PSF modeling using a Graph manifold

Morgan A. Schmitz,
Jean-Luc Starck, Fred Ngolè
morgan.schmitz@cea.fr

French-Chinese Days on Weak Lensing

04/10/2018

Introduction

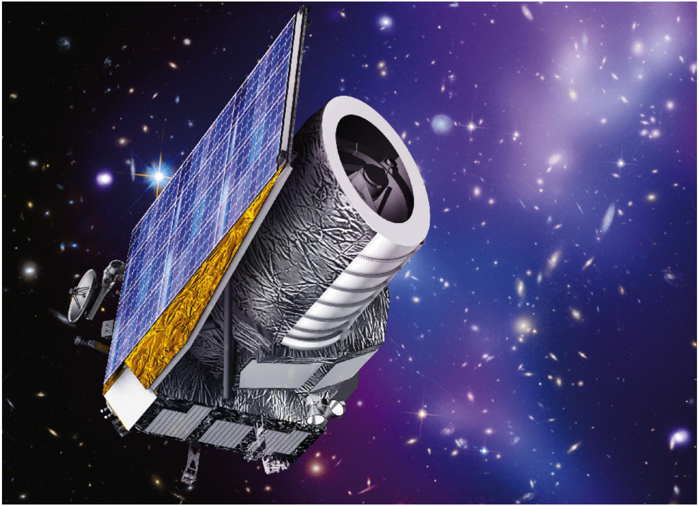
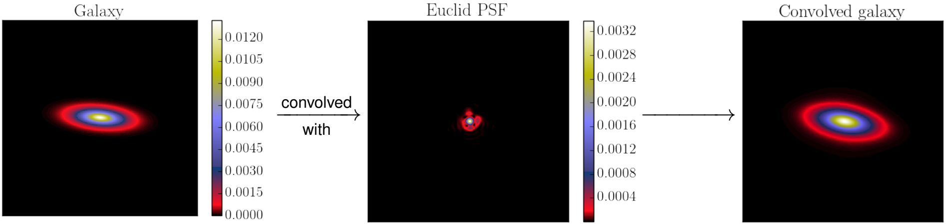
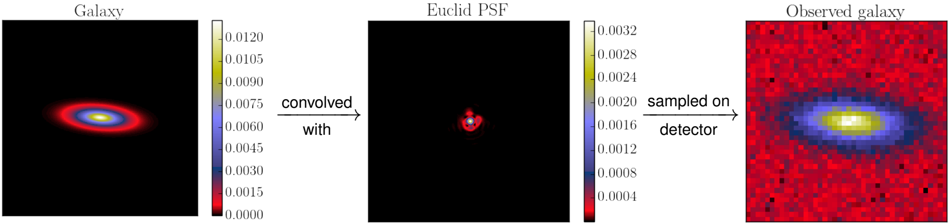


Figure: The Euclid mission (source: ESA)

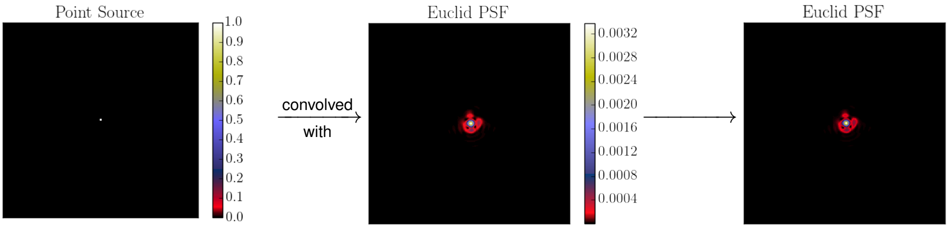
Introduction
Galaxy



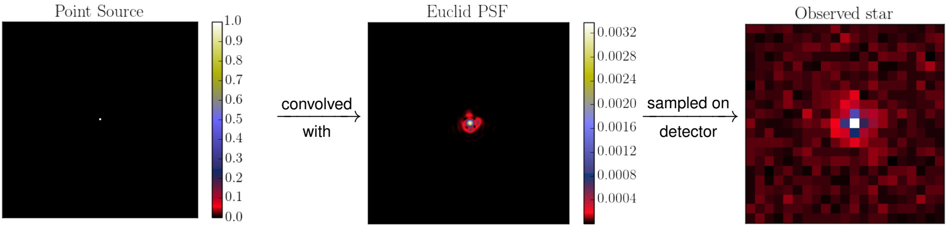
Introduction
Galaxy



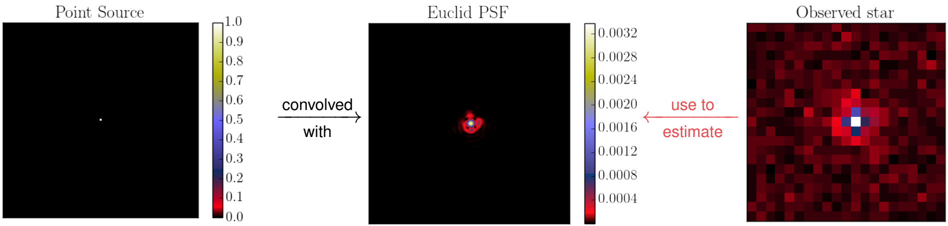
Introduction
Star



Introduction
Star



Introduction
Star



Introduction
Overview

- Euclid PSF field recovery entails:
  - PSF estimation at star positions
  - Spatial interpolation to galaxy positions
- We study the impact of errors made during those steps on shape measurement with image simulations

PSF estimation

Preprocessing

- $p \times p$ postage stamps around detected objects are extracted (*e.g.* with SExtractor) along with their positions $\mathcal{U} := (x_i, y_i)_i$ within field of view (FOV)
- Star-galaxy separation (*e.g.* through size-magnitude, Gaia catalog, ...) leads to clean sample of stars
- We consider each star image as a flat vector $Y_i \in \mathbb{R}^{p^2}$

PSF estimation
Observation model

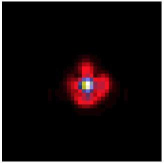
- Given galaxy positions $\mathcal{U}_G = \{(x_1, y_1), (x_2, y_2), \dots\}$, we want an estimator $\hat{H}(x_j, y_j)$ of the true PSF $H(x_j, y_j)$
- Observations are noisy, undersampled stars $(Y_i)_{i\in\mathcal{U}_S}$ at positions $\mathcal{U}_S \neq \mathcal{U}_G$:

$$Y_i = M_i H(x_i, y_i) + N_i$$

- N_i is white gaussian noise, M_i is degradation operator

PSF estimation
Degradation operator

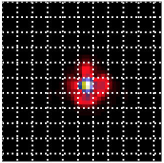
- M_i contains both decimation and object-specific shift:



H

PSF estimation
Degradation operator

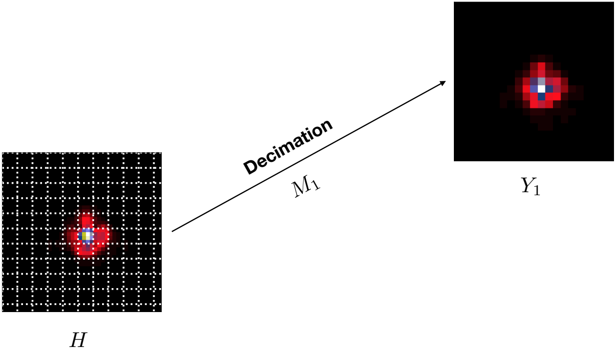
- M_i contains both decimation and object-specific shift:



H

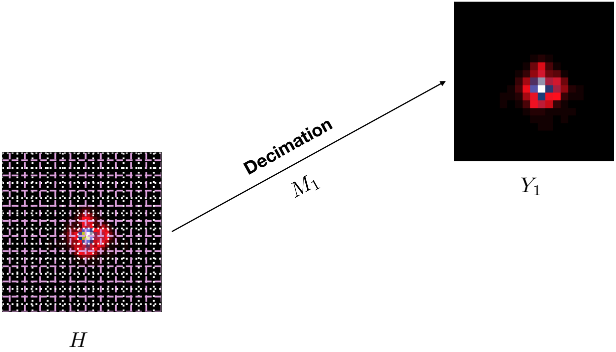
PSF estimation
Degradation operator

- M_i contains both decimation and object-specific shift:



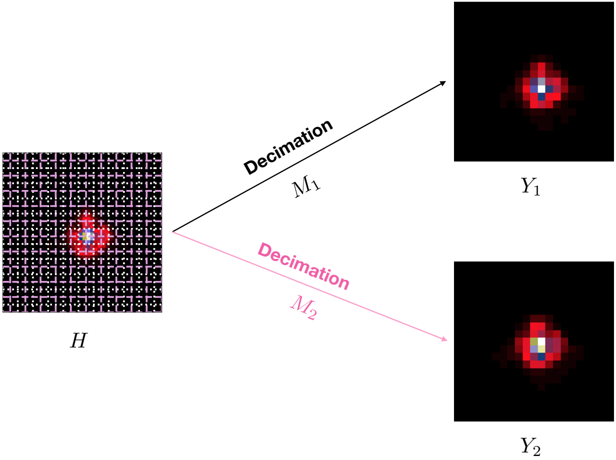
PSF estimation
Degradation operator

- M_i contains both decimation and object-specific shift:



PSF estimation
Degradation operator

- M_i contains both decimation and object-specific shift:



PSF estimation
Observation model

Observation model

$$Y = MH + N$$

where

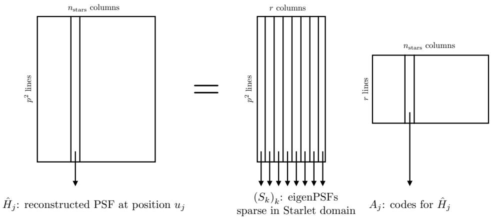
- $Y = (Y_i)_{i \in \{1,\dots,n_{\text{stars}}\}}$, $M = (M_i)_{i \in \{1,\dots,n_{\text{stars}}\}}$, $N = (N_i)_{i \in \{1,\dots,n_{\text{stars}}\}}$
- $H = (H(x_i, y_i))_{(x_i,y_i) \in \mathcal{U}_S}$
- We must both:
  - Counteract the effect of M (superresolution);
  - Interpolate from $\mathcal{U}_S$ to $\mathcal{U}_G$ (spatial interpolation).

Superresolution
Data-driven approaches

Learn Dictionary S and codes (A_i) such that $Y_i \approx M_i(SA_i)$

- PSFEx (Bertin, 2011)
- RCA (Ngolè et al., 2016): Enforce several constraints on S and $A = \alpha V^\top$ to reflect known properties of the PSF

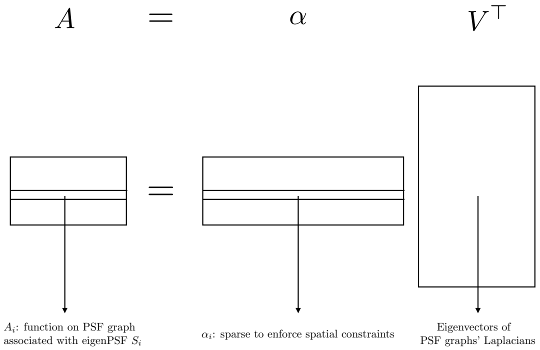
$$\hat{H} \quad = \quad S \quad A$$



Superresolution
Data-driven approaches

Learn Dictionary S and codes (A_i) such that $Y_i \approx M_i(SA_i)$

- PSFEx (Bertin, 2011)
- RCA (Ngolè et al., 2016): Enforce several constraints on S and $A = \alpha V^\top$ to reflect known properties of the PSF



Superresolution

Resolved Component Analysis (RCA)

$$\min_{S,\alpha} \frac{1}{2}\|Y - MS\alpha V^\top\|_2^2 + \iota_\Omega(\alpha) + \iota_+(S,\alpha) + \sum_{i=1}^{r} \|w_i \odot \Phi s_i\|_1 \tag{1}$$

- Dimensionality reduction: PSF variations across the field are smooth enough (at least locally) to be captured through a small number of eigenPSFs $r << p$;
- Graph constraints: the smaller the difference between two PSFs' positions u_i, u_j, the smaller the difference between their representations $\hat{H}_i, \hat{H}_j$ should be;
- Positivity: the PSF should only contain positive pixel values
- Sparsity: the PSF should have a sparse representation in an appropriate basis.

Superresolution
Resolved Component Analysis (RCA)

$$\min_{S,\alpha} \frac{1}{2}\|Y - MS\alpha V^\top\|_2^2 + \iota_\Omega(\alpha) + \iota_+(S,\alpha) + \sum_{i=1}^{r} \|w_i \odot \Phi s_i\|_1 \quad (1)$$

- Dimensionality reduction: PSF variations across the field are smooth enough (at least locally) to be captured through a small number of eigenPSFs $r << p$;
- Graph constraints: the smaller the difference between two PSFs' positions u_i, u_j, the smaller the difference between their representations $\hat{H}_i, \hat{H}_j$ should be;
- Positivity: the PSF should only contain positive pixel values
- Sparsity: the PSF should have a sparse representation in an appropriate basis.

Superresolution

Resolved Component Analysis (RCA)

$$\min_{S,\alpha} \frac{1}{2}\|Y - MS\alpha V^\top\|_2^2 + \iota_\Omega(\alpha) + \iota_+(S,\alpha) + \sum_{i=1}^{r} \|w_i \odot \Phi s_i\|_1 \tag{1}$$

- Dimensionality reduction: PSF variations across the field are smooth enough (at least locally) to be captured through a small number of eigenPSFs $r << p$;
- Graph constraints: the smaller the difference between two PSFs' positions u_i, u_j, the smaller the difference between their representations $\hat{H}_i, \hat{H}_j$ should be;
- Positivity: the PSF should only contain positive pixel values
- Sparsity: the PSF should have a sparse representation in an appropriate basis.

Superresolution

Resolved Component Analysis (RCA)

$$\min_{S,\alpha} \frac{1}{2}\|Y - MS\alpha V^\top\|_2^2 + \iota_\Omega(\alpha) + \iota_+(S,\alpha) + \sum_{i=1}^{r} \|w_i \odot \Phi s_i\|_1 \tag{1}$$

- Dimensionality reduction: PSF variations across the field are smooth enough (at least locally) to be captured through a small number of eigenPSFs $r << p$;
- Graph constraints: the smaller the difference between two PSFs' positions u_i, u_j, the smaller the difference between their representations $\hat{H}_i, \hat{H}_j$ should be;
- Positivity: the PSF should only contain positive pixel values
- Sparsity: the PSF should have a sparse representation in an appropriate basis.

Superresolution

Resolved Component Analysis (RCA)

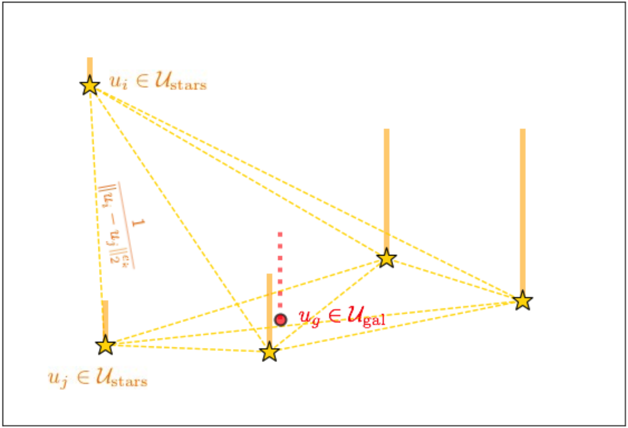
$$\min_{S,\alpha} \frac{1}{2}\|Y - MS\alpha V^\top\|_2^2 + \iota_\Omega(\alpha) + \iota_+(S,\alpha) + \sum_{i=1}^{r} \|w_i \odot \Phi s_i\|_1 \tag{1}$$

- Dimensionality reduction: PSF variations across the field are smooth enough (at least locally) to be captured through a small number of eigenPSFs $r << p$;
- Graph constraints: the smaller the difference between two PSFs' positions u_i, u_j, the smaller the difference between their representations $\hat{H}_i, \hat{H}_j$ should be;
- Positivity: the PSF should only contain positive pixel values
- Sparsity: the PSF should have a sparse representation in an appropriate basis.

Graph constraints

- Spatial variations can either be localized or occur slowly across the whole field
- Each eigenPSF S_k should capture information linked to a certain spatial frequency
- Consider a set of graphs with edges between points i and j weighted by $1/\|u_i - u_j\|_2^{e_k}$
- We enforce the spatial constraints by further factorizing A by the set of eigenvectors of our graphs' Laplacians, and forcing the coefficients to be sparse

Graph constraints
Illustration

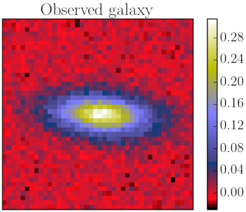


Spatial Interpolation

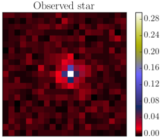
- RCA gives us estimators of the PSF at star positions $\hat{H}(x_i, y_i) = S\alpha_i V^\top$
- Spatial interpolation must then be carried out to obtain desired PSFs at galaxy positions $\left(\hat{H}(x_j, y_j)\right)_{j\in\mathcal{U}_G}$
- Dictionary S and graph eigenvectors $V^\top$ contain information about the PSF and its spatial variations by very construction
- Perform spatial interpolation *within* the RCA-learned graphs to stay within the correct manifold

Simulation set-up
Galaxies

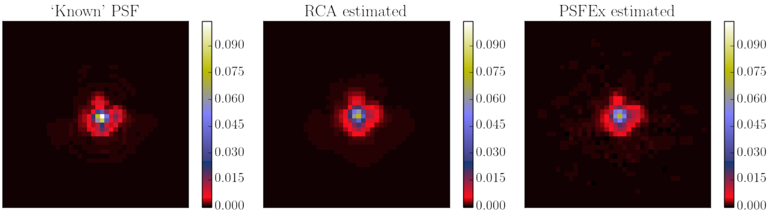
- Simulate GREAT3-like galaxies and convolve them with true, unobserved Euclid PSF with GalSim (Rowe et al., 2015): ~2,000,000 galaxies with 204 different shear values
- Sample them at twice Euclid resolution and add noise



Simulation set-up
Stars and PSF models



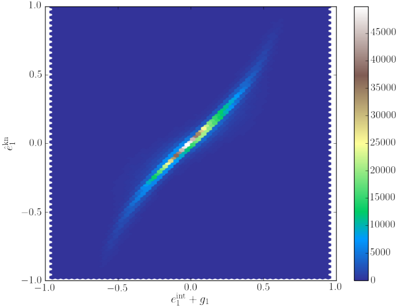
- Estimate PSF: "known", RCA+RBF, PSFex
- Apply actual shape measurement method with all three



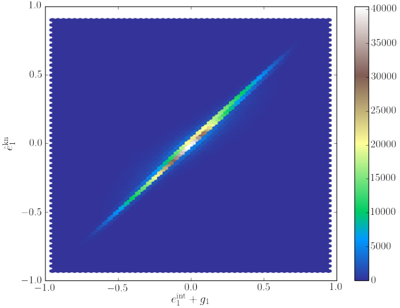
Shape measurement

- Two types of shape measurement approaches: KSB (Kaiser et al., 1995; Hirata & Seljak, 2003) and im3shape (Zuntz et al., 2013)
- Yields per-object estimator of the shape: $\hat{e}_j \approx e_j^{\mathrm{int}} + g_j$
- For each data-driven model of PSF, we can look at *relative* error on the shape itself: $\langle (|\hat{e}^{\mathrm{kn}}| - |\hat{e}|)^2 \rangle$
- Or estimate shear and look at shear bias: $\hat{g} = \langle \hat{e} \rangle \approx \langle e^{\mathrm{int}} \rangle + \langle g \rangle = g$

Shape measurement
Ellipticity distributions

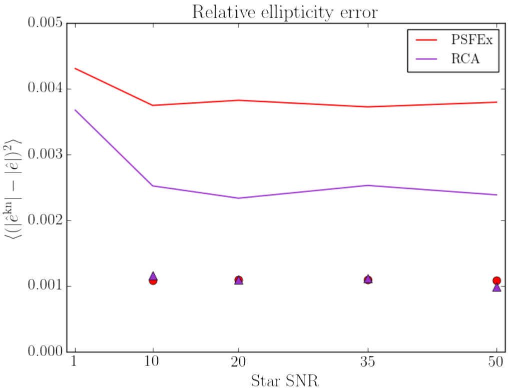


(a) KSB



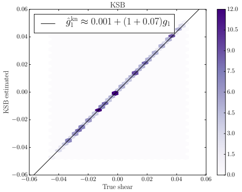
(b) im3shape

Impact on shape measurement
Relative ellipticity error

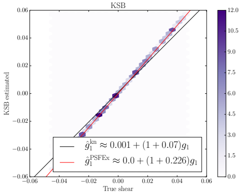


- KSB: RCA yields ~35% improvement over PSFEx
- Model fitting less sensitive to PSF modelling (no model bias!)

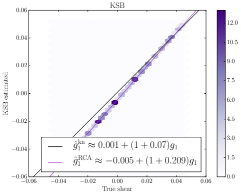
Impact on shear bias
KSB



(a) "Known" PSF



(b) PSFEx estimated



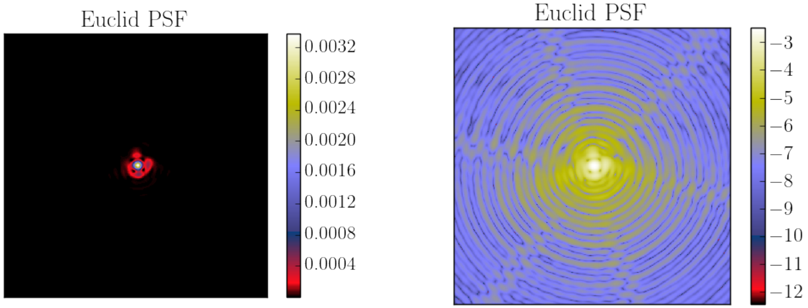
(c) RCA estimated

Summary

- Data-driven PSF estimation: build PSF model using only images
- Model fitting seems less sensible to PSF misspecifications
- On moments-based method, RCA-based approach yields 35% relative improvement over PSFEx in shape measurement quality
- Perspective: chromatic dependency (see Rebeca's talk tomorrow)
- Acknowledgements and funding:

morgan.schmitz@cea.fr
http://www.cosmostat.org/people/mschmitz/

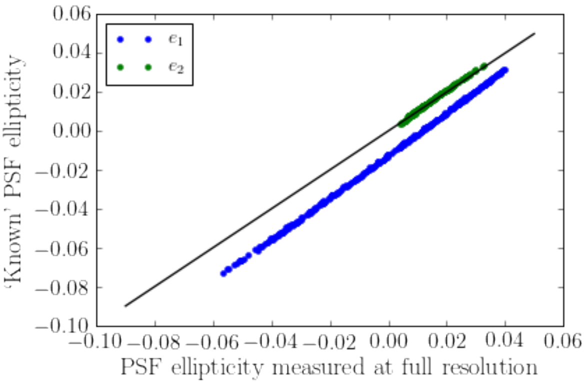
Euclid PSFs
Natural and log



Paulin propagation

- Paulin-Henriksson et al., 2008, eq (13):

$$\delta e = \hat{e} - e = (e - e_{\mathrm{PSF}}) \left(\frac{R^2_{\mathrm{PSF}}}{R^2_{\mathrm{gal}}} \right) \frac{\delta R^2_{\mathrm{PSF}}}{R^2_{\mathrm{PSF}}} - \left(\frac{R^2_{\mathrm{PSF}}}{R^2_{\mathrm{gal}}} \right) \delta e_{\mathrm{PSF}}$$



Appendix

4 Data-driven and optical PSF models

5 Superresolution & spatial interpolation

6 Impact on shape measurement (additional results)

Introduction

Data-driven PSF estimation: motivation

- PSF estimation can be carried out:
  - by fitting physically-motivated model to observed stars
  - by relying solely on the data (this talk)
- Always a good idea to have two independent methods for a specific problem:
  - Validation
  - Combination
- Data-driven approach may capture effects missed by physical models
- And should be less sensitive to unexpected surprises

Appendix

Superresolution

Data-driven approaches

Learn Dictionary S and codes (A_i) such that $Y_i \approx \mathcal{F}(SA_i)$

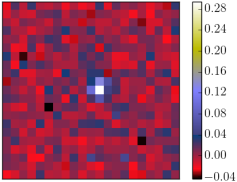
- PSFEx (Bertin, 2011):

$$A_i = \left(x_i^p y_j^q\right)_{p+q \leq d}$$

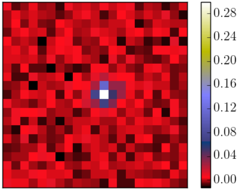
- RCA (Ngolè et al., 2016): Enforce several constraints on S and $A = \alpha V^\top$ to reflect known properties of the PSF

Observed stars

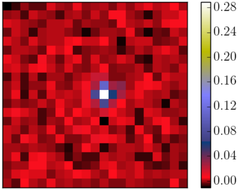
Noise levels



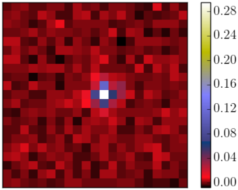
(a) Star at SNR 1



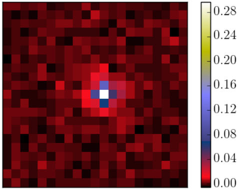
(b) Star at SNR 10



(c) Star at SNR 20



(d) Star at SNR 35



(e) Star at SNR 50

Superresolution

Usual approach

- Several exposures of same object available: random intra-pixel shifts lead to capture of different parts of the information
- Use that information to perform superresolution
- SPRITE (Ngolè et al., 2015): use sparsity to superresolve and denoise Euclid-like PSF
- Problem: different Euclid exposures of same star might not measure the same $H(x, y)$

Spatial Interpolation

Radial basis function interpolation

- Radial basis functions (RBF): kernels Φ that only depend on the distance between two observations
- Typically, $\Phi(A_{ik}, A_{ik'}) = \varphi(\|u_k - u_{k'}\|)$
- Thin-lens kernel: $\varphi : r \mapsto r^2 \log(r)$
- Use star neighborhood U_j of galaxy position u_j to estimate

$$\hat{A}_{ik} = \sum_{u \in U_j} w_u \varphi\left(\|u - u_k\|\right)$$

Appendix

Impact on shape measurement
Absolute ellipticity error

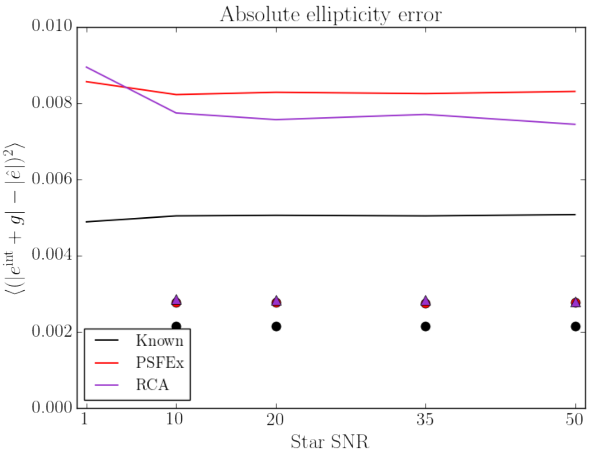


Figure: Absolute ellipticity errors as a function of star SNR

Impact on shape measurement
Relative ellipticity error (1st component)

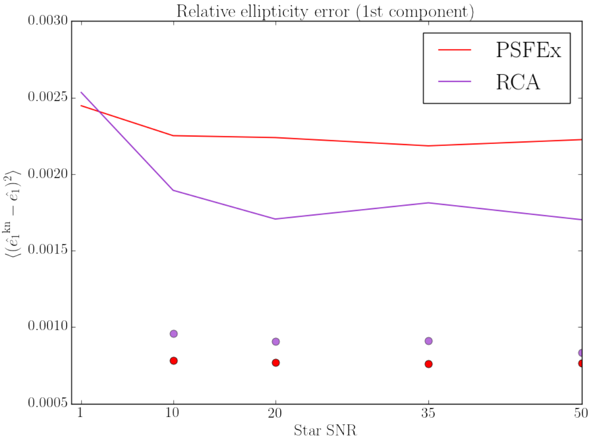


Figure: Relative e_1 errors as a function of star SNR

Impact on shape measurement

Relative ellipticity error (2nd component)

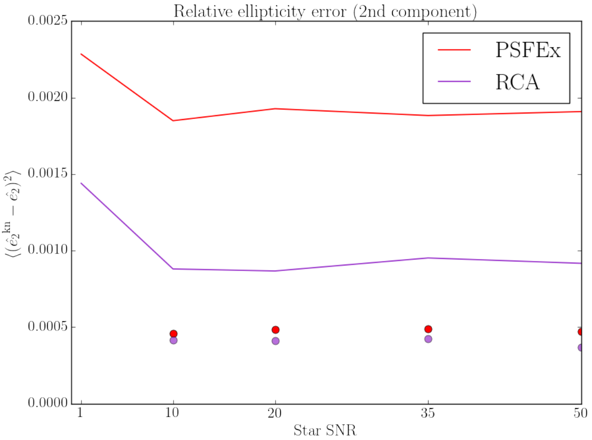


Figure: Relative e_2 errors as a function of star SNR

Impact on shape measurement
Multiplicative bias per SNR

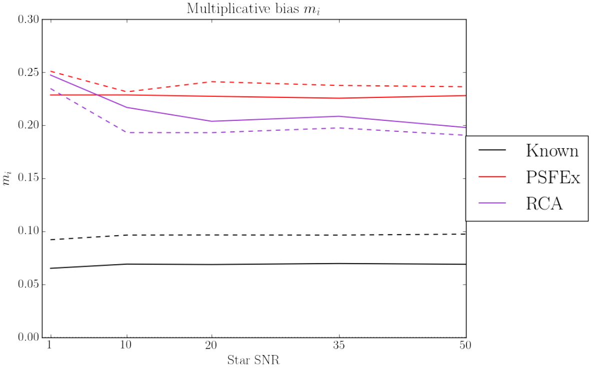


Figure: m as a function of star SNR

Impact on shape measurement
Additive bias per SNR

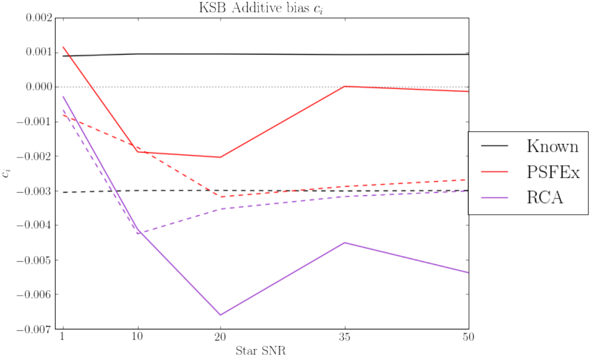
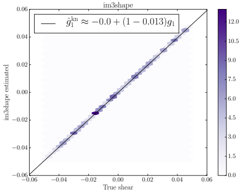
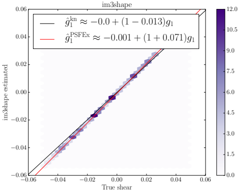


Figure: c as a function of star SNR

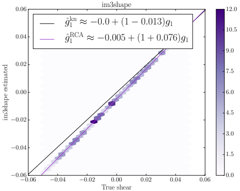
Impact on shear bias
im3shape



(a) "Known" PSF



(b) PSFEx estimated



(c) RCA estimated