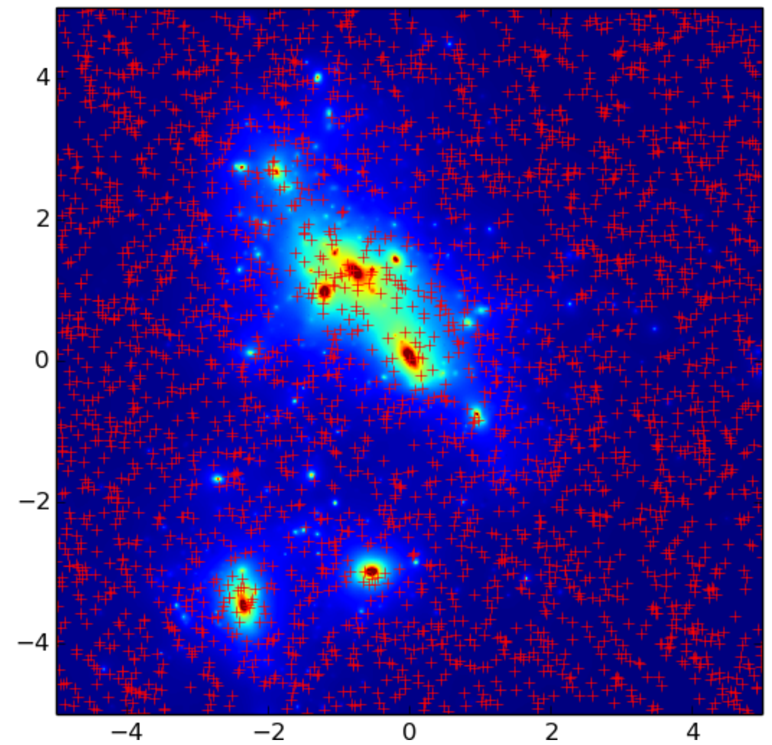


$$\kappa = F^* P F M \gamma$$

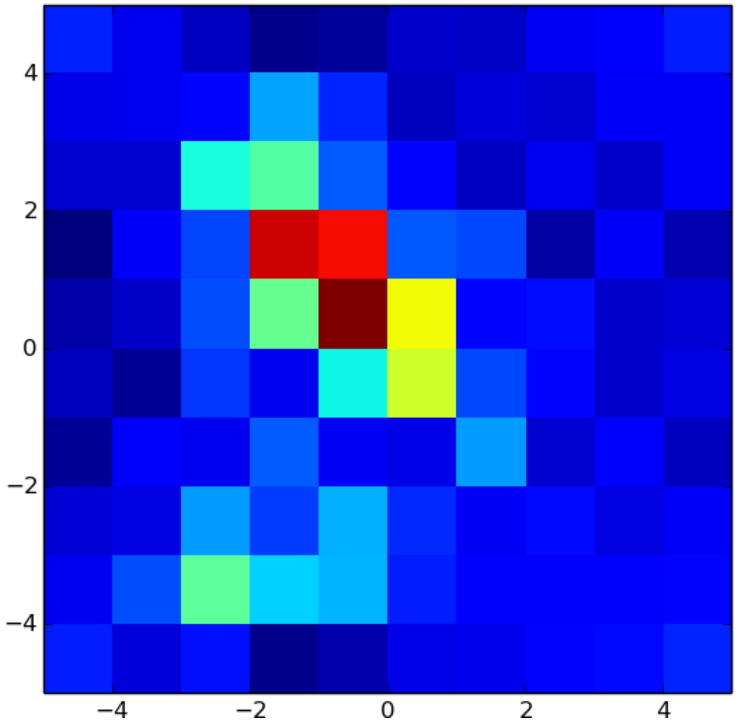
Handling Missing Data (no noise): Binning+Smoothing



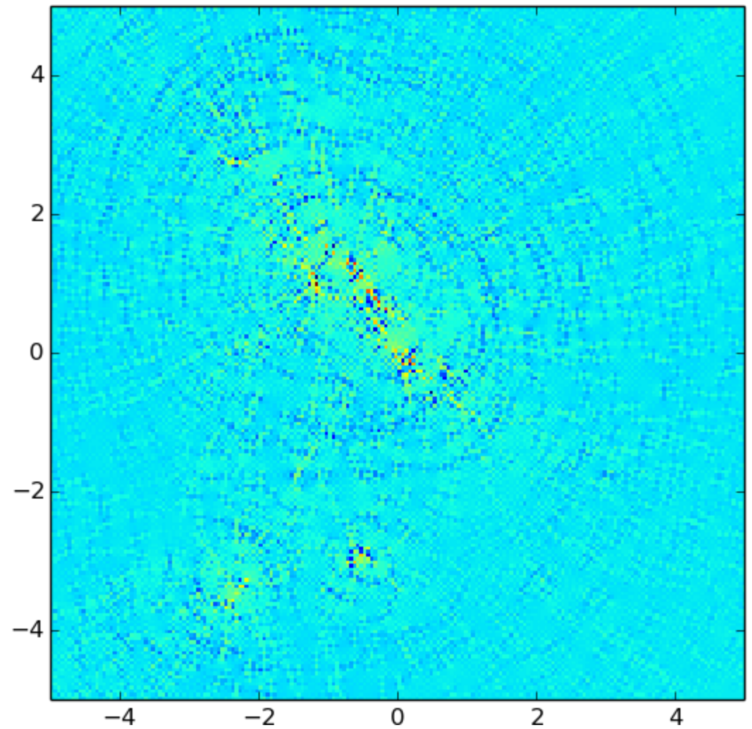
Input



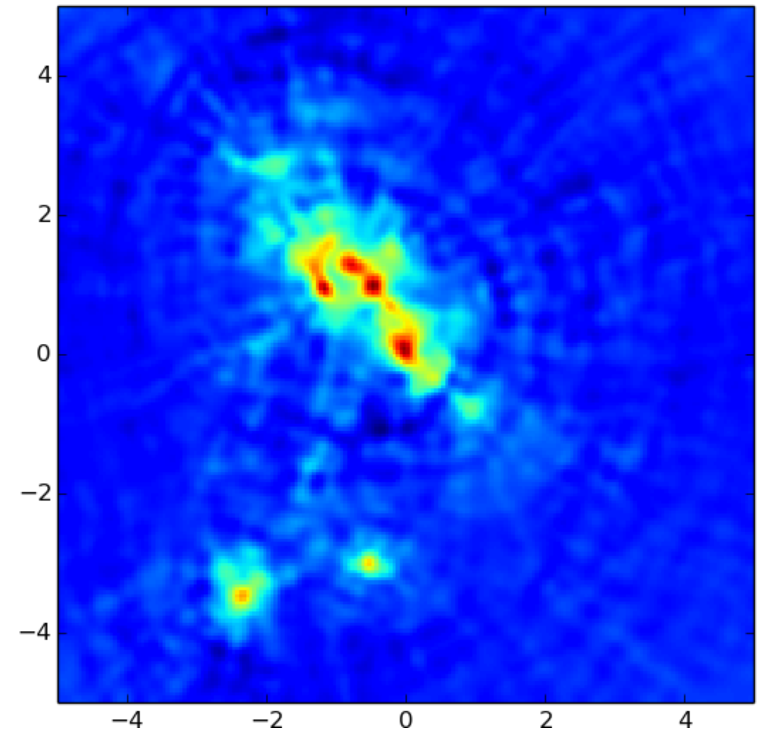
Galaxy catalogue with 30 gal/arcmin^2



Kaiser-Squires with 1' bins



Kaiser-Squires with 0.05' bins



KS with 0.05' bins + 0.1’ smoothing

Mass mapping as an inverse problem

Binned data: $\gamma = F^* P F \kappa$

Unbinned data: $\gamma = T^* P F \kappa$

T = Non Equispaced Discrete Fourier Transform (NDFT)

$$\boxed{\min_{\kappa} \frac{1}{2} \parallel \gamma - \mathbf{P}\kappa \parallel_2^2}$$

with $\mathbf{P} = T^* P F$

But there is no unique and stable solution, it is an ill posed inverse problem.

Inverse Problems & Regularization

$$Y = HX + N$$

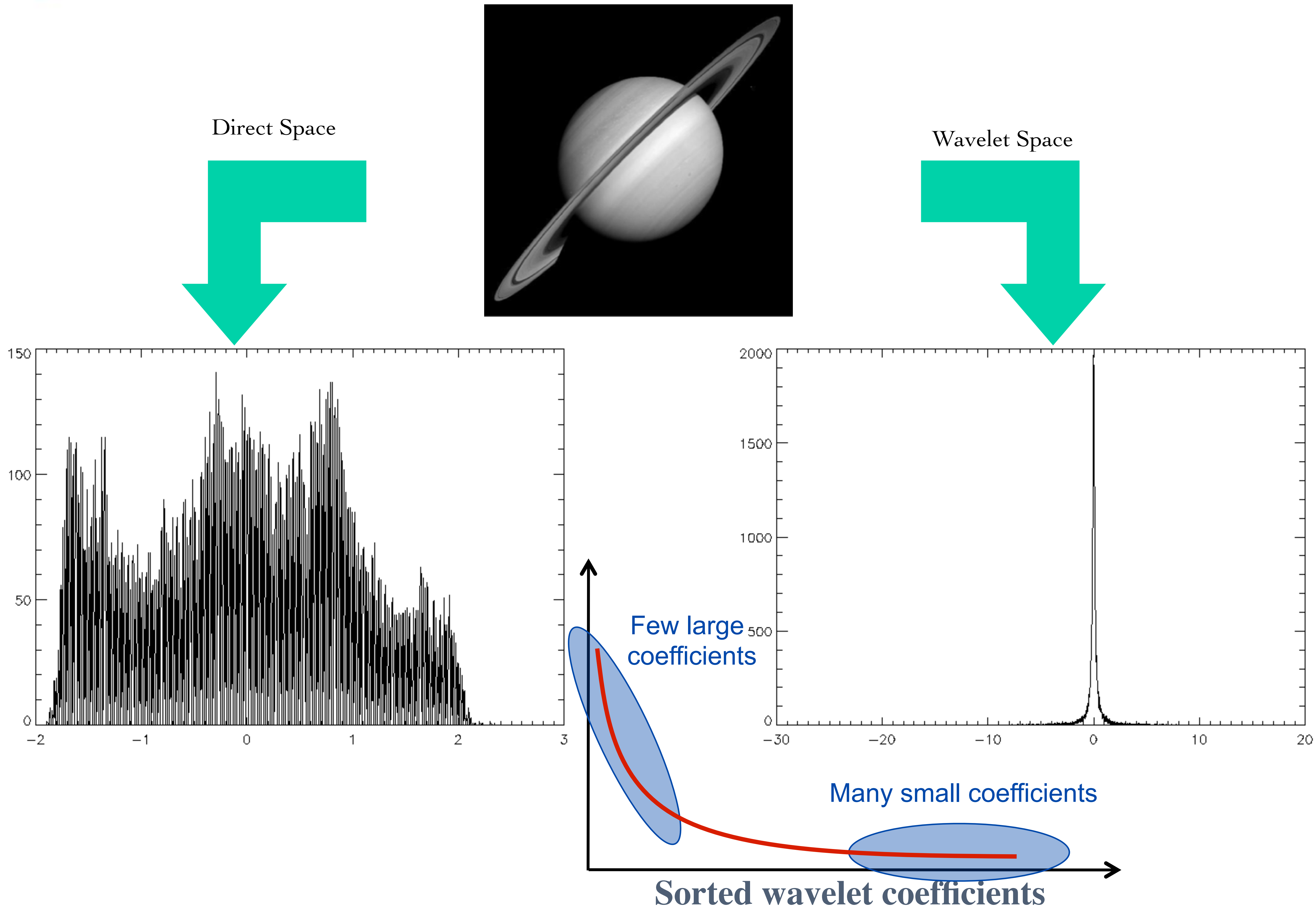
$$\min_X ||Y - HX||^2 \quad + \quad \mathcal{C}(X)$$

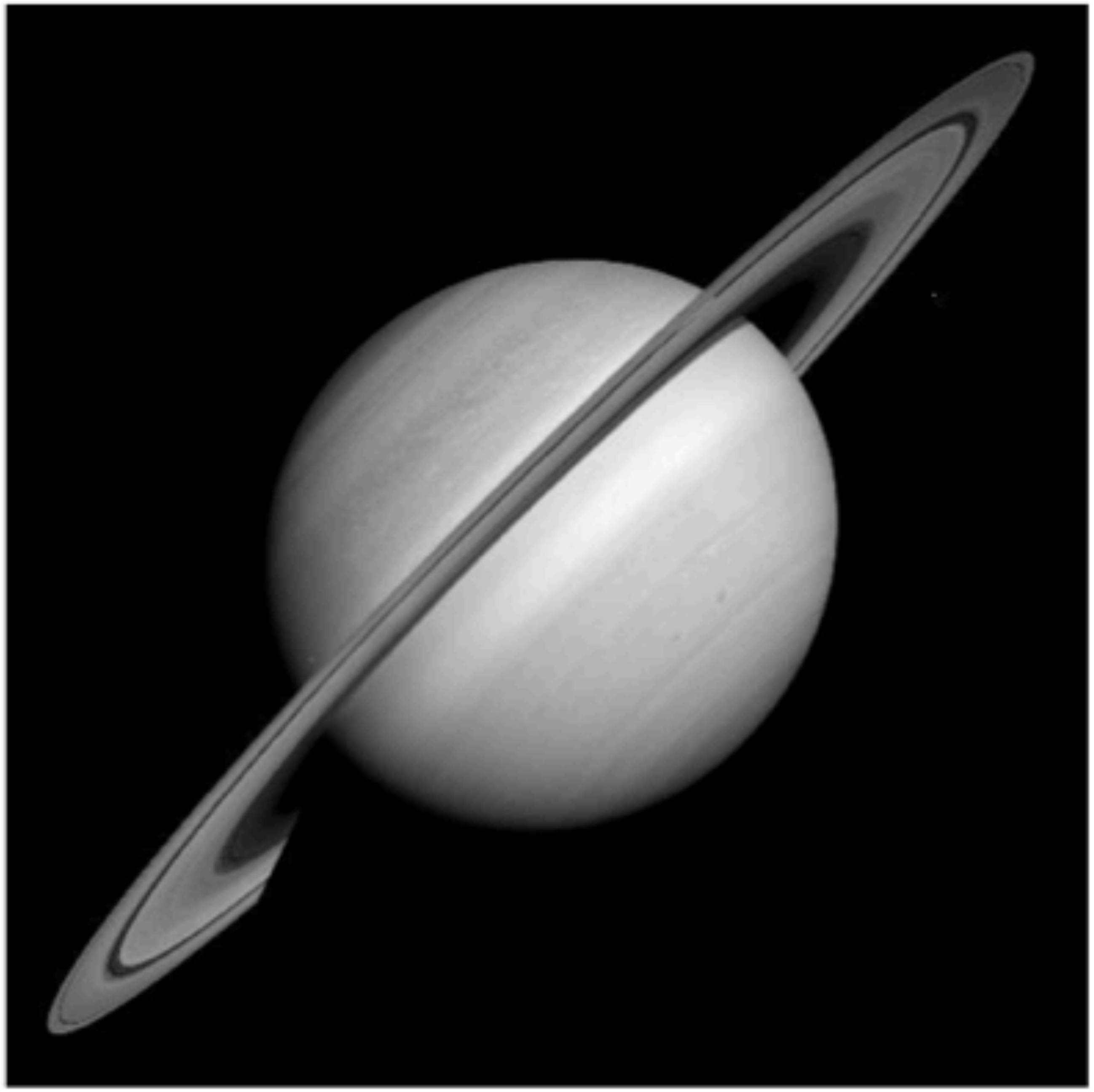
Physical Knowledge on X (ex: Gaussian Random Field, etc).
==> Gaussian smoothing, Wiener reconstruction, etc
==> Log normal distribution prior

X properties are understood through a representative data set.
==> Machine Learning

Knowledge on the histogram of X in pixel space or in another one.
==> Positivity constraint, sparsity constraint, etc.

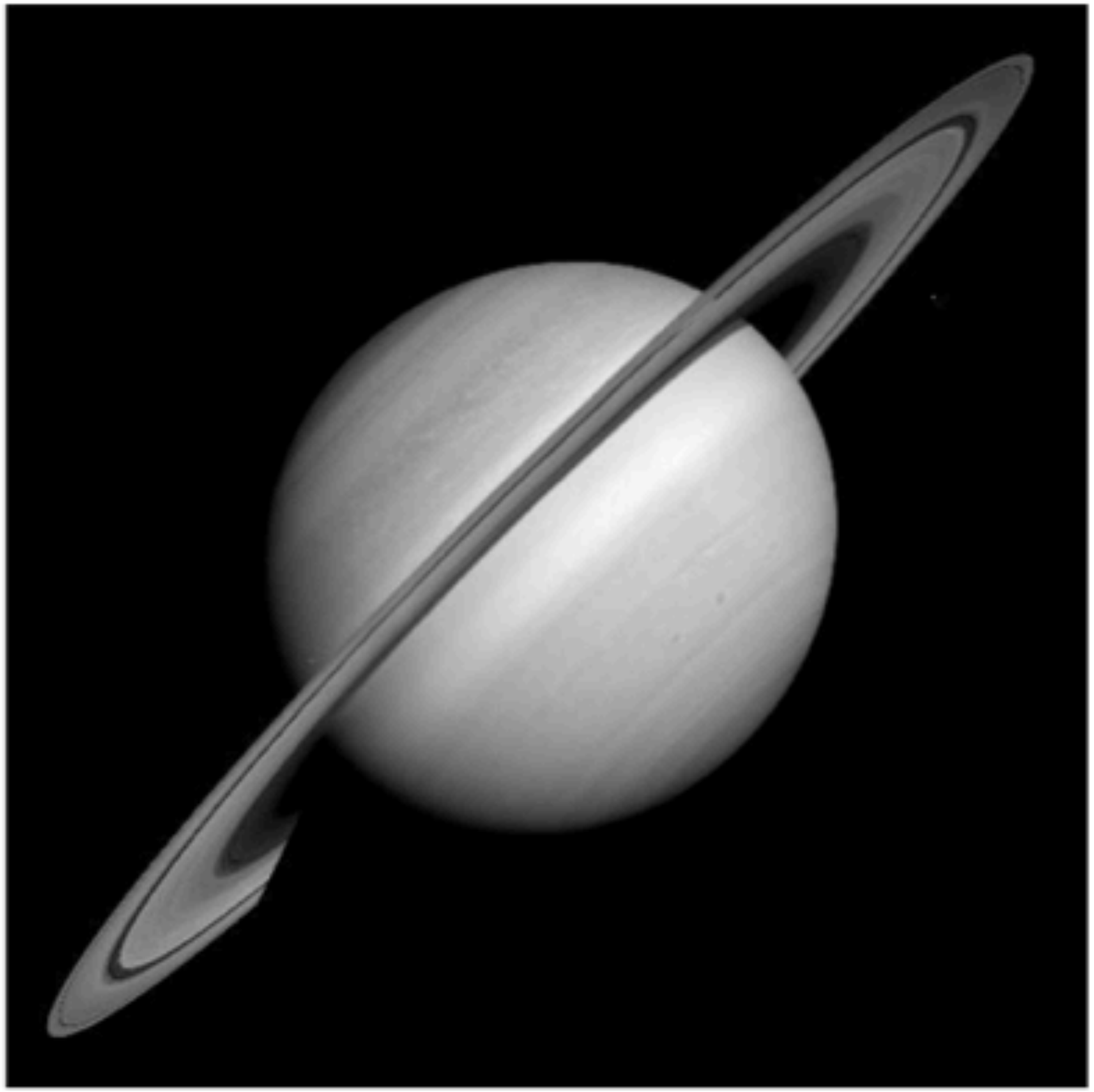
Sparse Representation





1% largest coefficients in real space
(the others are set to 0)

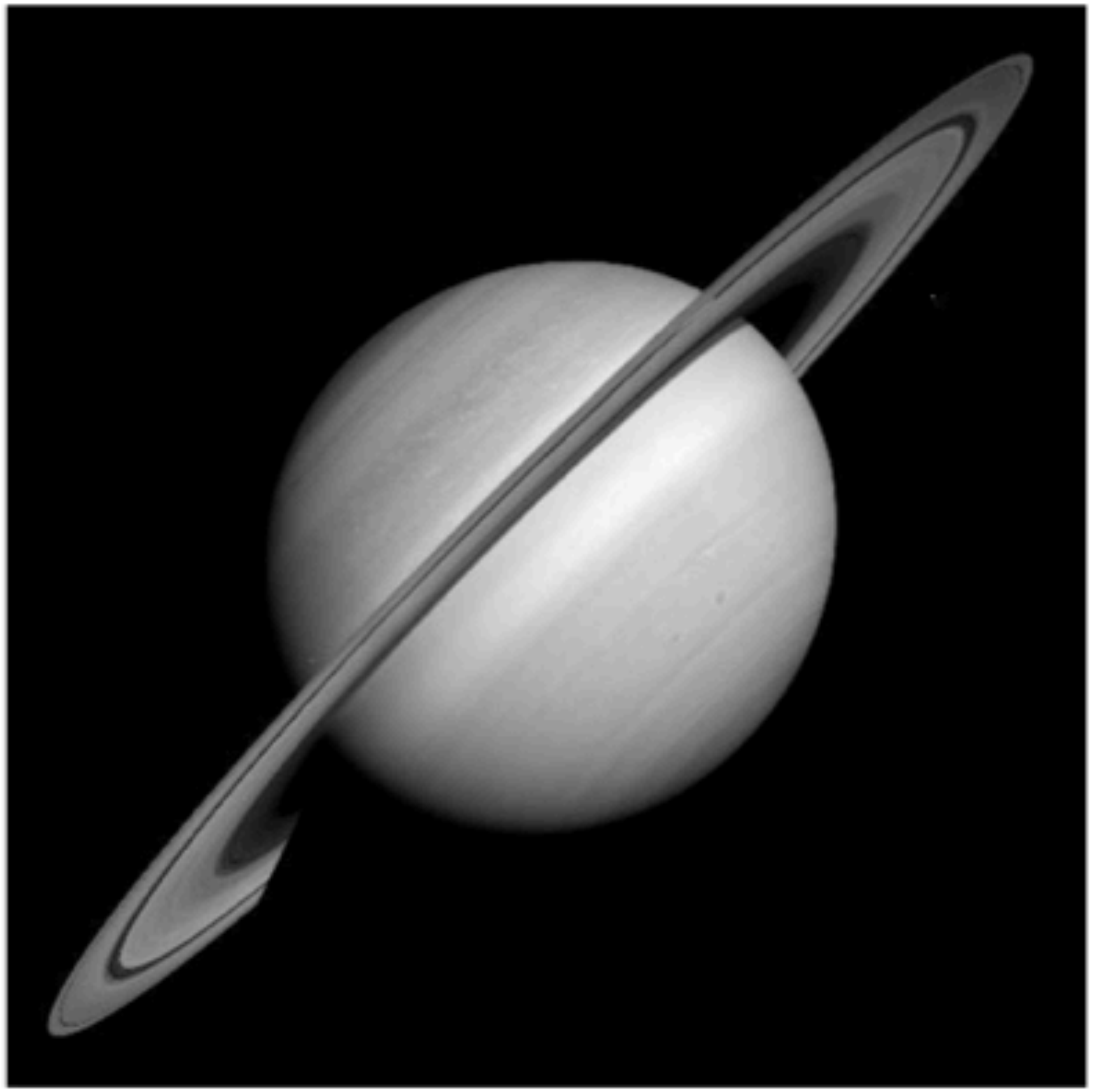
The top 1% of the coefficients concentrate only 8.66% of the energy. Not sparse…

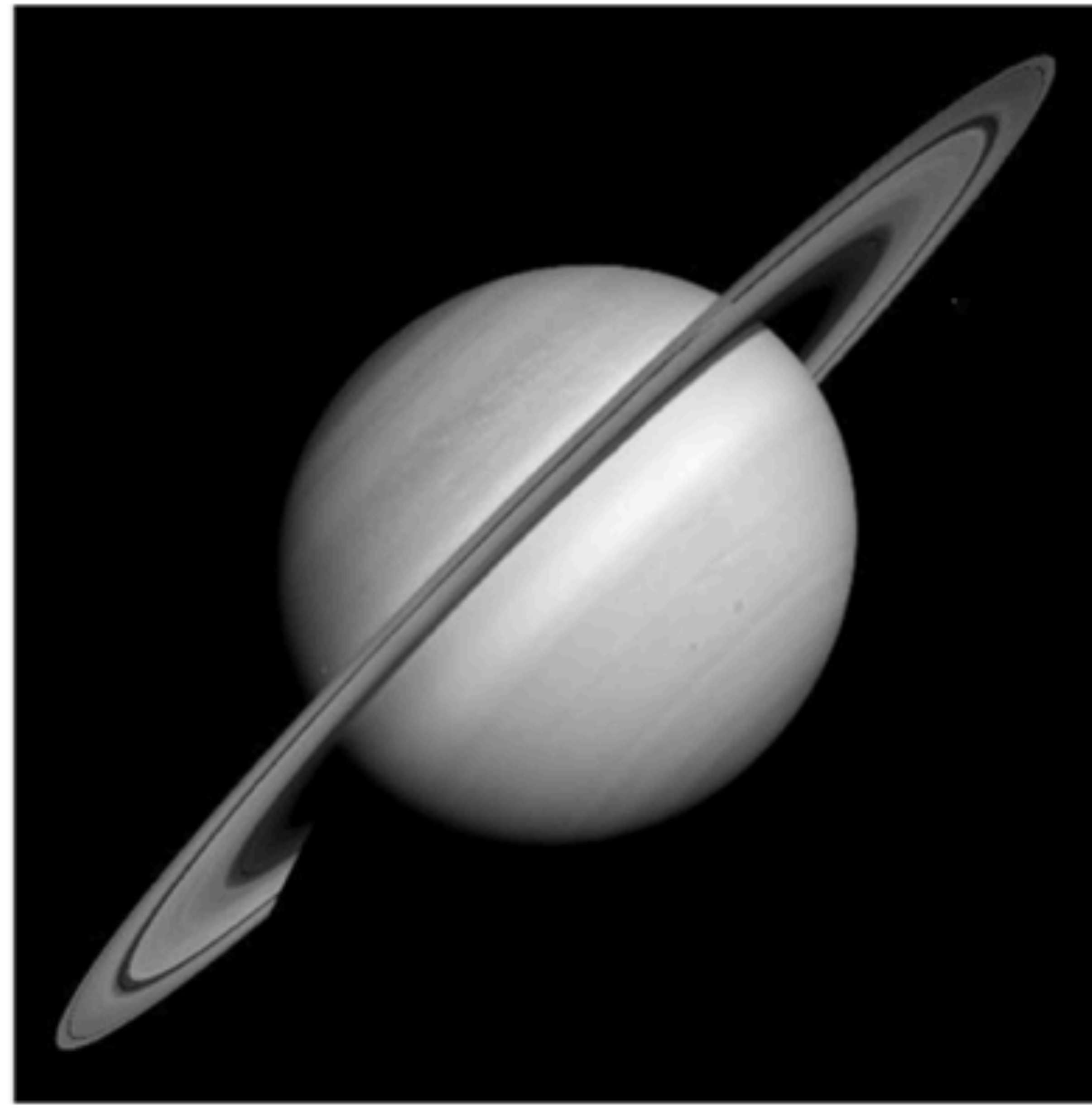


1% largest coefficients in wavelet space
(the others are set to 0)

The wavelet coefficients encode edges and large scale information.

Wavelet transform



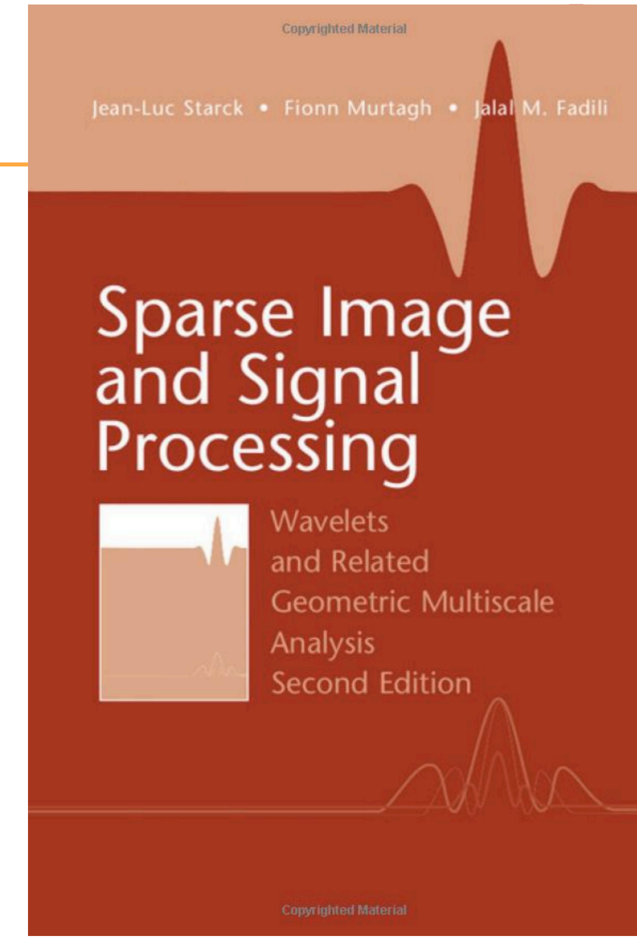


1% of the wavelet coefficients concentrate 99.96% of the energy: This can be used as a *prior.*

Reconstruction, after throwing away 99% of the wavelet coefficients

80%

Inverse Problems & Sparse Recovery



$$Y = HX + N$$

$$\min_X ||Y - HX||^2 + \mathcal{C}(\mathcal{X})$$

Sparse model: $X = \Phi\alpha$

$$\boxed{\min_\alpha ||Y - H\Phi\alpha||^2 + \lambda||\alpha||_p^p}$$

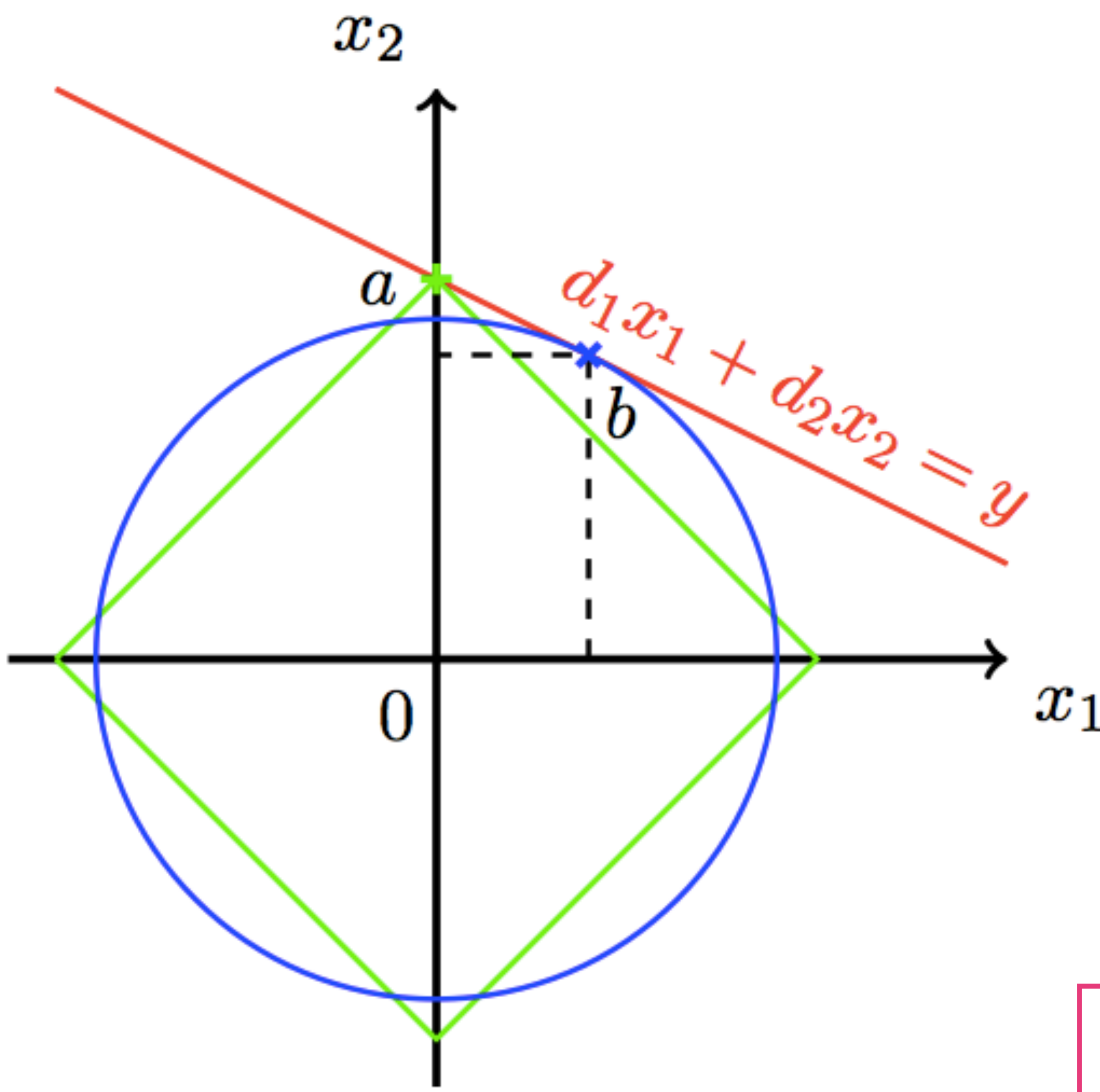
Optimization
(proximal theory)

Data Representation
(harmonic analysis, machine learning)

Noise Modeling
(Gaussian, Poissson, etc)

L1 Norm & Sparsity

$$\| X \|_p = \left(\sum_i \mid X_i \mid^p \right)^{\frac{1}{p}}$$



$$\boxed{p < 2}$$

Mass mapping as an inverse problem

F. Lanusse, J.-L. Starck, A. Leonard, and S . Pires, High Resolution Weak Lensing Mass Mapping combining Shear and Flexion, A&A, 2016.

Binned data: $\gamma = F^* P F \kappa$

Unbinned data: $\gamma = T^* P F \kappa$

$$\mathbf{P} = T^* P F$$

T = Non Equispaced Discrete Fourier Transform (NDFT)

$$\boxed{\min_{\kappa} \frac{1}{2} \parallel \gamma - \mathbf{P}\kappa \parallel_2^2 \quad + \quad \mathcal{C}(\kappa)}$$

$$g = \frac{\gamma}{1-\kappa} \longrightarrow \boxed{\min_{\kappa} \frac{1}{2} \parallel (1-\kappa) g - \mathbf{P}\kappa \parallel_2^2 \quad + \quad \mathcal{C}(\kappa)}$$

The 2D Glimpse Algorithm

$$\min_{\kappa} \frac{1}{2} F(\kappa) + \lambda \parallel \boldsymbol{\Phi}^t \kappa \parallel_1 \text{ with } F(\kappa) = \frac{1}{2} \parallel (1-\kappa) g - \mathbf{P}\kappa \parallel_2^2$$

Primal-dual splitting:

$$\boxed{\begin{cases} \kappa^{(n+1)} & = & \kappa^{(n)} + \tau \left(\nabla F(\kappa^{(n)}) + \boldsymbol{\Phi}\alpha^{(n)} \right) \\ \alpha^{(n+1)} & = & (\mathrm{Id} - \mathrm{ST}_\lambda) \left(\alpha^{(n+1)} + \boldsymbol{\Phi}^t (2\kappa^{(n+1)} - \kappa^{(n)}) \right) \end{cases}}$$

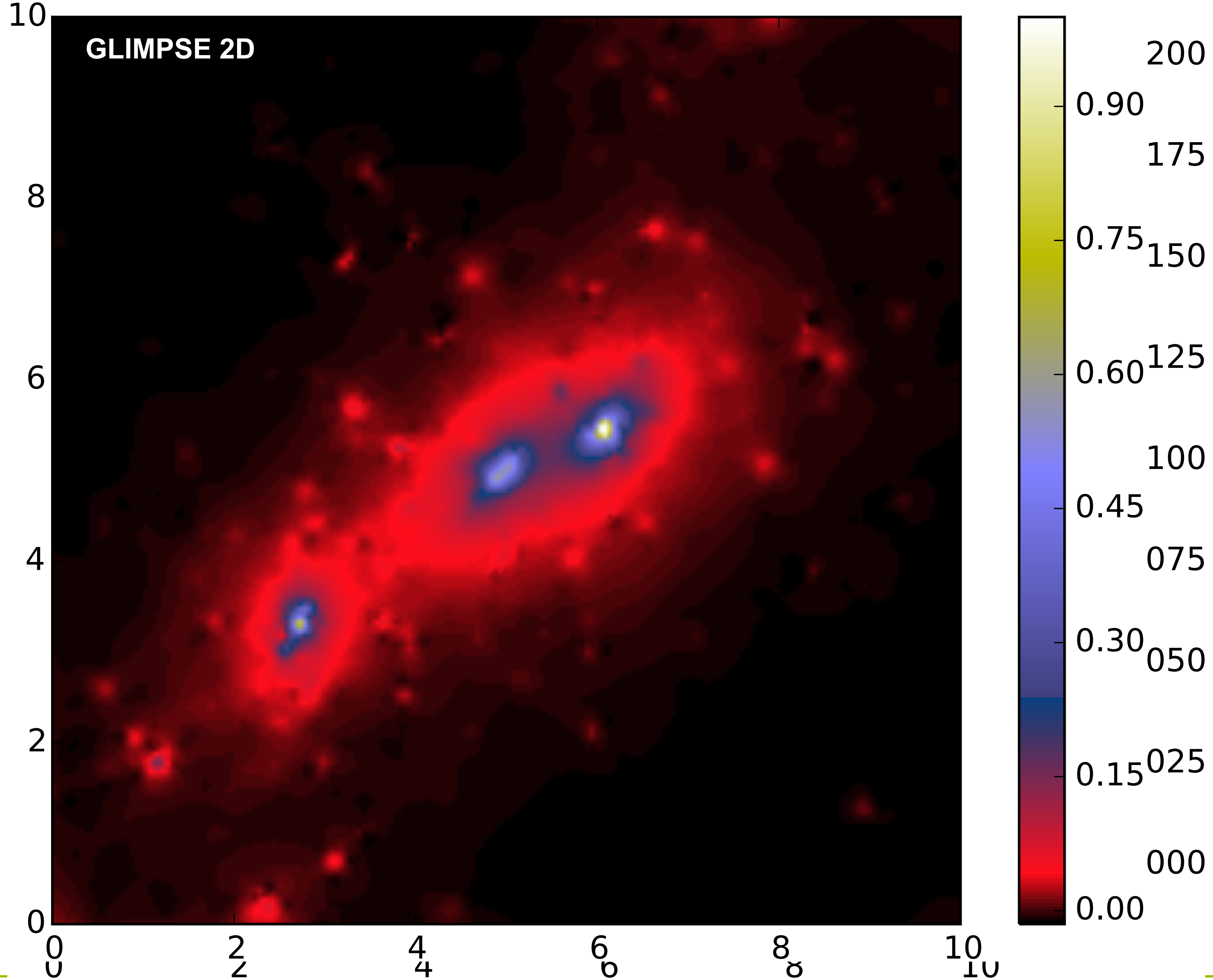
Condat-Vu algorithm, 2013

- Fast and flexible algorithm

A few remarks:
• Recovers the convergence from the reduced shear
• **P** can be defined with and without binning the shear
• **P** can be ill-posed in case of missing data
• Sparse regularization of noise and missing data
We use isotropic wavelets, well adapted to the recovery of clusters.

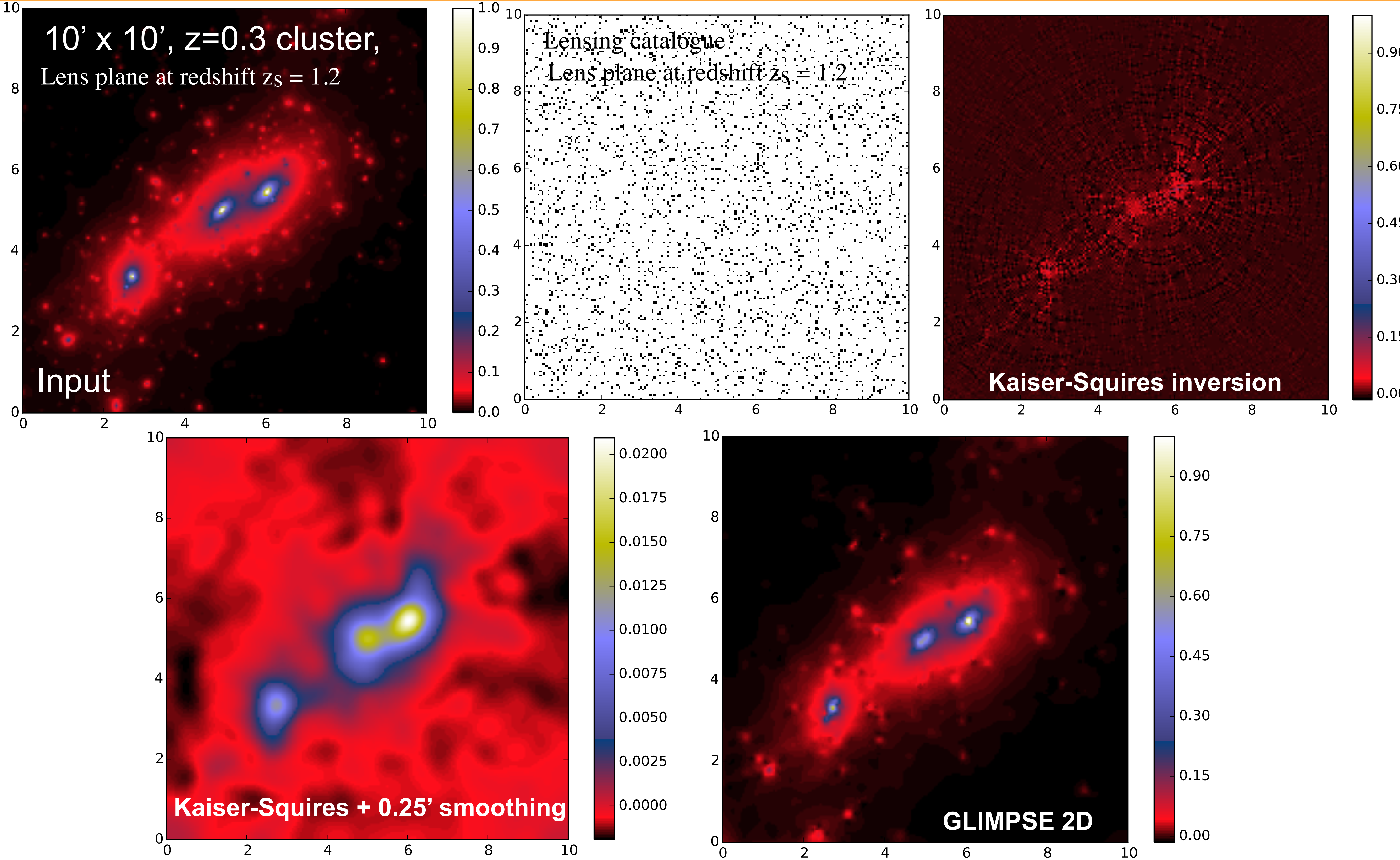
- Sparsity constraint λ estimated locally by noise simulations $\Longrightarrow$ Accounts for **survey geometry**, **varying noise levels**

Example with 93 % of missing data



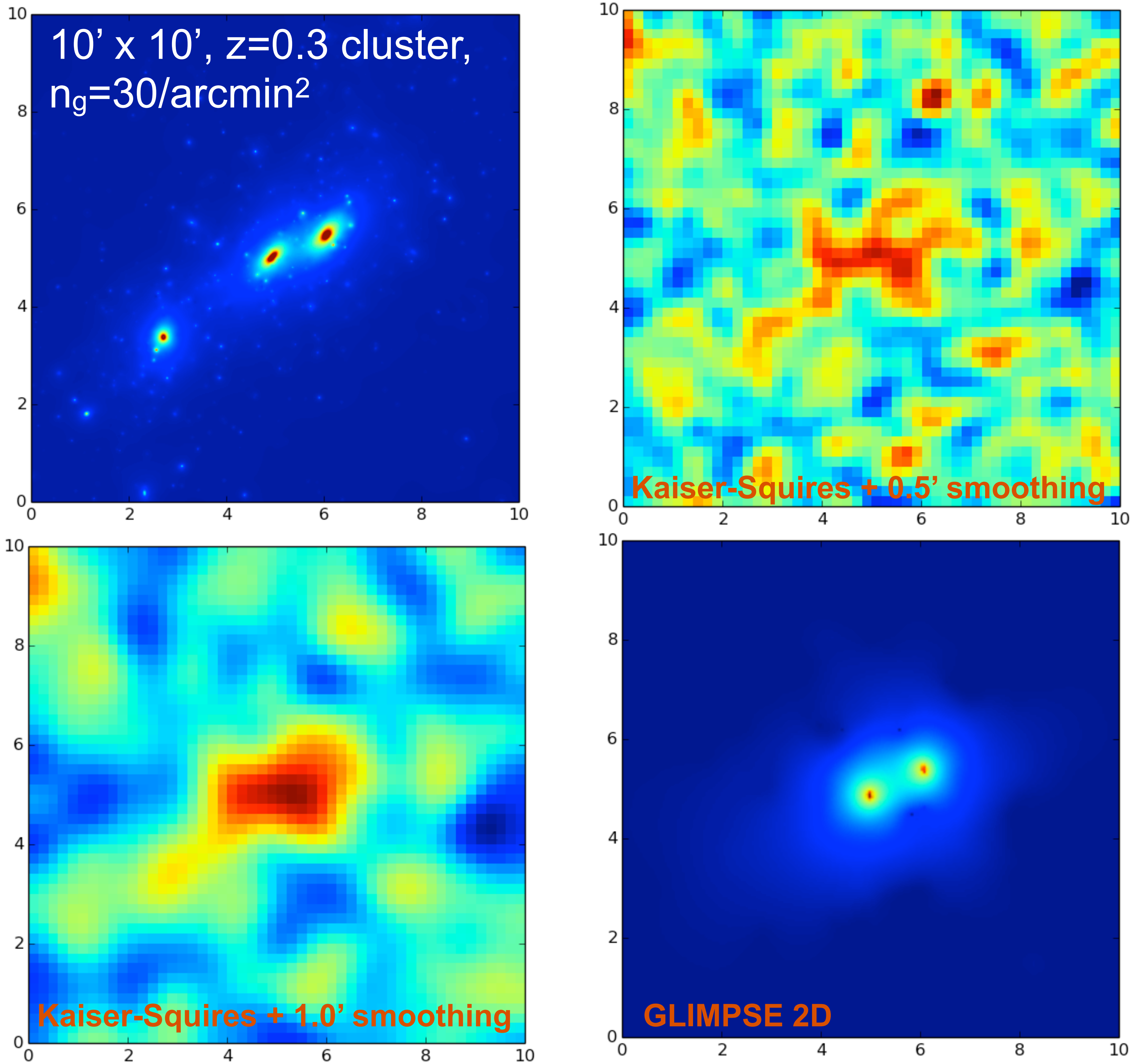
Galaxy distribution: **93% of missing pixels**, corresponding to 30 galaxies per square arcminute

Example with 93 % of missing data



Galaxy distribution: **93% of missing pixels**, corresponding to 30 galaxies per square arcminute

Missing Data + Noise



Flexion + Redshift Information

We can integrate flexion in our reconstruction framework

=> **Jointly** fit shear and flexion

$$\min_{\kappa} \frac{1}{2} \parallel (1-\kappa) \begin{bmatrix} g \\ F \end{bmatrix} - \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} \kappa \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^t \kappa \parallel_1$$

=> **Jointly** fit shear and flexion with redshift information

$$\min_{\kappa} \frac{1}{2} \parallel (1-\mathbf{Z}\kappa) \begin{bmatrix} g \\ F \end{bmatrix} - \mathbf{Z} \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} \kappa \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^t \kappa \parallel_1$$

with $\mathbf{Z} = \Sigma_{critic}^{\infty} / \Sigma_{critic}(z_i)$

$$\Sigma_{crit}^{\infty} = \lim_{z \to \infty} \Sigma_{crit}(z)$$

$$\Sigma_{crit}(z_s) = \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

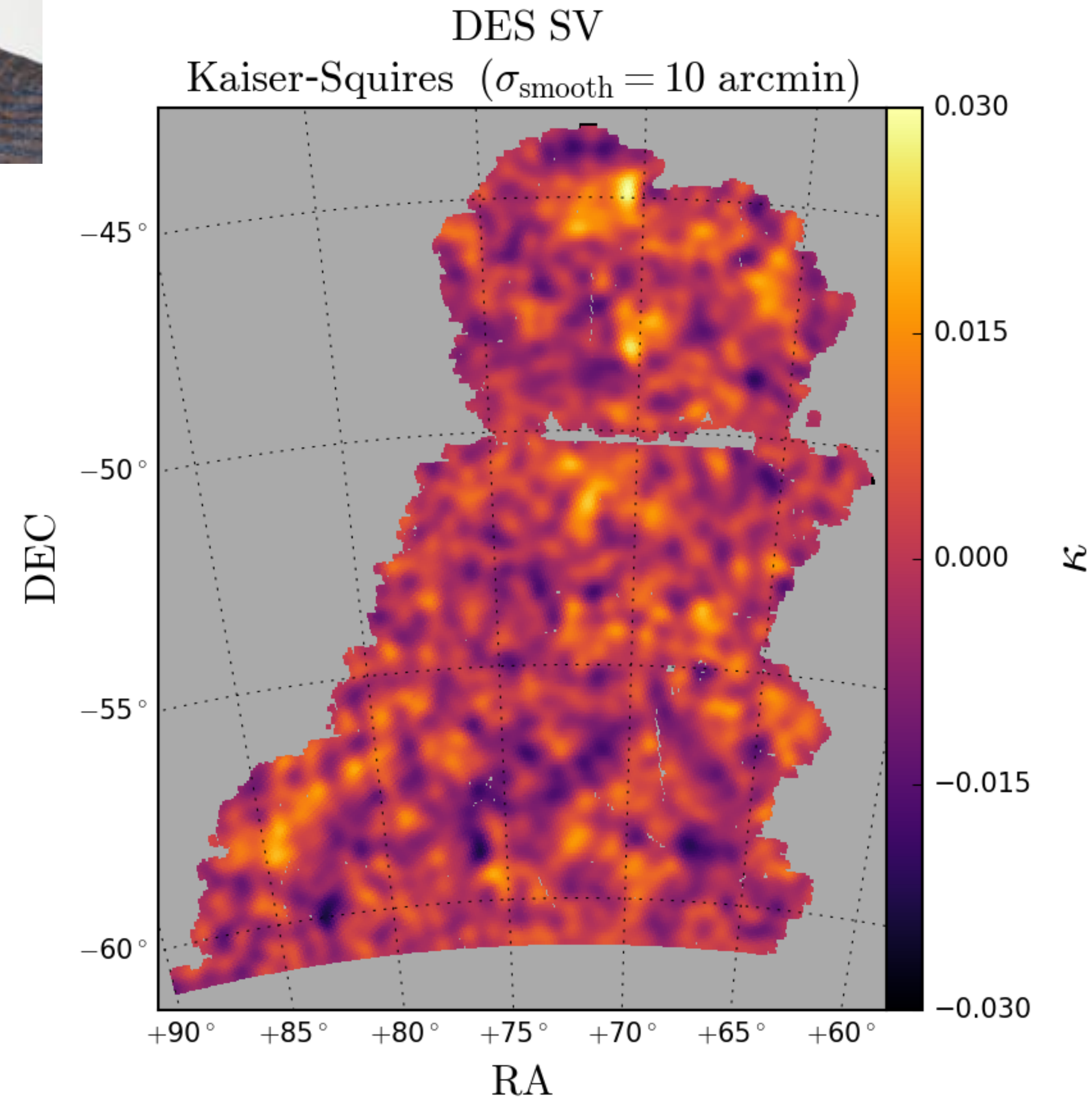
Individual redshifts have two benefits:

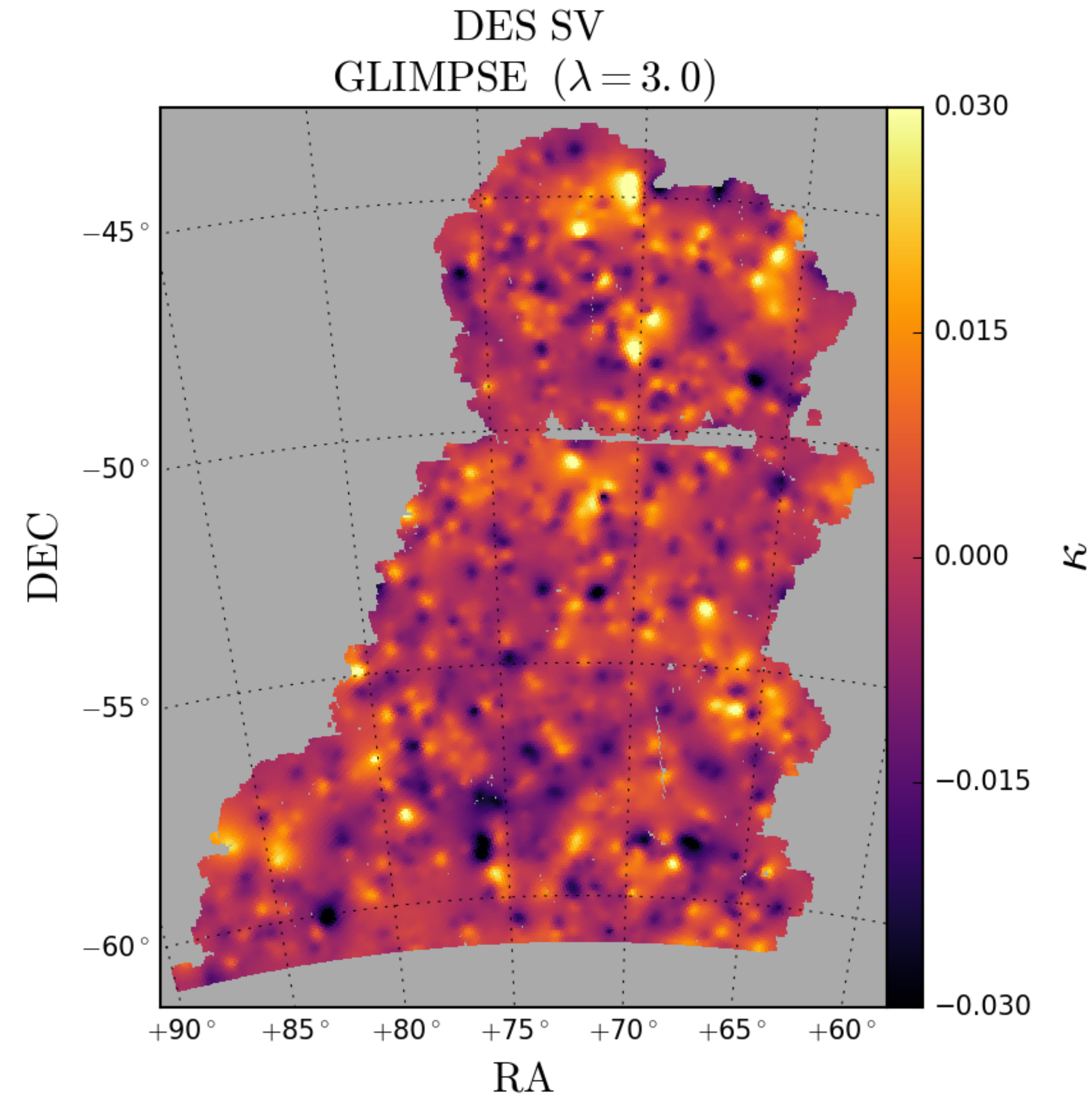
- Directly map the **surface mass density** of the lens
- Mitigate the **mass-sheet degeneracy** when κ becomes significant (Bradac, Lombard and Schneider, 2004)

Glimpse2D on public DES SV data

Niall Jeffrey et al. 2018, MNRAS, arXiv:1801.08945

139 deg²





The maximum signal-to-noise value of peak statistic increased by a factor of 9 using GLIMPSE.

Mass Mapping & Cluster A520

A. Peel, F. Lanusse, J.-L. Starck, « SPARSE RECONSTRUCTION OF THE MERGING A520 CLUSTER SYSTEM », **ApJ**, 847, 1, id. 23, 2017.

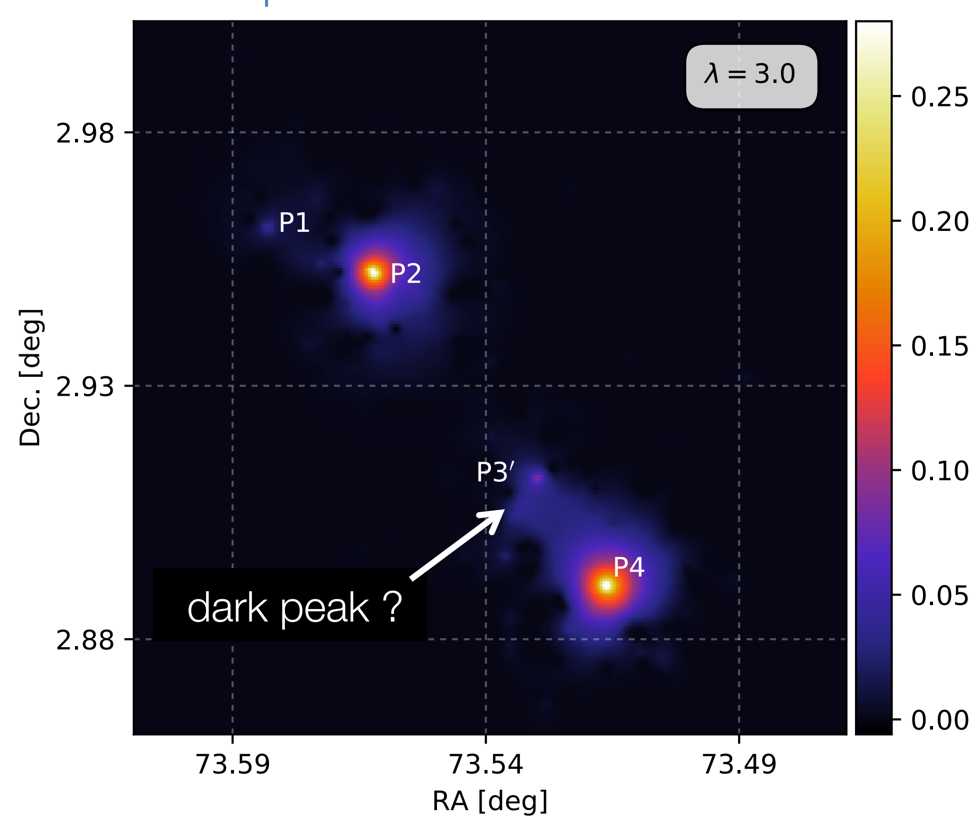
A puzzling case

Abell 520 (the “cosmic train wreck”) is a dynamically complex merging cluster system. Previous weak-lensing studies disagree about the presence of a mysterious dark mass peak—if real, it would challenge our current understanding of dark matter.

Sparsity-based mass mapping

We generated new mass maps of A520 using Glimpse2D*, a novel technique based on a sparsity prior.

Glimpse2D mass reconstruction



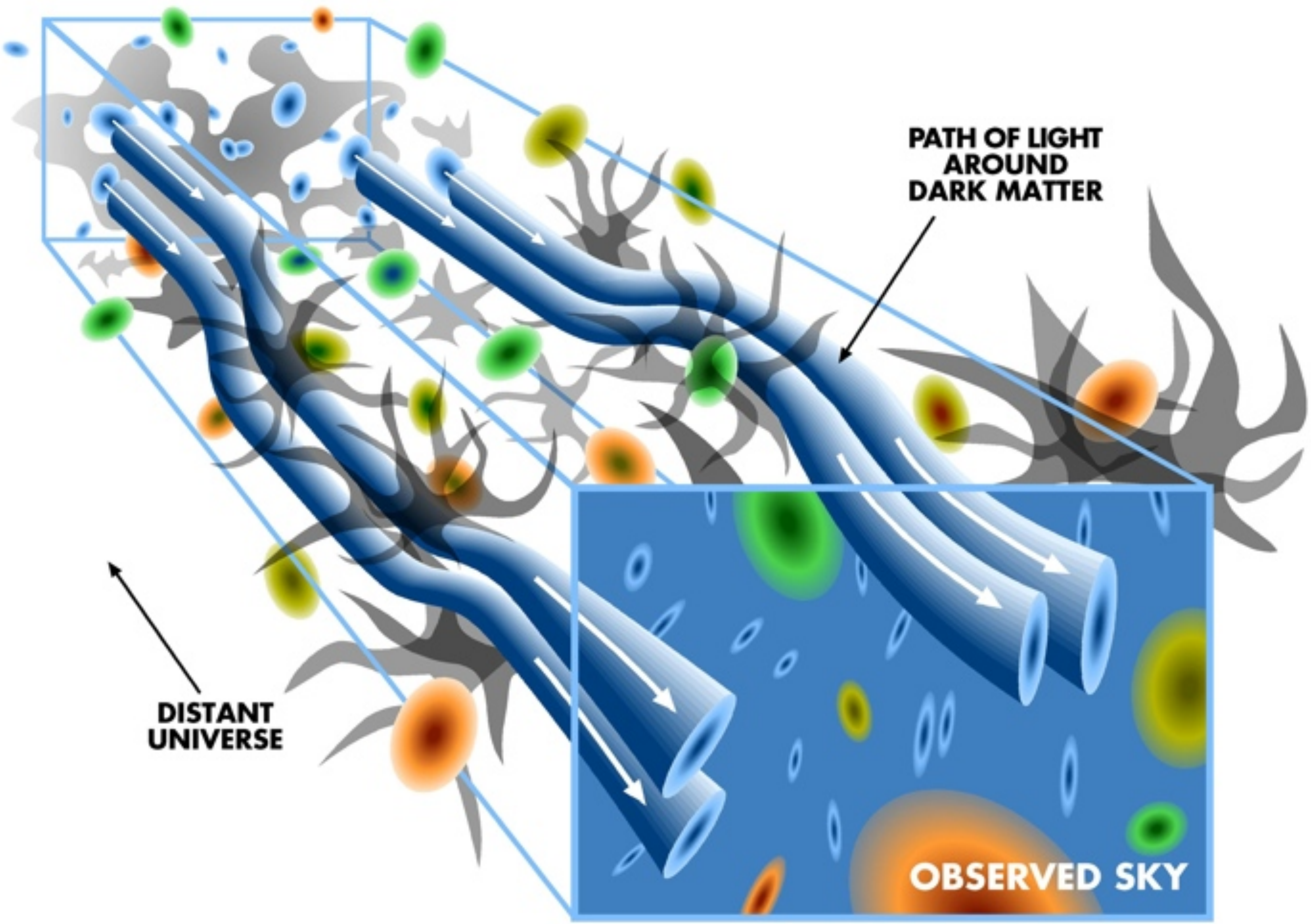
Upper limits on the significance of the P3’ structure of **2.3σ and 1.0σ** for the J14 and C12 catalogs,

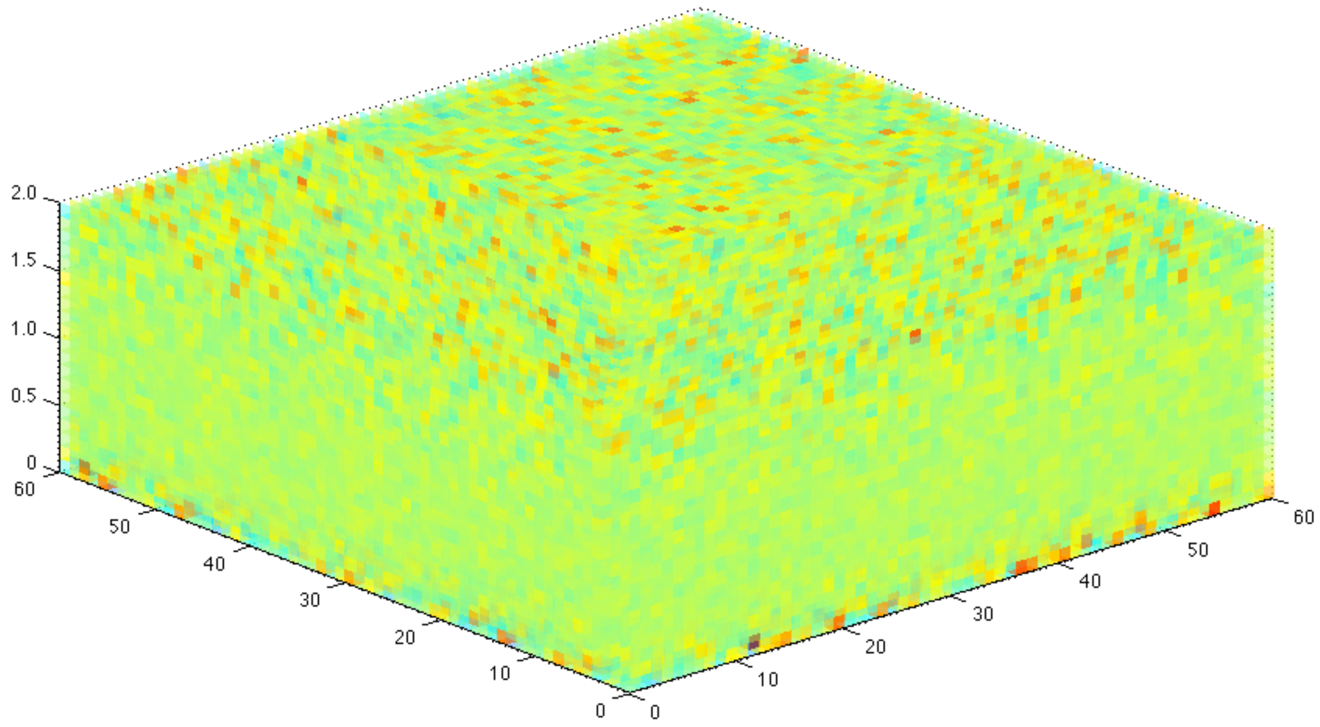
Result

Based on a statistical noise analysis, we cannot confirm the existence of the dark peak.

*http://www.cosmostat.org/software/glimpse

3D Weak Lensing




2.0
1.5
1.0
0.5
0
60
50
40
30
20
10
0
0
10
20
30
40
50
60

3D Mass Mapping

$$\gamma(\boldsymbol{\theta}) = \frac{1}{\pi} \int d^2\boldsymbol{\theta}' \mathcal{D}(\boldsymbol{\theta} - \boldsymbol{\theta}') \kappa(\boldsymbol{\theta}')$$

Kappa (or convergence) is a dimensionless surface mass density of the lens

$$\kappa(\boldsymbol{\theta}, w) = \frac{3H_0^2\Omega_M}{2c^2} \int_0^w dw' \frac{f_K(w') f_K(w - w')}{f_K(w)} \frac{\delta[f_K(w')\boldsymbol{\theta}, w']}{a(w')} ,$$

f_K is the angular diameter distance, which is a function of the comoving radial distance r and the curvature K.

$$\boldsymbol{\gamma} = \mathbf{P}_{\gamma\kappa}\boldsymbol{\kappa} + \boldsymbol{n}_\gamma ,$$

$$\kappa = Q\delta + n$$

$$\boxed{\gamma = \mathbf{R}\delta + n}$$

- Galaxies are not intrinsically circular: intrinsic ellipticity ~ 0.2-0.3; gravitational shear ~ 0.02
- Reconstructions require knowledge of distances to galaxies

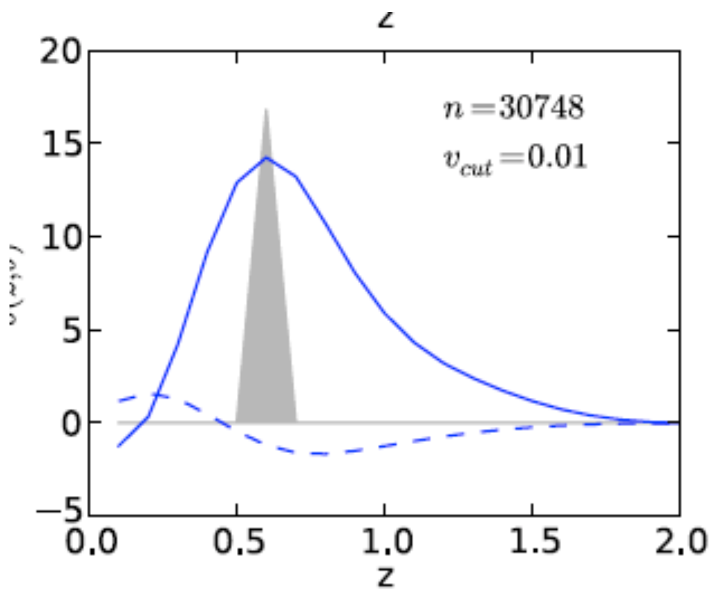
Linear Methods

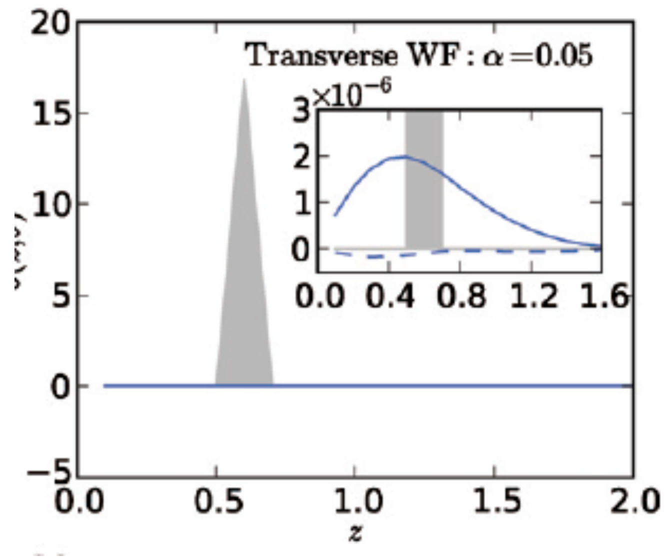
- ✧ Assume uncorrelated Gaussian noise*
- ✧ Linear methods
  - ✧ Wiener/inverse variance filter (Simon et al., 2009)
  - ✧ $$\hat{s}_{MV} = [\alpha \mathbf{1} + \mathbf{S}\mathbf{R}^{\dagger}\mathbf{\Sigma}^{-1}\mathbf{R}]^{-1}\mathbf{S}\mathbf{R}^{\dagger}\mathbf{\Sigma}^{-1}\boldsymbol{d} \; .$$
  - ✧ SVD decomposition & thresholding (VanderPlas et al., 2011)

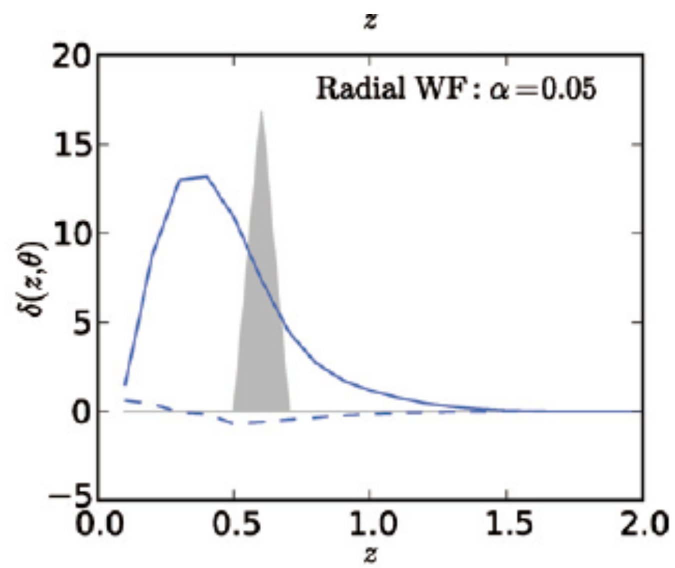
$$\hat{s}_{IV} = \mathbf{V}\mathbf{\Lambda}^{-1}\mathbf{U}^{\dagger}\mathbf{\Sigma}^{-1/2}\boldsymbol{d} \; ,$$

Reconstruction resolution limited by resolution of data

Linear Methods

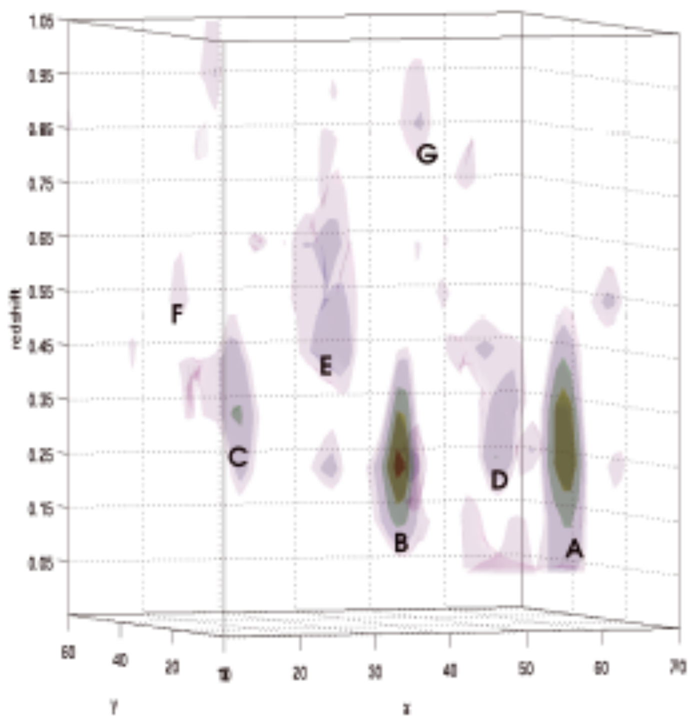




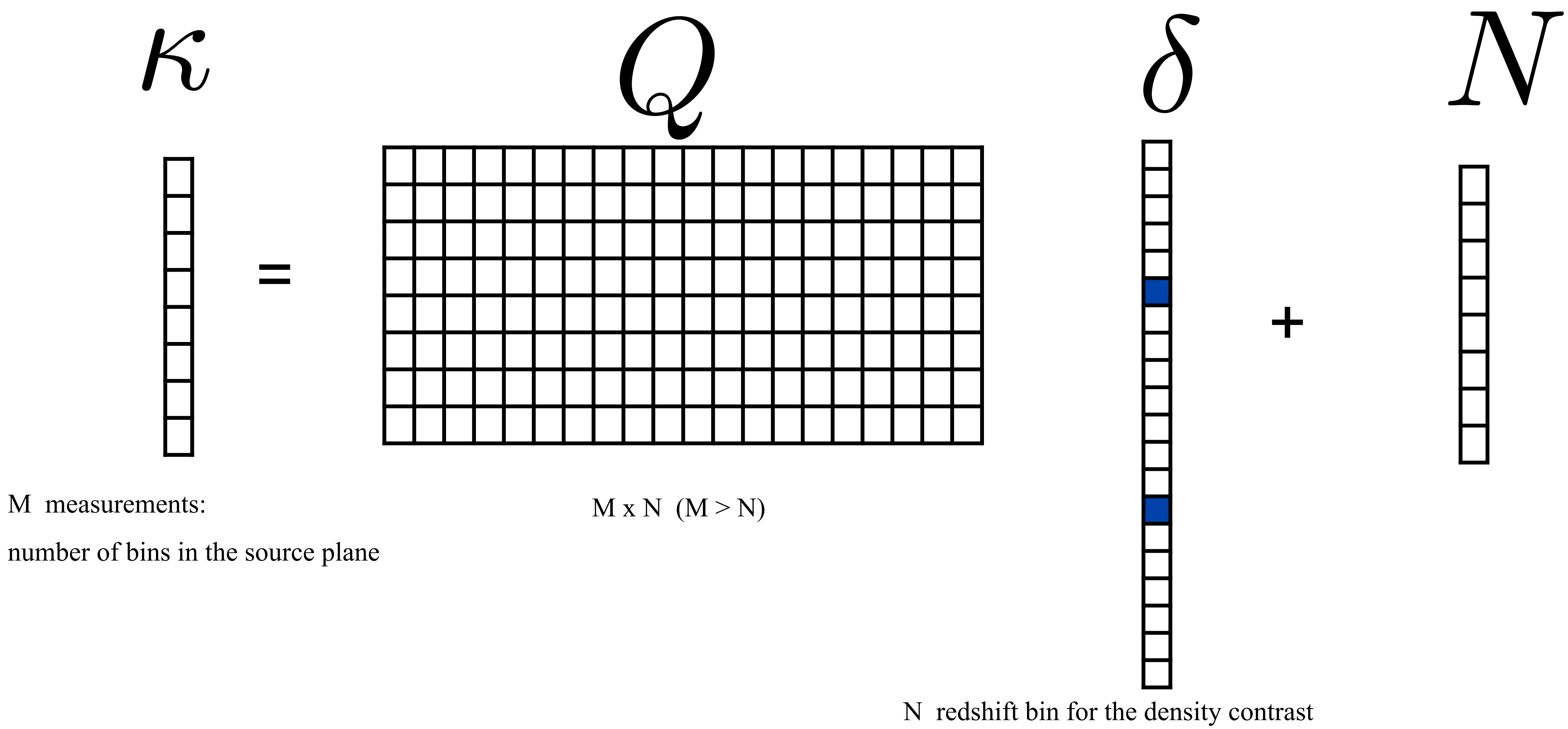


Target Areas for Improvement

- ✧Redshift bias in location of detected peaks
- ✧Smearing along the line of sight
- ✧Damping of the reconstruction
- ✧Sensitivity at high redshift
- ✧Improving resolution in reconstructions



3D Mass Mapping



δ is sparse.

Q spreads out the information in δ along κ bins.

More unkown than measurements

Weak Lensing & 3D Matter Distribution

A. Leonard, F.X. Dupe, and J.-L. Starck, "A Compressed Sensing Approach to 3D Weak Lensing", **Astronomy and Astrophysics** , 539, A85, 2012.

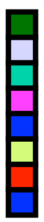
A. Leonard, F. Lanusse, J-L. Starck, GLIMPSE: Accurate 3D weak lensing reconstruction usiing sparsity, Astronomy and Astrophysics, 2014

$$\gamma = P\kappa$$
$$\kappa = Q\delta$$

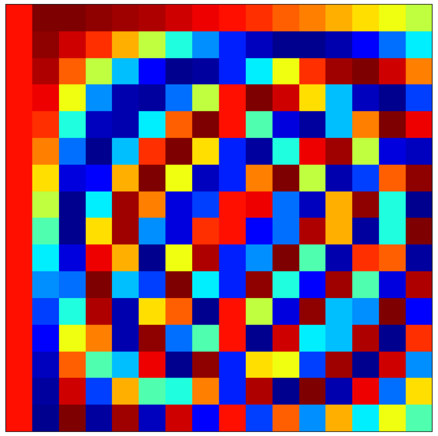
$$\delta = \Phi\alpha \rightarrow \gamma = PQ\Phi\alpha = R\Phi\alpha$$

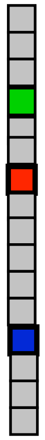
$$R = PQ \qquad \alpha$$

$$\gamma$$



$=$





Shear Measurements
M measurements: number of bins in the source plan x number of pixels at for given bin

Density constrast wavelet coefficients

Related to Compressed Sensing theorem

==> Use Sparse recovery and Proximal optimization theory

$$\min_{\alpha} \| \alpha \|_1 \quad s.t. \ \frac{1}{2} \| \gamma - R\Phi\alpha \|^2_{\Sigma^{-1}} \leq \epsilon$$

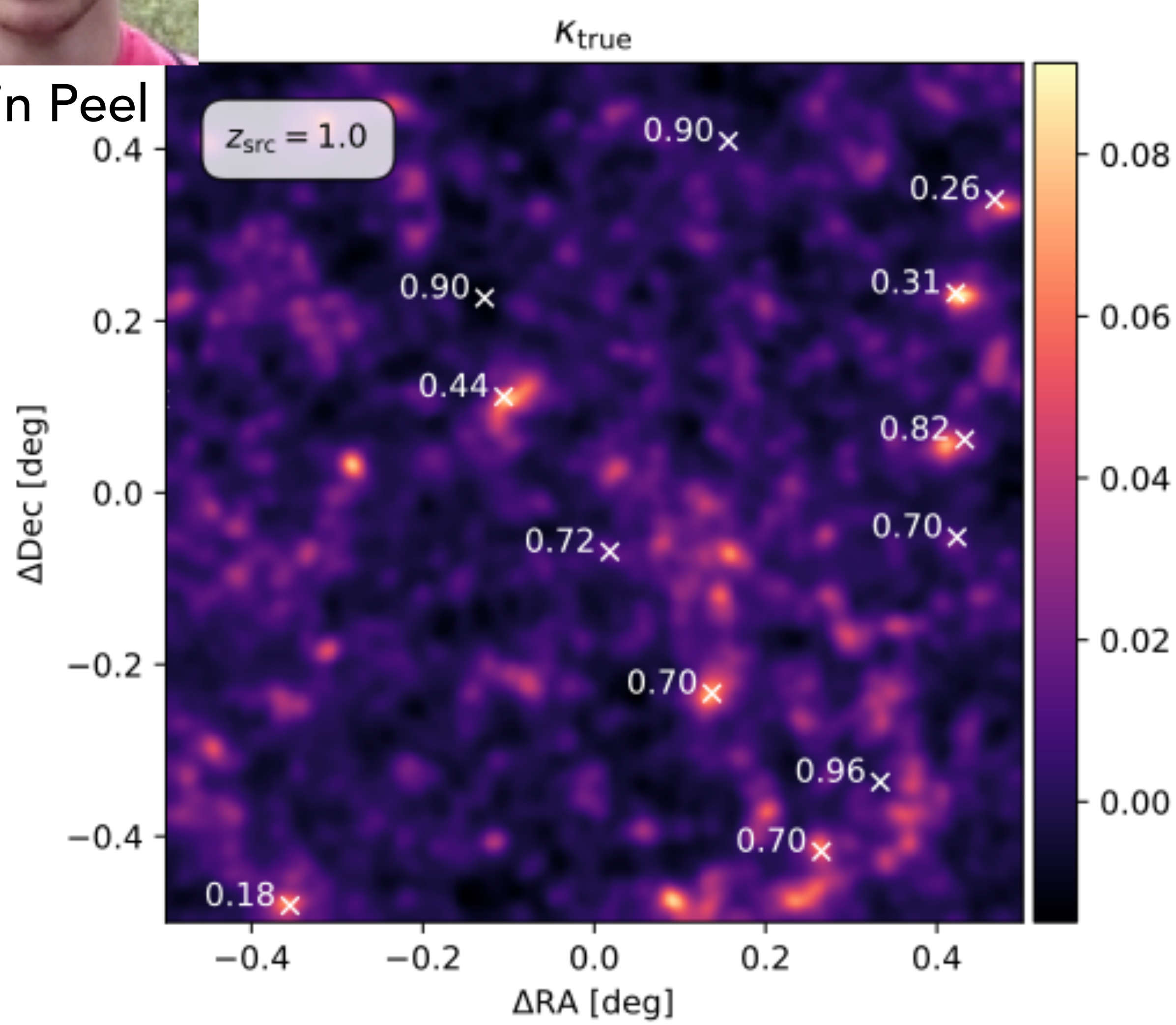
$$\delta = \Phi\alpha$$

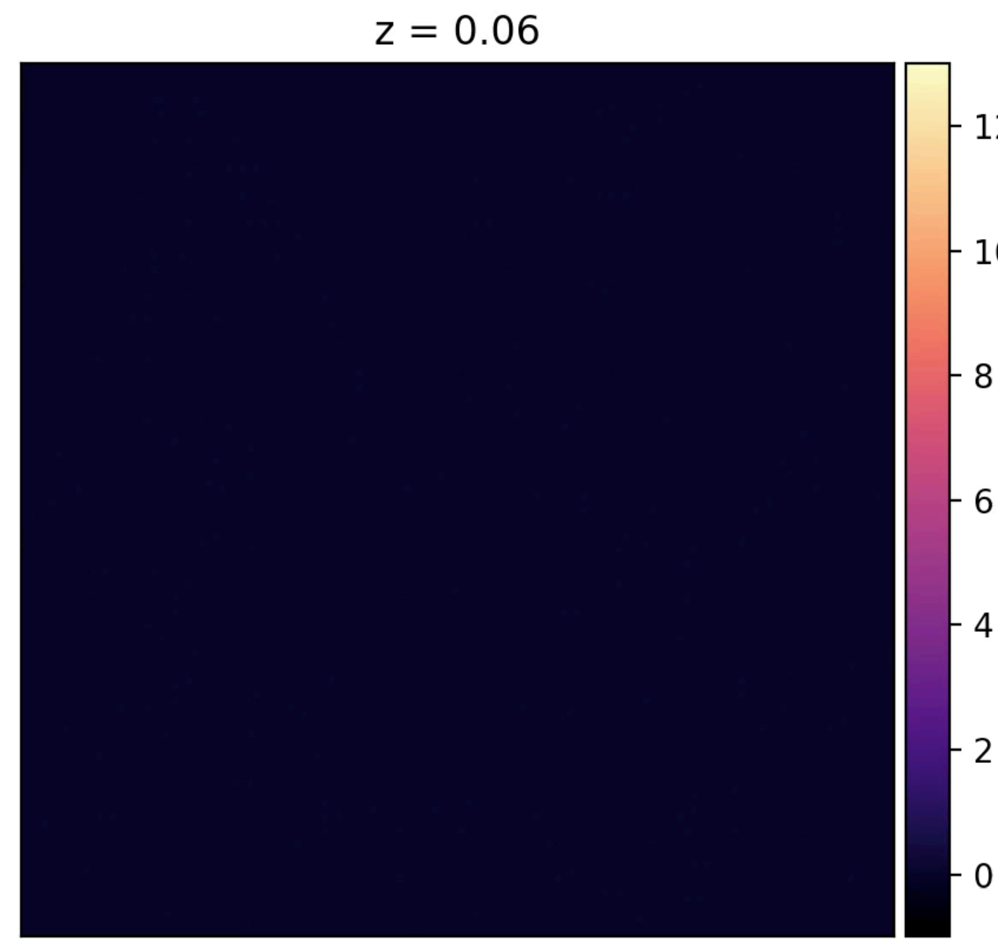
Φ = 2D Wavelet Transform on each redshift bin

Glimpse3D on Euclid calibration mock

Austin Peel

1 deg² example tile





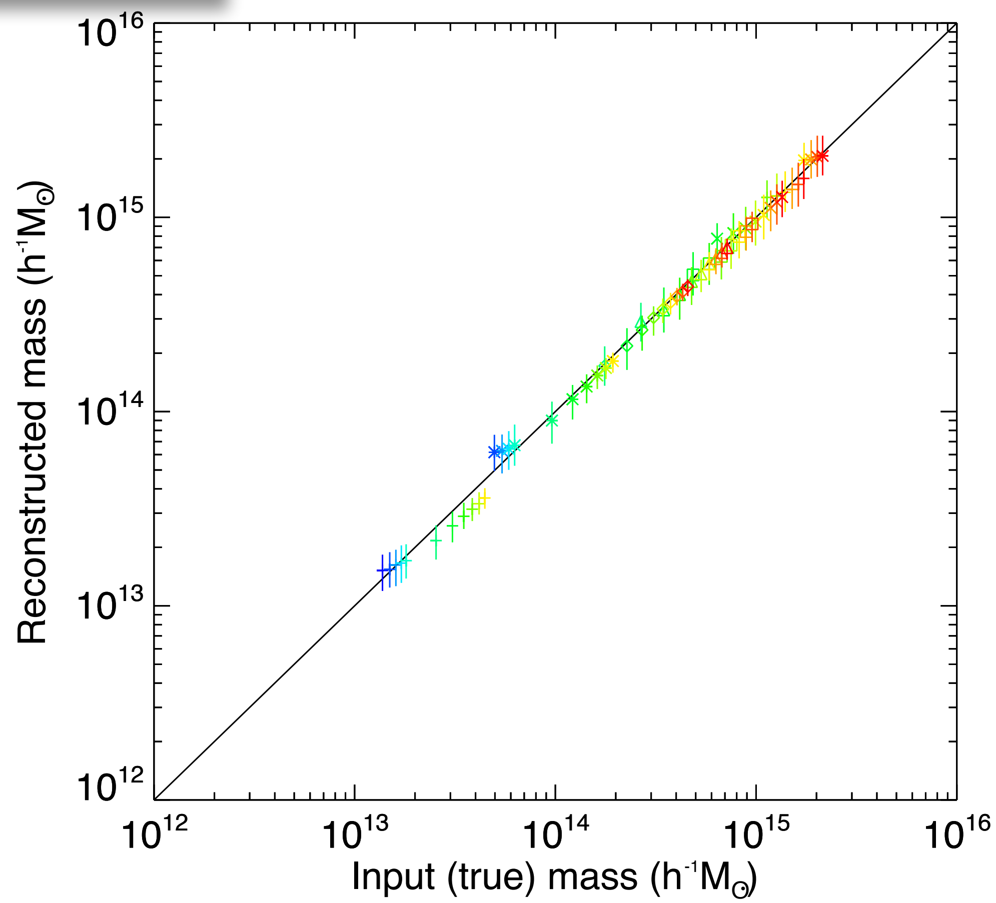
1 pix = **0.23** arcmin

no Glimpse3D smoothing

density reconstruction on redshift slices

Mass Estimation

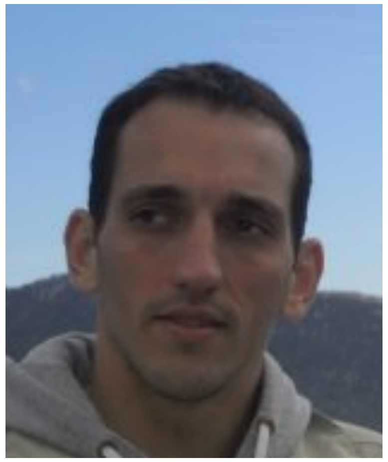
Cluster Masses from 3D Weak



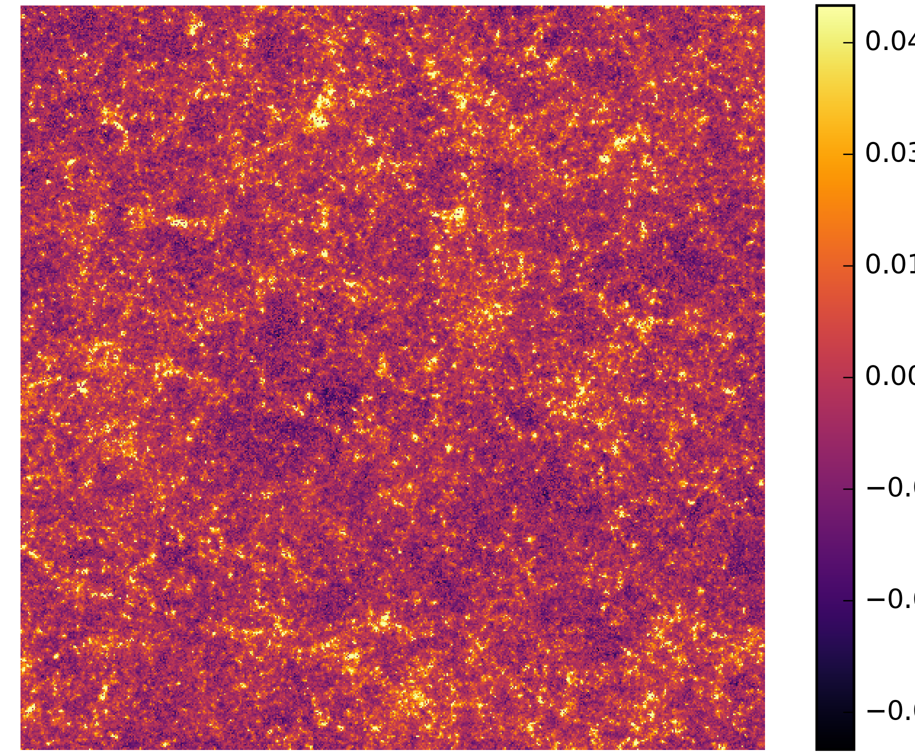
- GLIMPSE 3D reconstructions provide a direct, unbiased & nonparametric estimate of the cluster mass (Leonard, Lanusse & Starck 2014, MNRAS, 440, 1281)
- Masses estimated integrating the density in the central 4 x 4 arcmin
- Error bars reflect the standard deviation in mass estimates 1000 Monte Carlo simulations of each cluster
- Cluster masses $2 \times 10^{13} h^{-1} M_\odot \leq M_{vir} \leq 10^{15} h^{-1} M_\odot$
- Cluster redshifts $0.05 \leq z \leq 0.75$

•A. Leonard, F. Lanusse, & J.-L. Starck, "Weak lensing reconstructions in 2D & 3D: implications for cluster studies", MNRAS, 449, 1146–1157, 2015.
•A. Leonard, F. Lanusse & J.-L. Starck, A&A, “GLIMPSE: Accurate 3D weak lensing reconstructions using sparsity”, 2014.

A New Model



$$\boldsymbol{\kappa} = \boldsymbol{\kappa}_{\mathrm{NG}} + \boldsymbol{\kappa}_{\mathrm{G}}.$$



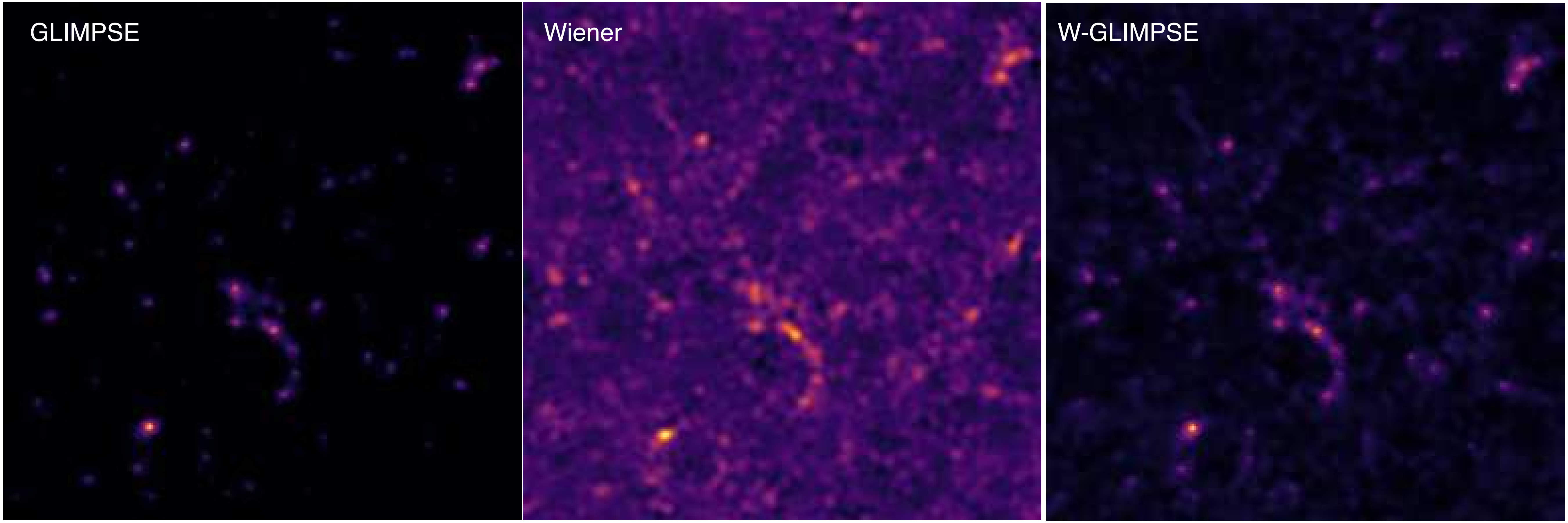
$$\boldsymbol{\gamma} = \mathbf{P}\boldsymbol{\kappa} + \mathbf{n}$$

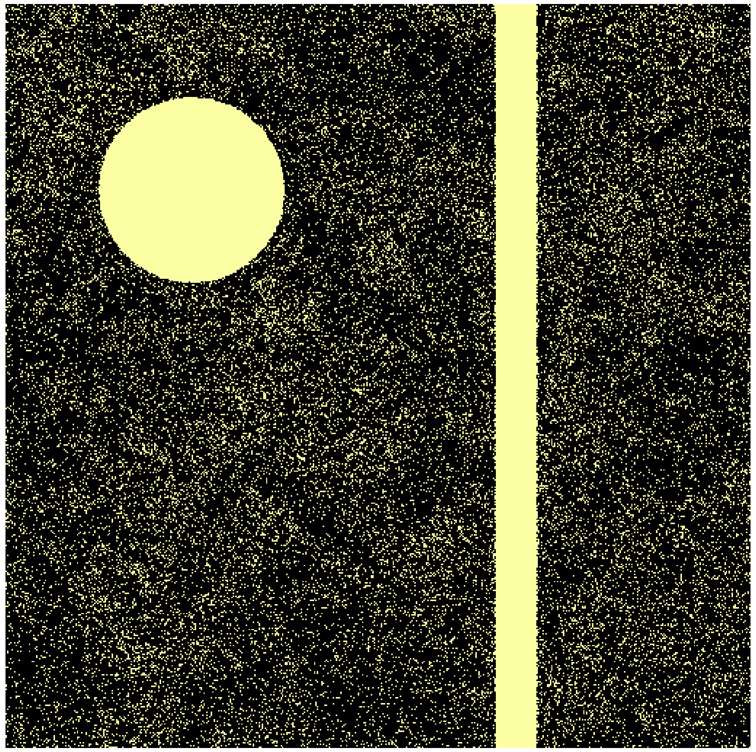
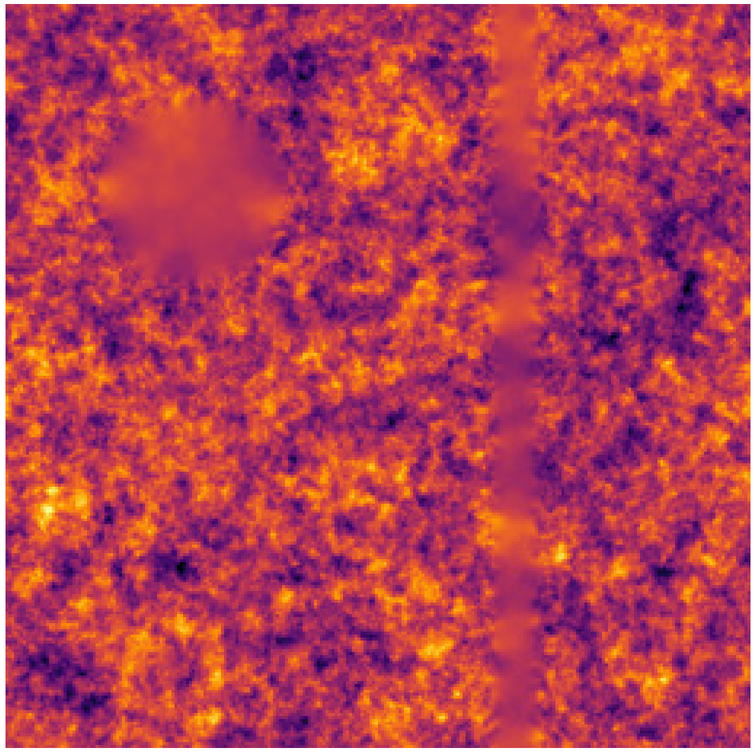
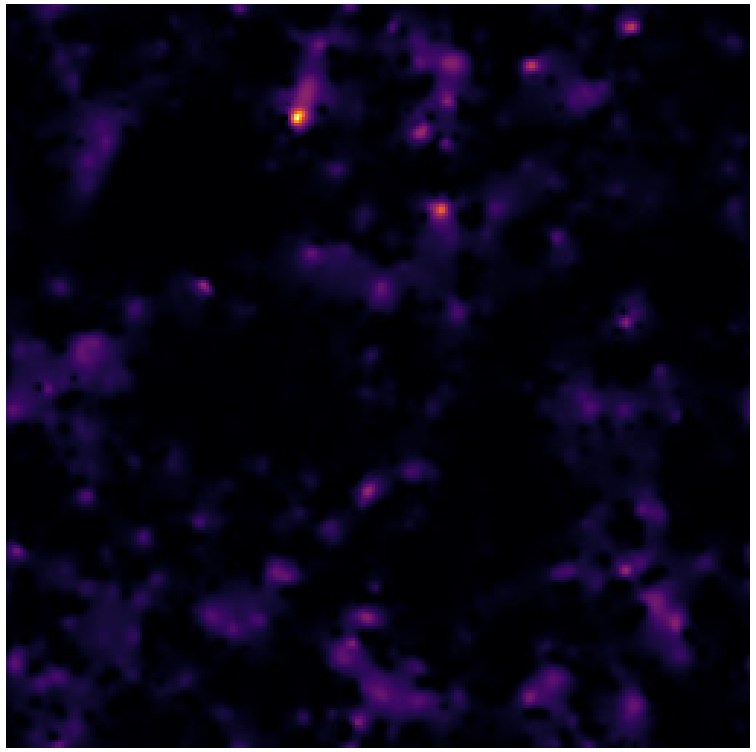
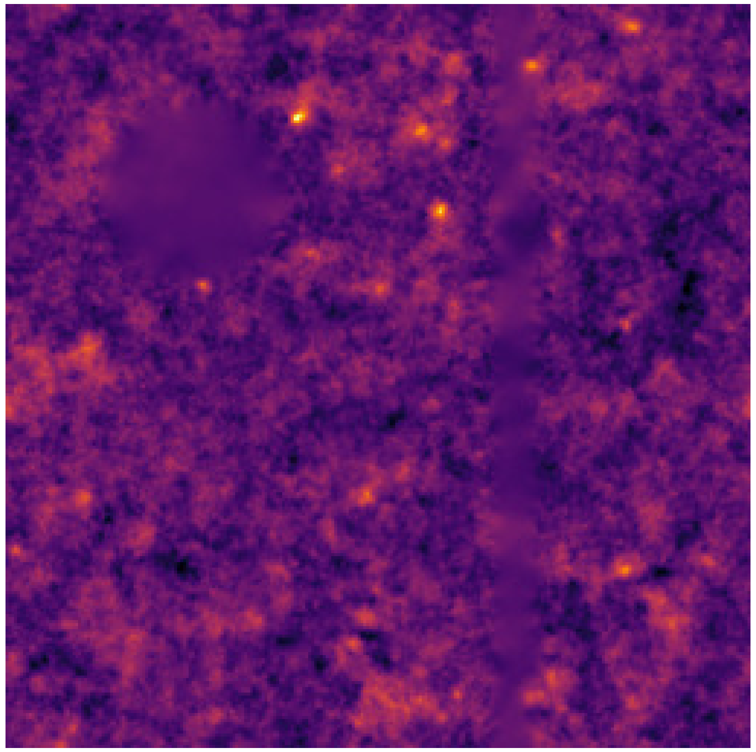
$$\boldsymbol{\gamma} = \mathbf{P}\boldsymbol{\kappa}_{\mathrm{NG}} + \mathbf{n}' \quad \text{with} \quad \mathbf{n}' = \mathbf{P}\boldsymbol{\kappa}_{\mathrm{G}} + \mathbf{n}$$

$$\min_{\boldsymbol{\kappa}_{NG}} \left\{ \frac{1}{2} \| \underbrace{\boldsymbol{\gamma} - \mathbf{P}\boldsymbol{\kappa}_{NG}}_{\boldsymbol{\gamma}_r} \|^2_{\boldsymbol{\Sigma}} + \lambda \| \mathbf{w} \odot \boldsymbol{\Phi}^* \boldsymbol{\kappa}_{NG} \|_1 + \mathcal{I}_{\mathbb{R}+}(\boldsymbol{\kappa}_{NG}) \right\}$$

$$\min_{\mathbf{W}, \boldsymbol{\kappa}_G} \left\{ \| \mathbf{W}\boldsymbol{\gamma}_r - \mathbf{P}\boldsymbol{\kappa}_G \|^2_{\mathbf{N}^{1/2}} \right\},$$

W-GLIMPSE



Conclusions

✓ **GLIMPSE2D: A new mass mapping algorithm, based on sparsity and proximal optimization theory:**

https://github.com/CosmoStat

⇒ Does not require angular binning of the ellipticities, accounts for reduced shear, and proper regularization of missing data.

⇒ Can include individual redshift PDFs of sources and flexion measurements if available

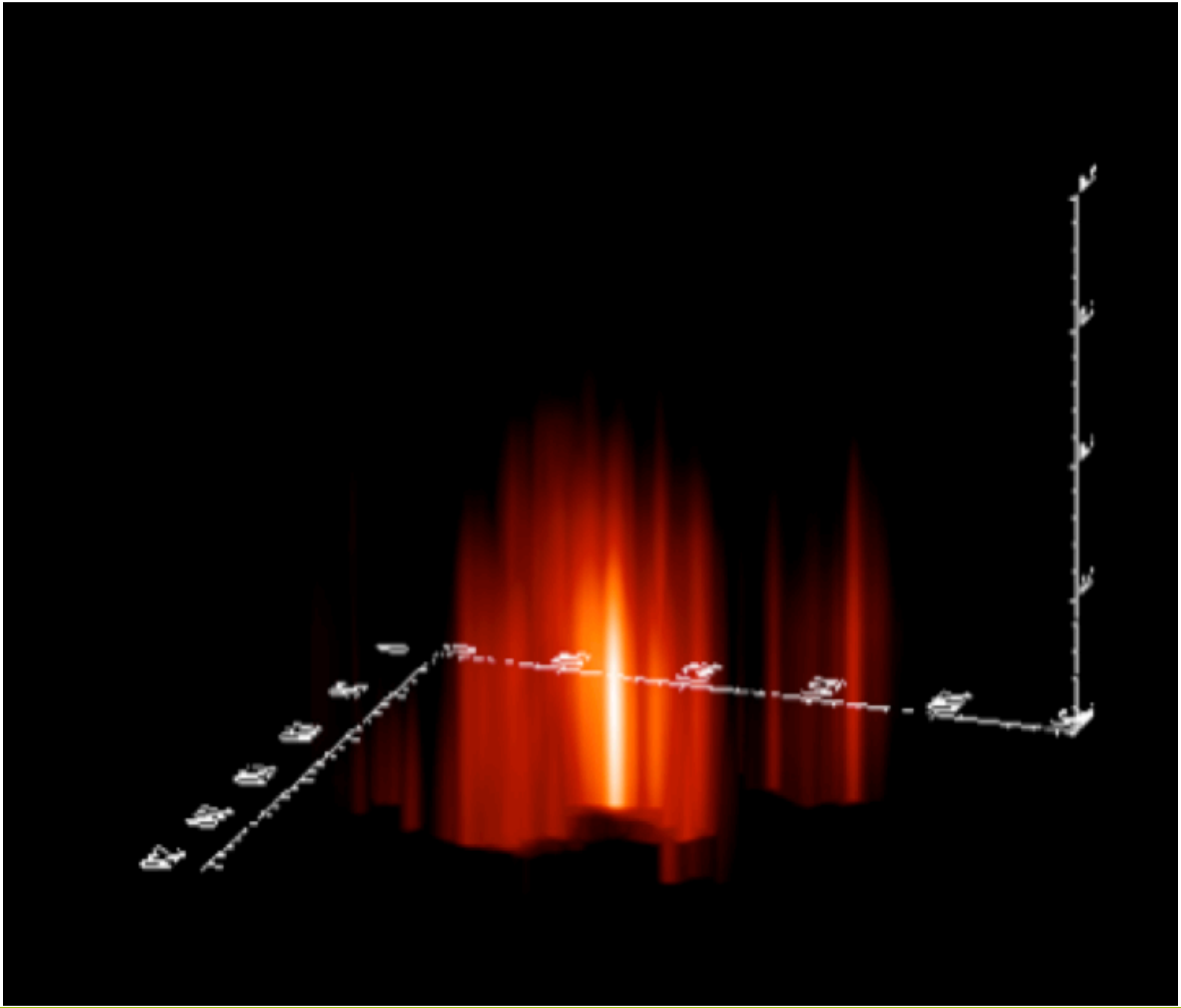
➡ **Bridge between low resolution weak lensing and high resolution strong lensing**

➡ **Can recover cluster substructures without strong lensing information**

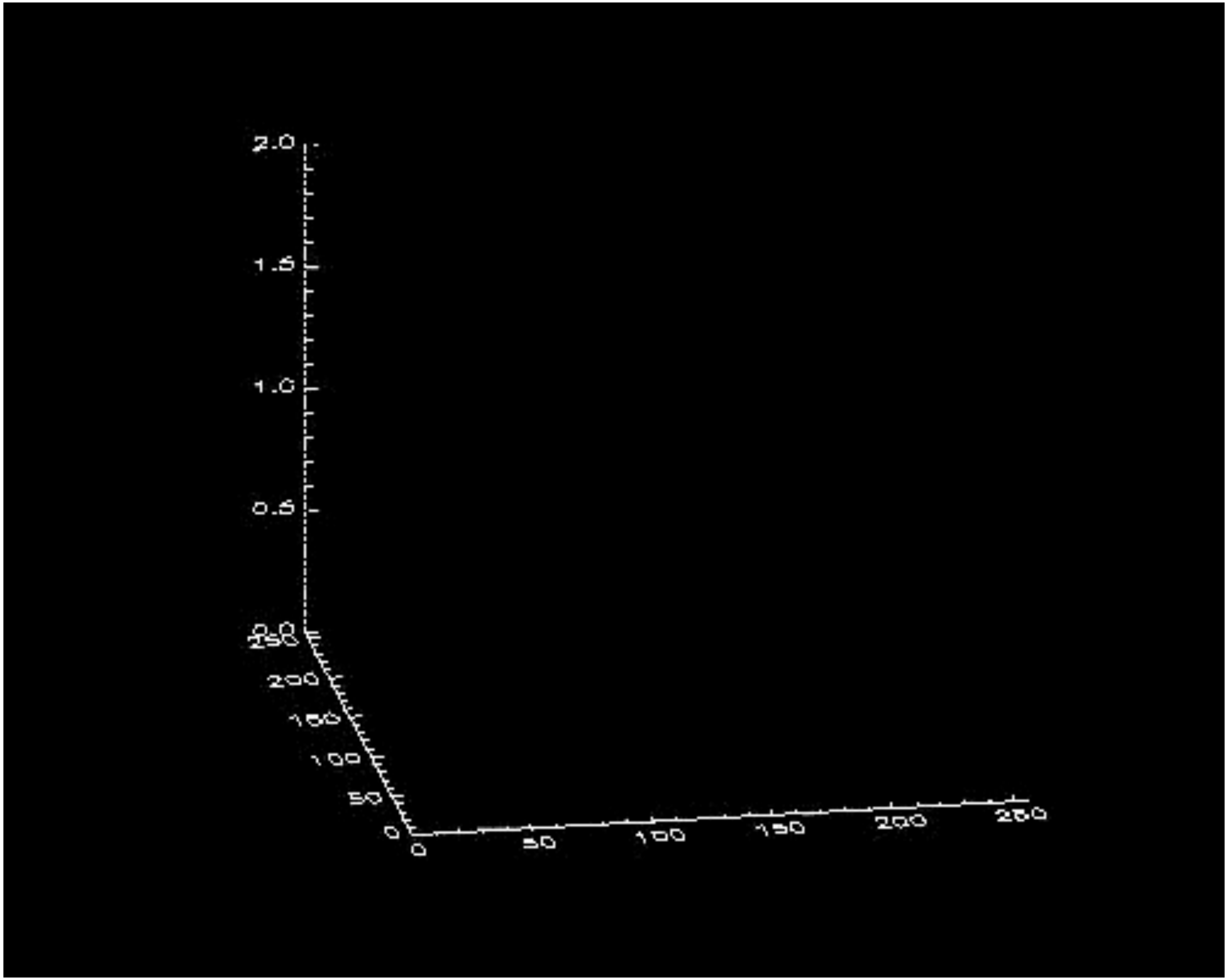
➡ **Ideal tool for investigating models of dark matter**

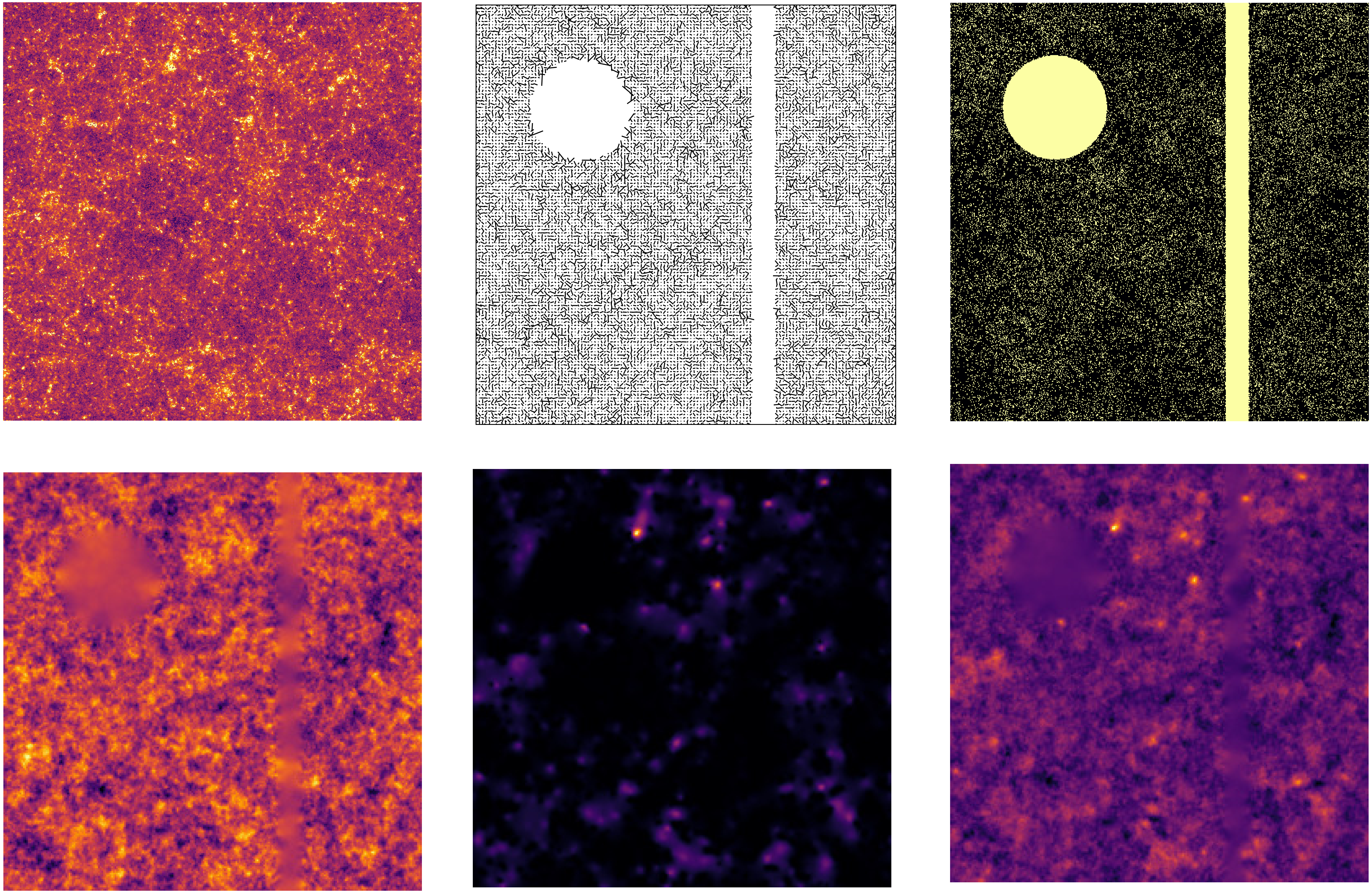
- F. Lanusse , J.-L. Starck, A. Leonard, and S . Pires, High Resolution Weak Lensing Mass Mapping combining Shear and Flexion, A&A, 2016.
- A. Peel, F. Lanusse, J.-L. Starck, Sparse Reconstruction of the merging A520 cluster system, ApJ, 847, 1, id. 23, 2017.

WL 3D Cosmo-Door is now open



WL 3D Cosmo-Door is now open





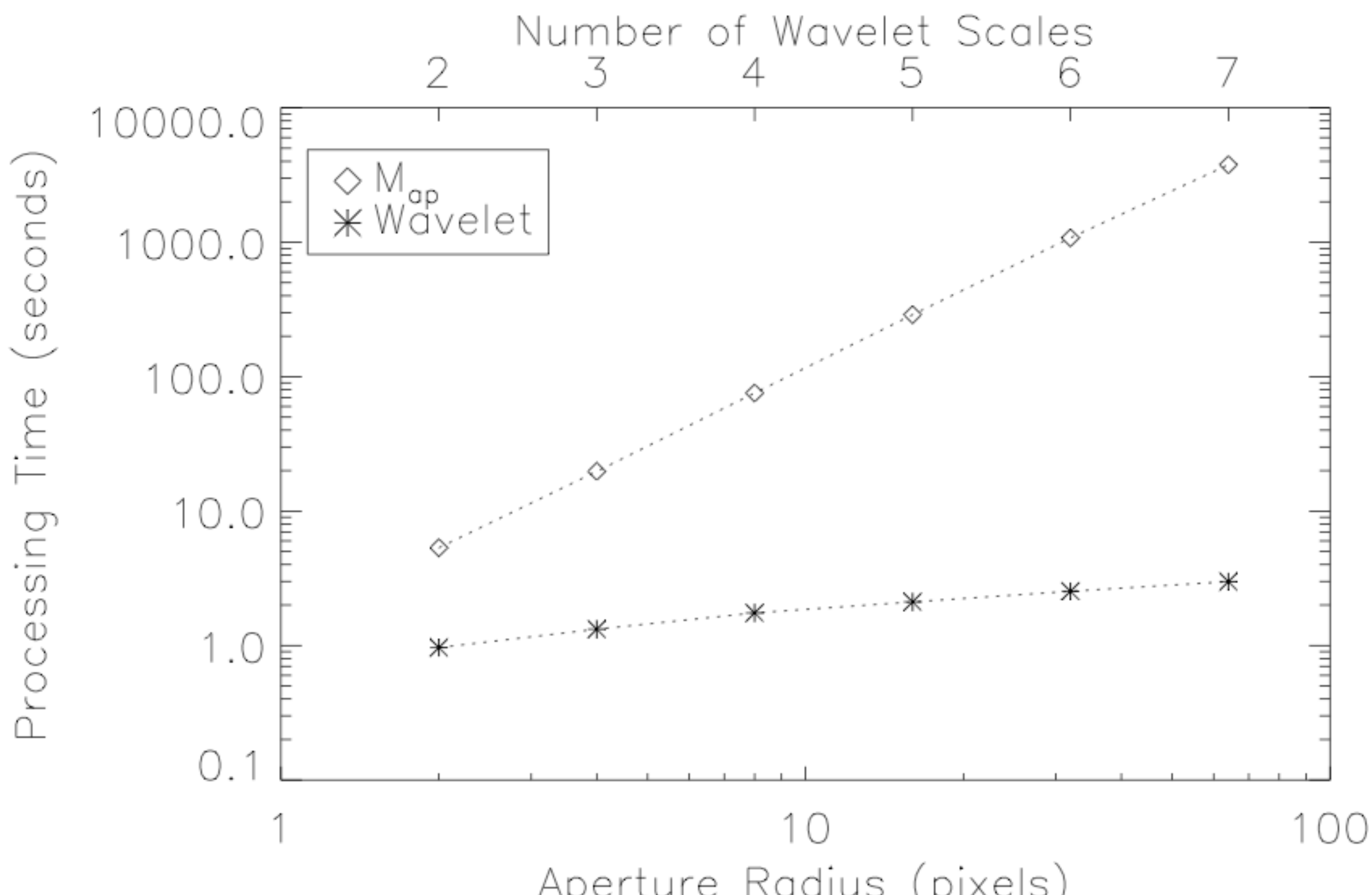
Aperture Mass or Convergence ?

$$M_{ap}(\theta) = (\boldsymbol{W}\kappa)_\theta$$

but wavelets presents many advantages:

- compensated and **compact** support filters
- **fast** calculation:
- **all scales** processed in one step.
- **reconstruction** is possible

==> image restoration for peak counting



- S. Pires, A. Leonard, J.-L. Starck, "Cosmological Parameters Constraint from Weak Lensing Data", **MNRAS**, 423, pp 983-992, 2012.
- A. Leonard, S. Pires, J.-L. Starck, "Fast Calculation of the Weak Lensing Aperture Mass Statistic", **MNRAS**, 423, pp 3405-3412, 2012.
- A. Leonard, J.-L. Starck, S. Pires , F.-X Dupe, Exploring the Components of the Universe Through Higher-Order Weak Lensing Statistics, Open Questions in Cosmology, Gonzalo J. Olmo (Ed.), InTech, 2012.

Aperture Mass and Wavelets

$$M_{ap}(\boldsymbol{\theta}) = \int d^2\boldsymbol{\vartheta}\ \gamma_t(\boldsymbol{\vartheta})Q(|\boldsymbol{\vartheta}|)$$

$$\boxed{M_{ap}(\theta) = (\boldsymbol{\Phi}^t\kappa)_\theta}$$

$\Rightarrow$ Wavelets filters are formally **indentical** to Mass aperture

A. Leonard, S. Pires, J.-L. Starck, "Fast Calculation of the Weak Lensing Aperture Mass Statistic", **MNRAS**, 423, pp 3405-3412, 2012.

but wavelets presents many advantages:
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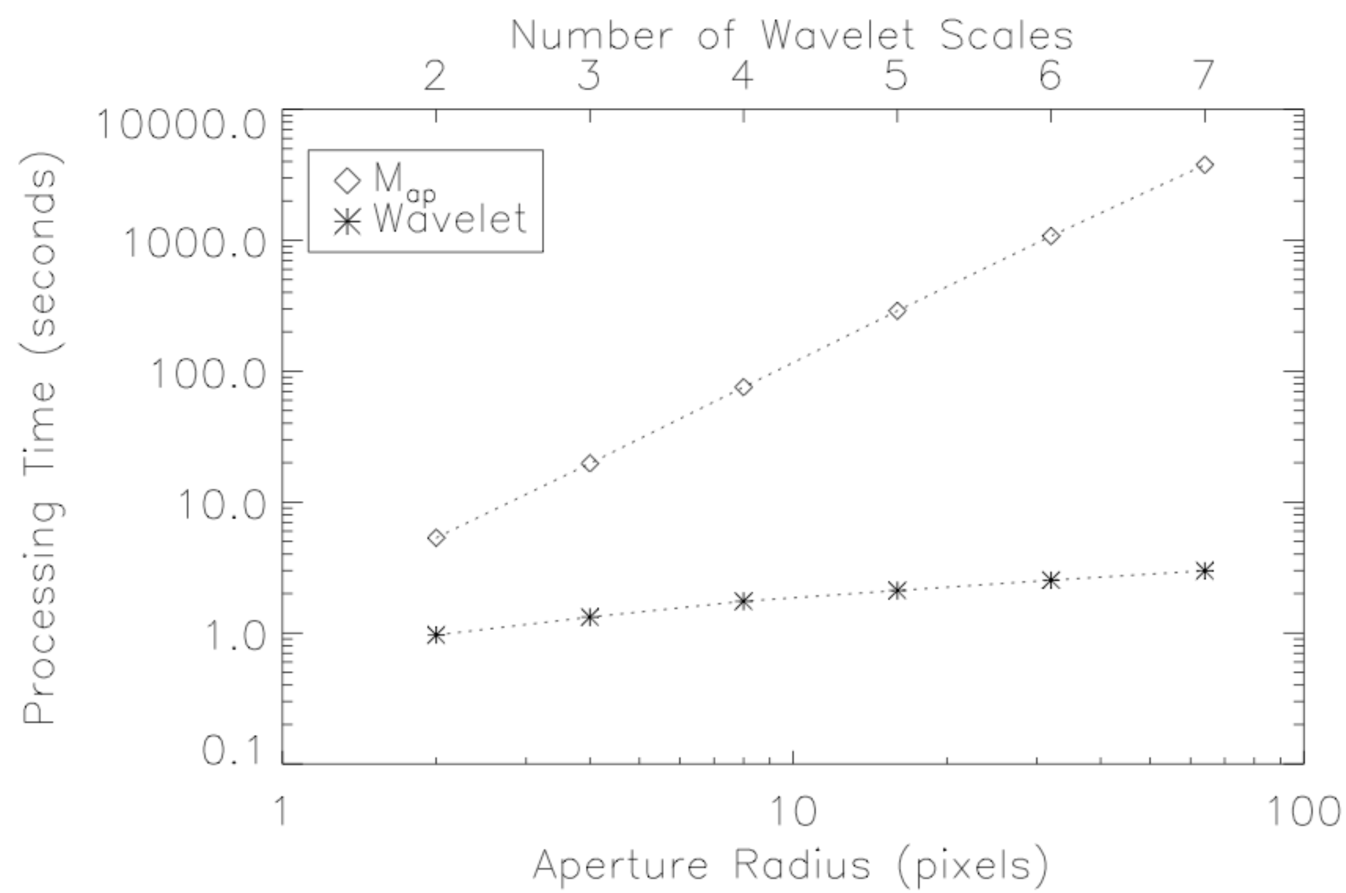
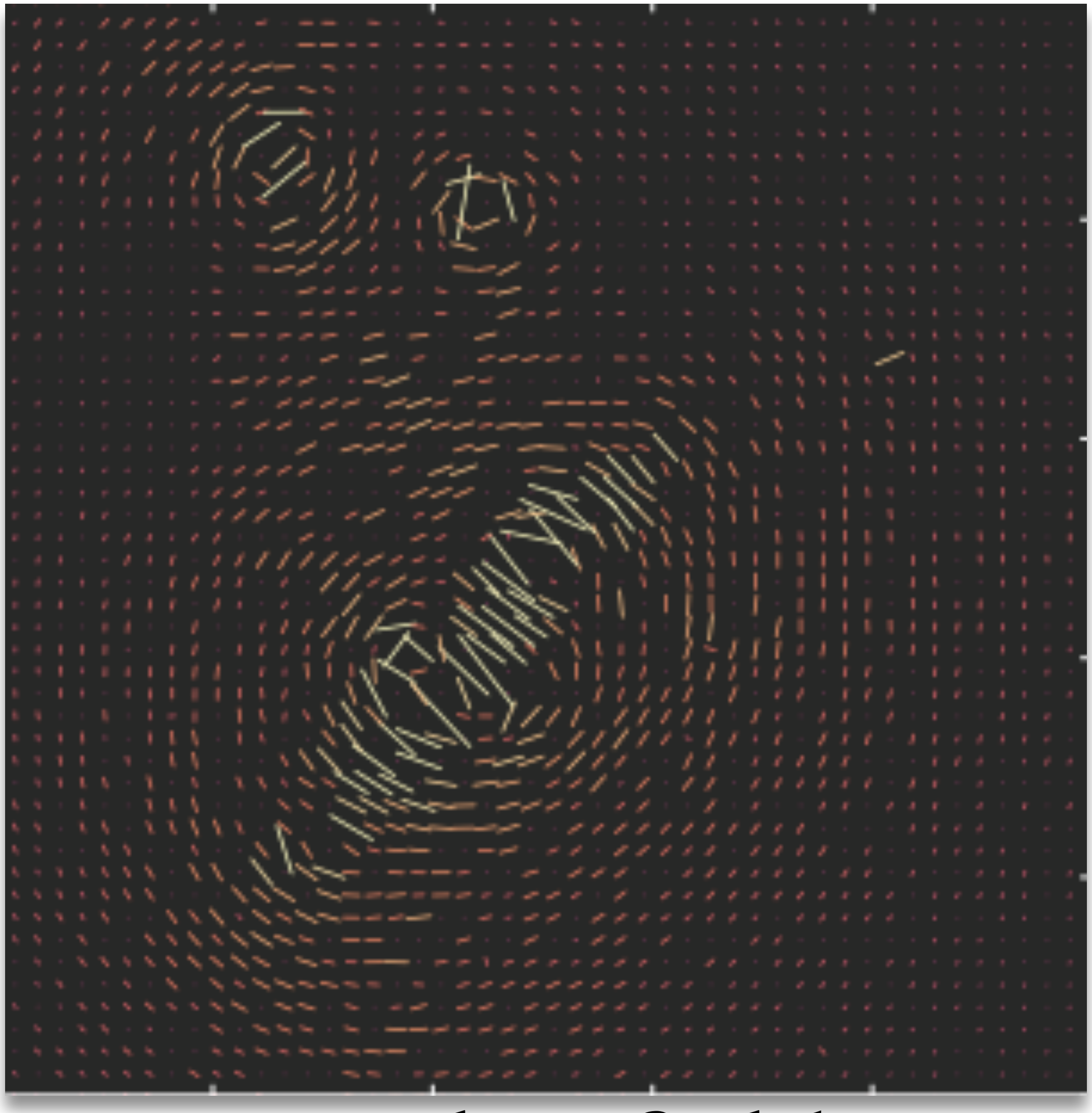


Shear

pixelated shear map



complex field

$$\gamma(\theta) = \gamma_1 + i\gamma_2$$

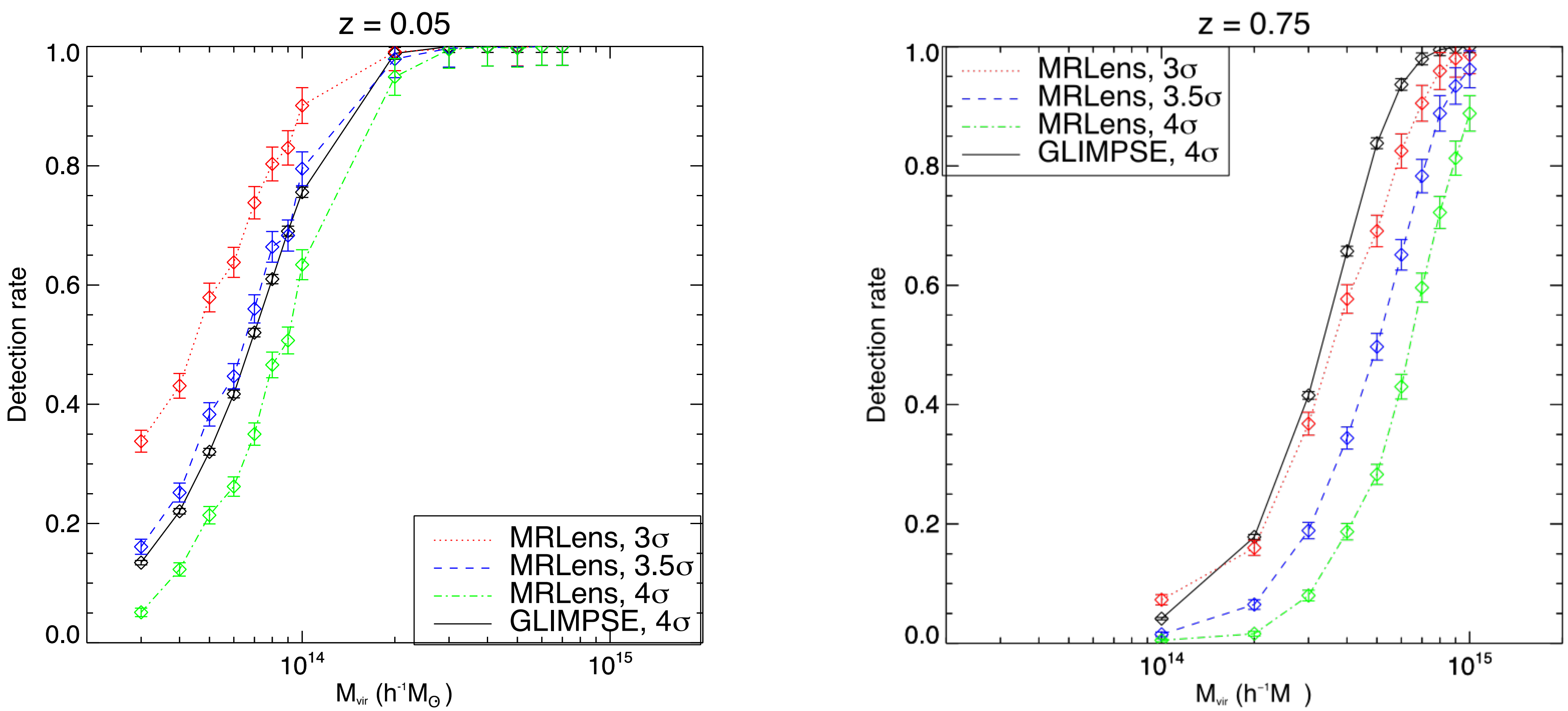
- Part I: Introduction to Weak Lensing Mass Mapping

- Part II: 2D Weak Lensing Mass Mapping

- Part III: 3D Mass Map Reconstruction

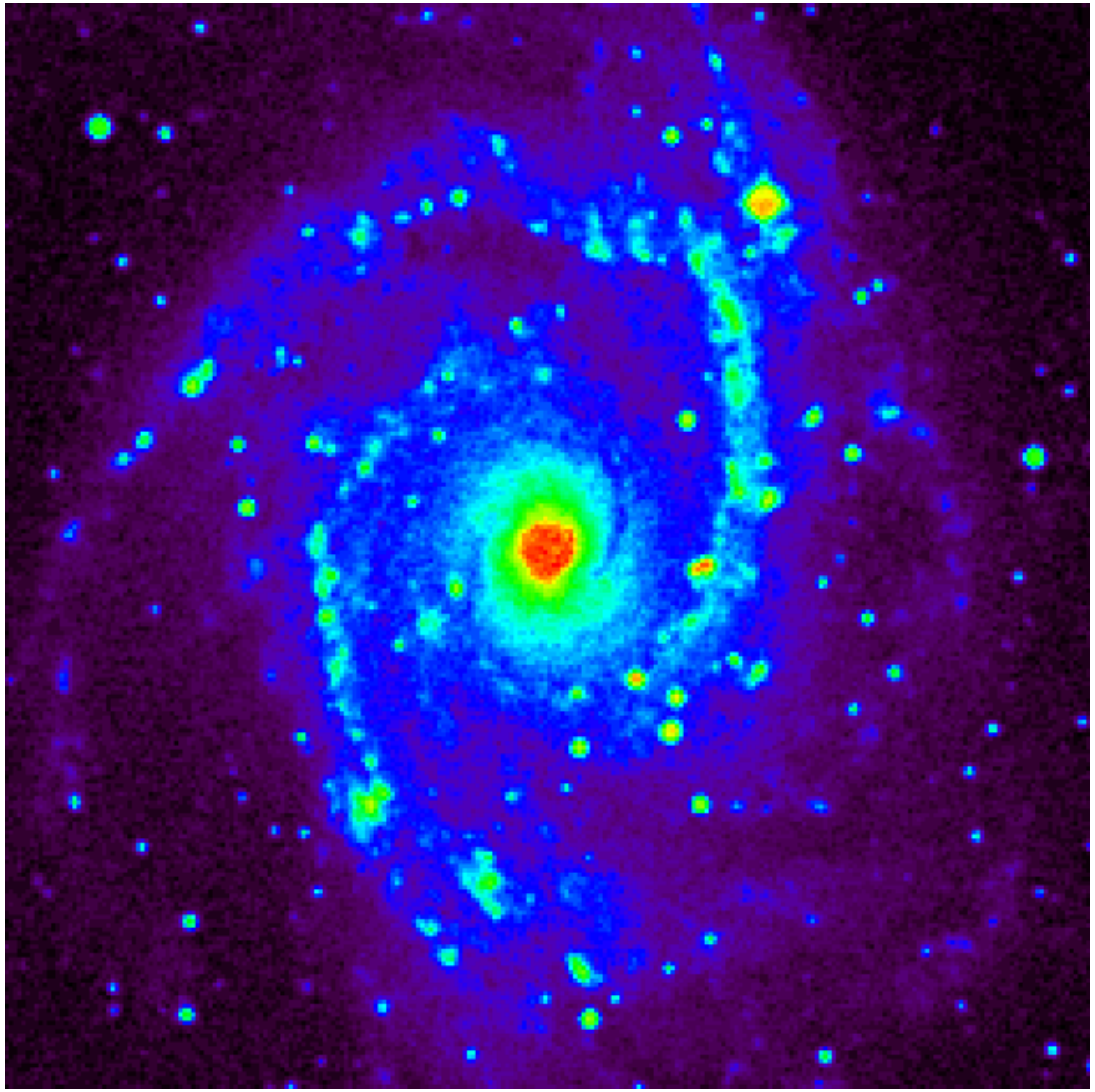
Cluster Detections: 2D vs 3D mapping

- 3D reconstructions (GLIMPSE) may offer an SNR advantage over 2D reconstructions (MRLens) for the detection of clusters.
- Improvement particularly significant at high redshift.

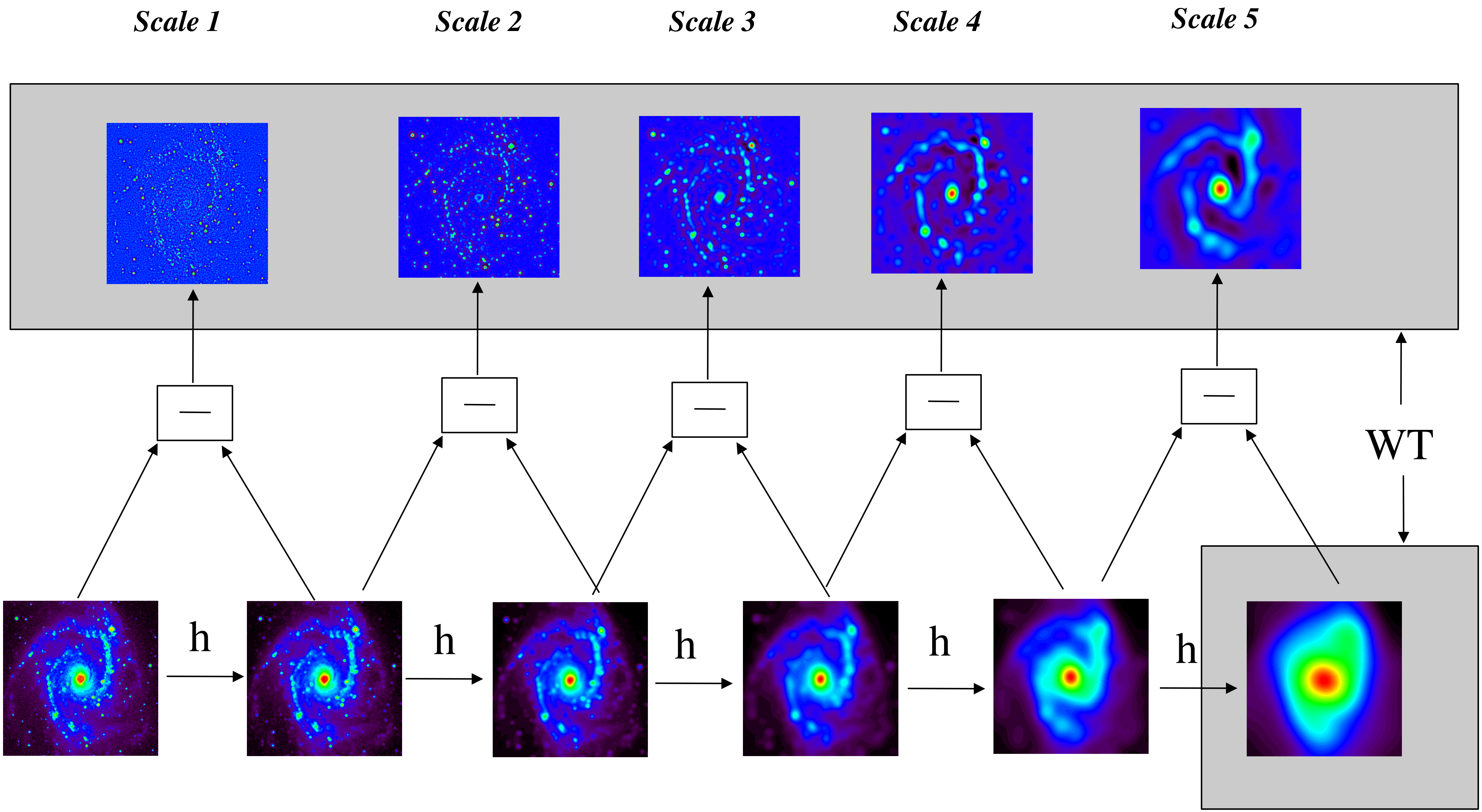


Leonard, Lanusse, & Starck 2015, MNRAS

NGC2997



ISOTROPIC UNDECIMATED WAVELET TRANSFORM

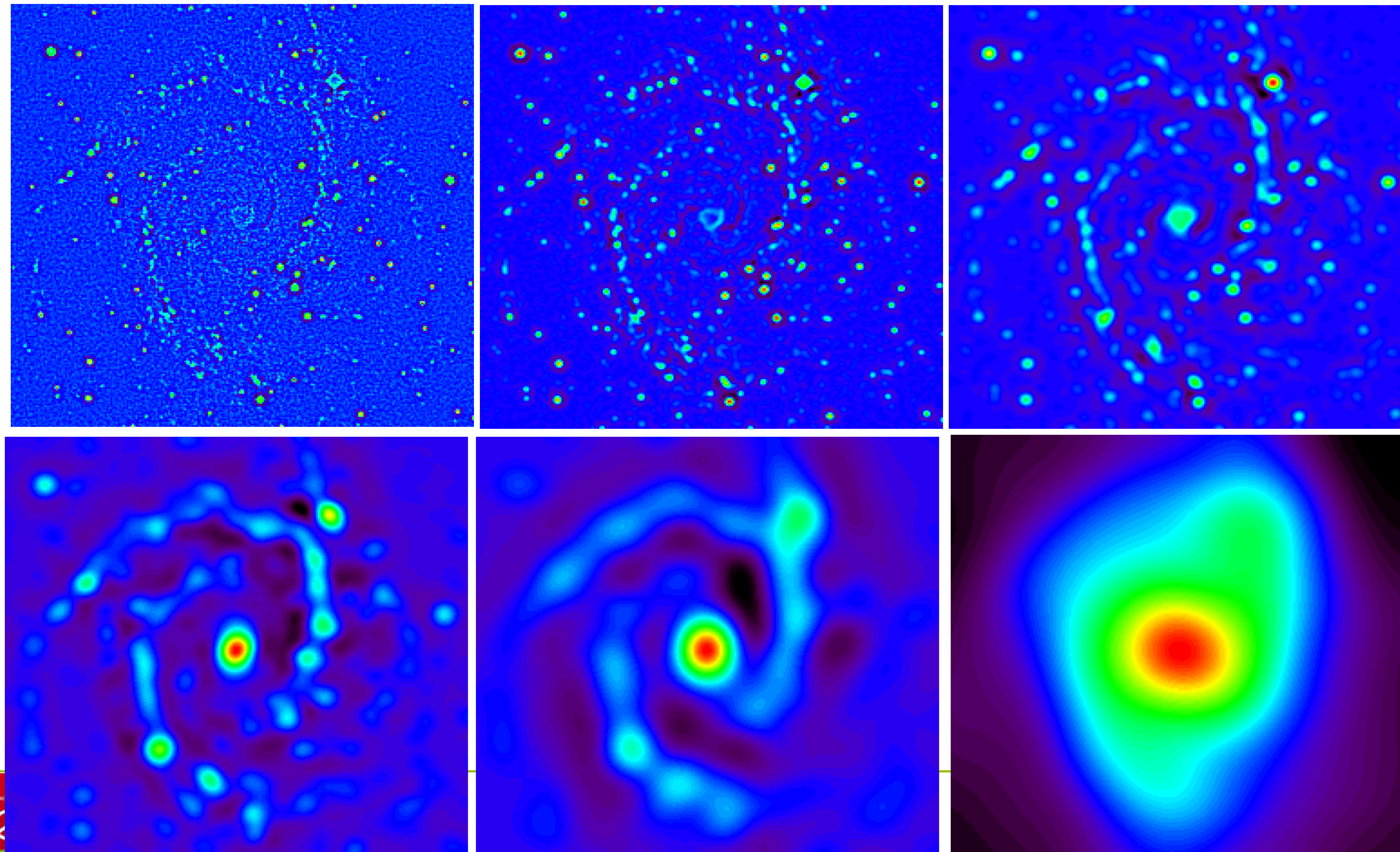


The STARLET Transform
Isotropic Undecimated Wavelet Transform (a trous algorithm)

$$\varphi = B_3 - \text{spline}, \quad \frac{1}{2}\psi\left(\frac{x}{2}\right) = \frac{1}{2}\varphi\left(\frac{x}{2}\right) - \varphi(x)$$

$$h = [1,4,6,4,1]/16, \quad g = \delta - h, \quad \tilde{h} = \tilde{g} = \delta$$

$$I(k,l) = c_{J,k,l} + \sum_{j=1}^{J} w_{j,k,l}$$



Redshift Information

Since, we are **not binning** the data, the framework can be expanded to include additional information, in particular **redshifts**.

==> Allows **cluster density mapping** (assuming knowledge of lens and sources redshifts).

$$\min_{\kappa} \frac{1}{2} \parallel (1 - \mathbf{Z}\kappa)g - \mathbf{Z}\mathbf{P}\kappa \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^t \kappa \parallel_1$$

$$\text{with } \mathbf{Z} = \Sigma^{\infty}_{critic} / \Sigma_{critic}(z_i)$$

$$\Sigma_{crit}(z_s) = \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

$$\Sigma^{\infty}_{crit} = \lim_{z \to \infty} \Sigma_{crit}(z)$$

Individual redshifts have two benefits:

- Directly map the **surface mass density** of the lens
- Mitigate the **mass-sheet degeneracy** when κ becomes significant (Bradac et al. 2004)

Simulations: 100 independent noise and galaxy distribution realisations

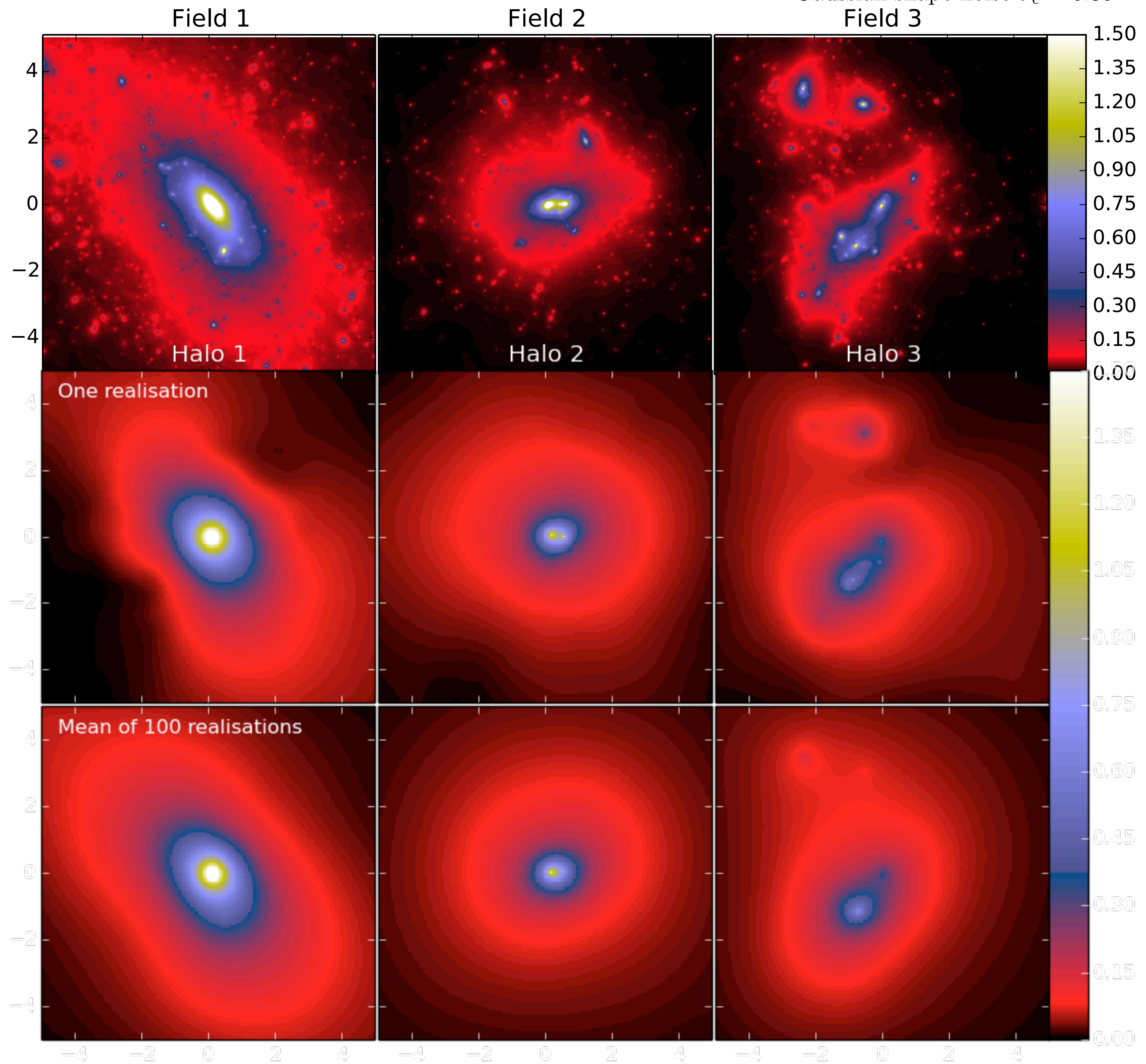
Density maps from Bolshoi simulations (Kyplin et al. 2011)

Virial masses $[10^{14}\ h^{-1}\mathrm{M}_{\odot}]$: 10.9 - 3.02 - 2.70

Cluster redshift: $z_{clus} = 0.30$

Field size: 10×10 arcmin$^2 \sim 2 \times 2\ h^{-2}\mathrm{Mpc}^2$

- Uniform angular distribution with $n_{gal} = 80$ gal/arcmin^2
- Redshift distribution $z_{med} = 1.75$
- Strongly lensed sources ($|g| > 1$) excluded
- Gaussian photo-z errors $\sigma_z = 0.05(1+z)$
- Gaussian shape noise $\sigma_\epsilon = 0.30$



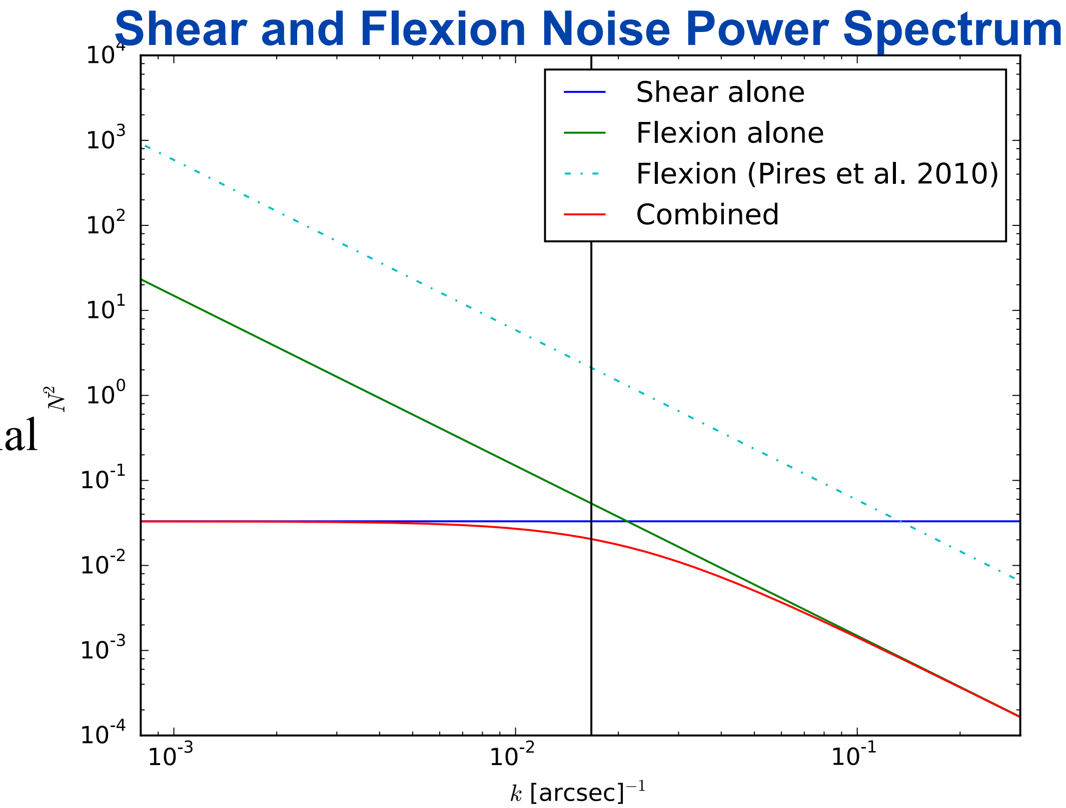
Flexion

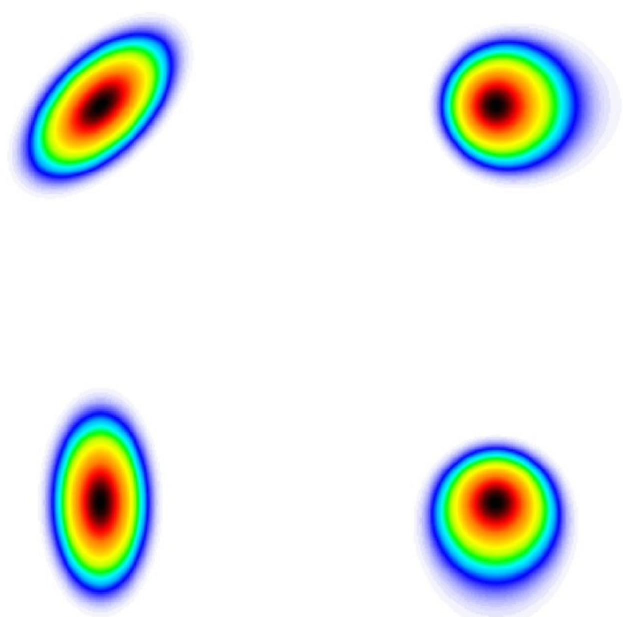
Shear is noise dominated on small scales ==> Substructures are lost

Small-scale substructure can be recovered from strong lensing when available.

Gravitational **Flexion** is useful in the intermediate regime.

Flexion gives information relative to the third order derivatives of the lensing potential





Shear (left) and first flexion (right) (Bartelmann 2010)

$$\mathcal{F} = \nabla \kappa$$

$$F = \frac{\mathcal{F}}{1-\kappa}$$

Flexion Information

We can integrate flexion in our reconstruction framework

=> **Jointly** fit shear and flexion

$$\min_{\kappa} \frac{1}{2} \parallel (1-\kappa) \begin{bmatrix} g \\ F \end{bmatrix} - \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} \kappa \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^t \kappa \parallel_1$$

=> **Jointly** fit shear and flexion with redshift information

$$\min_{\kappa} \frac{1}{2} \parallel (1-\mathbf{Z}\kappa) \begin{bmatrix} g \\ F \end{bmatrix} - \mathbf{Z} \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} \kappa \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^t \kappa \parallel_1$$

with $\mathbf{Z} = \Sigma_{critic}^{\infty} / \Sigma_{critic}(z_i)$

$$\Sigma_{crit}^{\infty} = \lim_{z \to \infty} \Sigma_{crit}(z)$$

$$\Sigma_{crit}(z_s) = \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

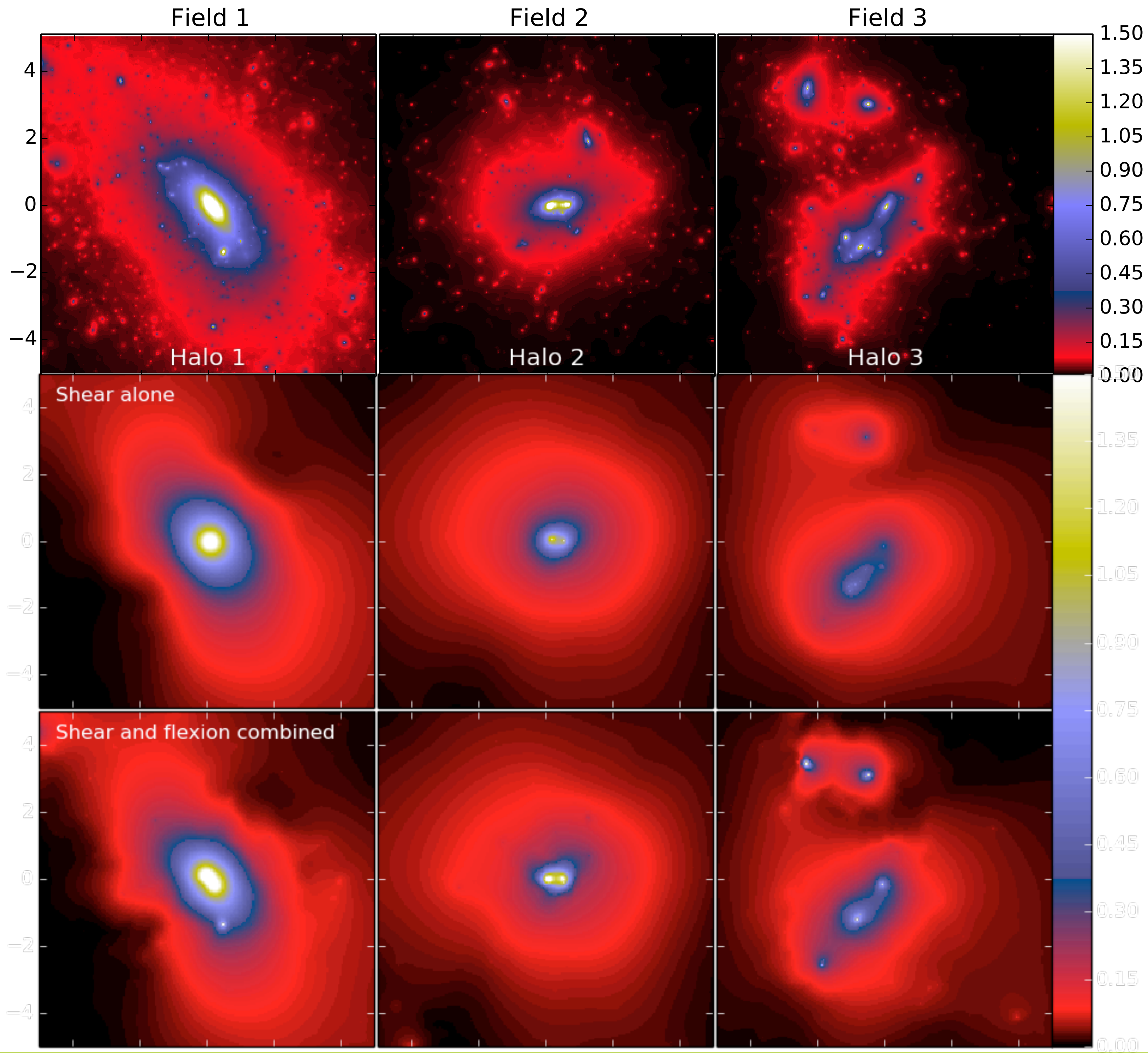
Individual redshifts have two benefits:

- Directly map the **surface mass density** of the lens
- Mitigate the **mass-sheet degeneracy** when κ becomes significant (Bradac et al. 2004)

Simulations with Flexion

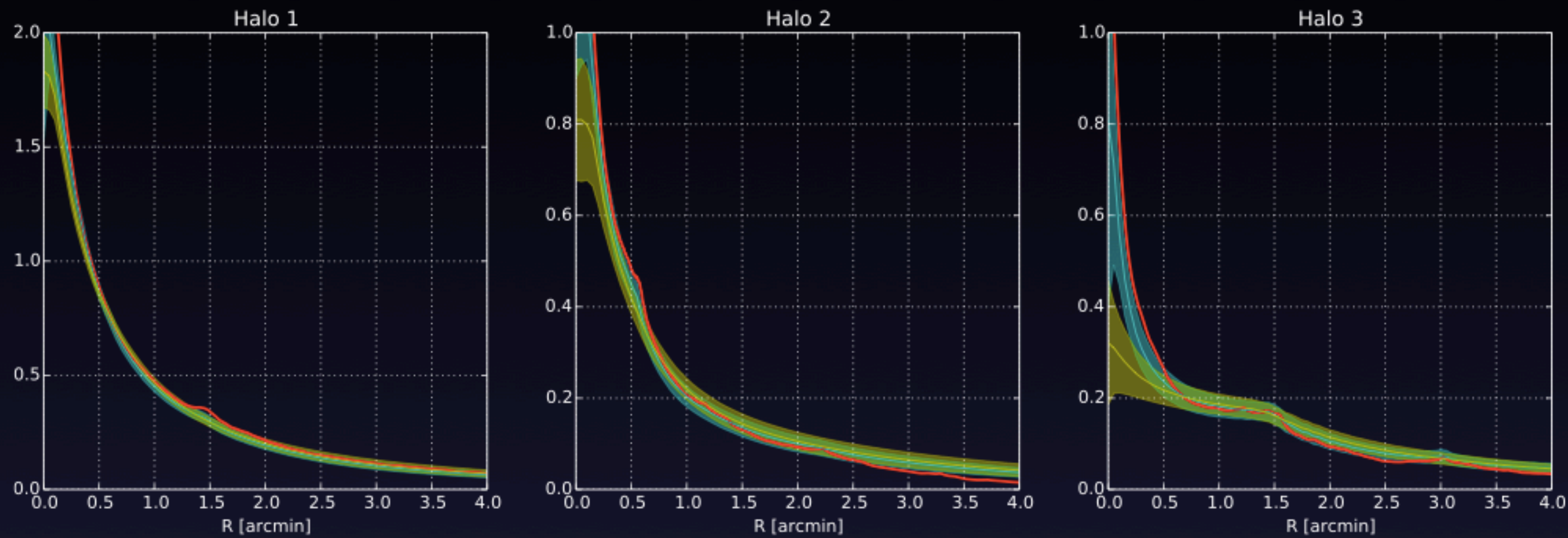
Simulate reduced flexion

Flexion noise $\sigma_F = 0.029$ arcsec^{-1} (Cain et al, 2011)



Reconstruction from one realisation

Flexion



Benefits of adding flexion:

- Improvement on the recovered profiles below 0.5 arcmin
- Recovery of small-scale substructure at the 10 arcsec scale

Weak Sparsity or Compressible Signals

A signal s (n samples) can be represented as sum of weighted elements of a given dictionary

Dictionary (basis, frame)

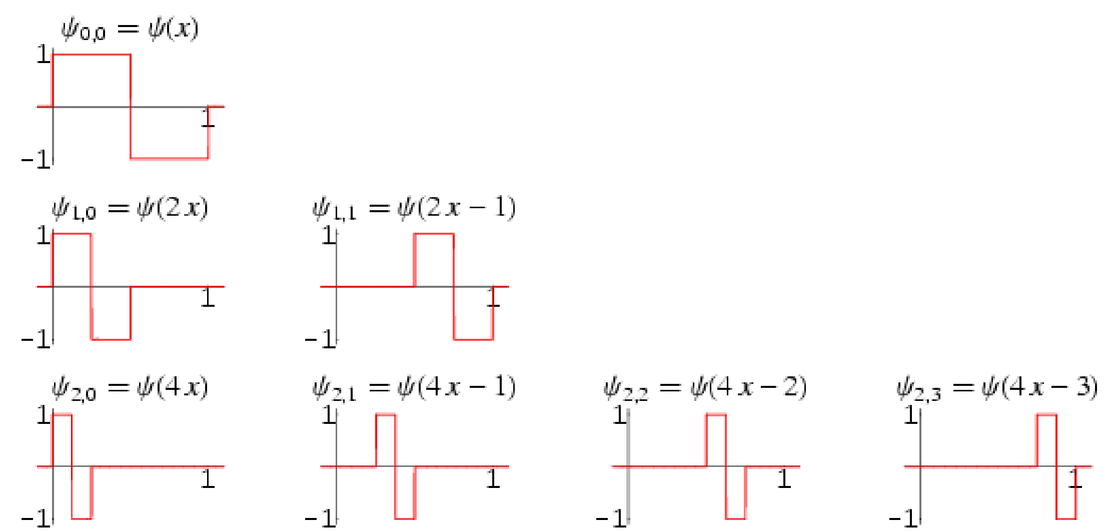
$$\Phi = \{\phi_1, \ldots, \phi_K\}$$

Atoms

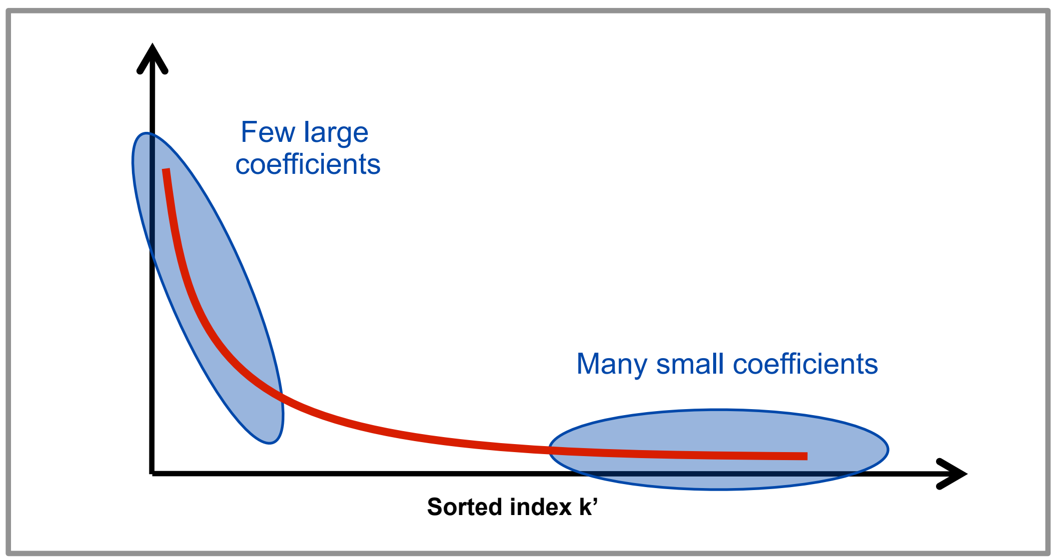
$$s = \sum_{k=1}^{K} \alpha_k \phi_k = \Phi\alpha$$

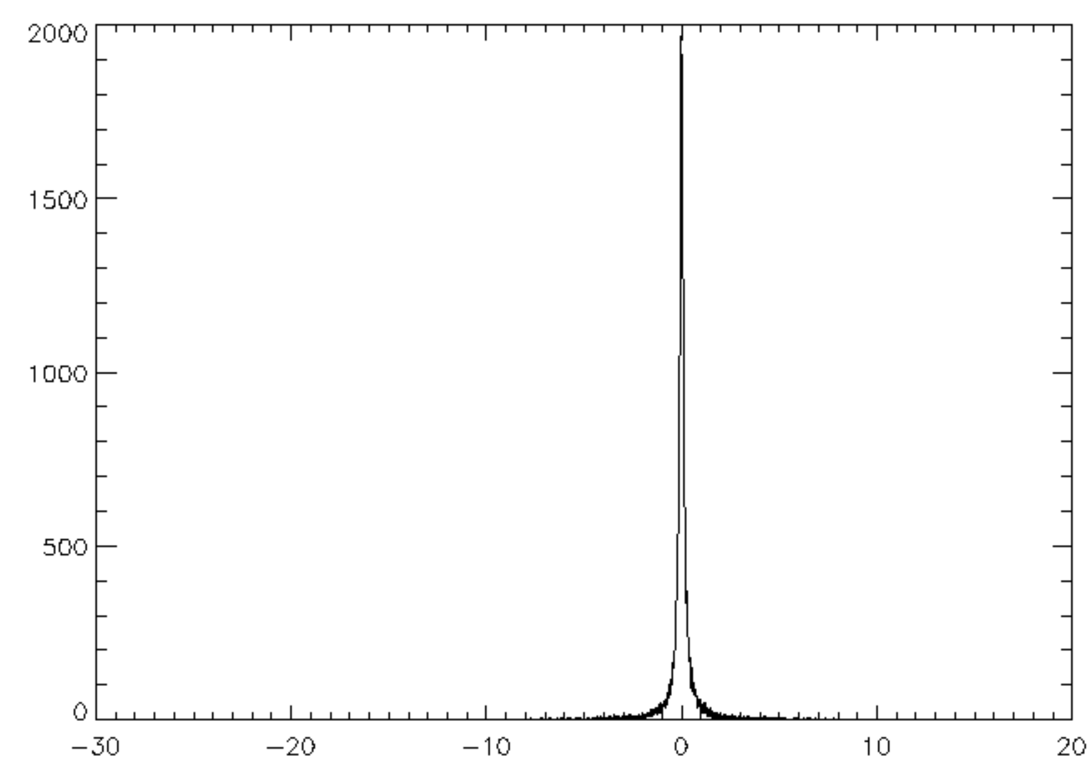
coefficients

Ex: Haar wavelet



$$f(x) = c_0 + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} c_{jk} \psi_{jk}(x).$$





- Fast calculation of the coefficients
- Analyze the signal through the statistical properties of the coefficients
- Approximation theory uses the sparsity of the coefficients