Weak Gravitational Lensing
TD

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Exercises

Coding

Limber equation (cycle 1)

Data analysis

Compute the shear two-point correlation function (2PCF) (cycle 1+2)

Calculations

Convergence and shear (cycle 1)

Galaxy-galaxy lensing (cycle 2)

Code up Limber equation in python I

Getting the 3D power spectrum

Run CLASS to get 3D power spectrum at various redshifts:

```
> mkdir output
> /path/to/class lcdm.ini
```

This creates files `output/test_z<N>_pk_nl.dat`, where `<N>` corresponds to redshift $0.1 \times N$, due to the definition of the redshift keyword in `lcdm.ini`:

```
z_pk = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1
```

Run python program:

```
> ./limber.py
```

This reads in CLASS output P_δ files, creates splines to interpolate to arbitrary k and stores them in an array (for the redshift).
It produces a test message:

```
Test: P_delta(k=0.5 h/Mpc, z=0.3) = 600.998245036
```

Check in the corresponding P_δ file whether this value makes sense.

Code up Limber equation in python II

Here is the code snippet:

```
from astropy.io import ascii

p_delta = []
for iz in range(nz):
    p_delta_name = '{0}z{1}_pk_nl.dat'.format(root, iz+1)
    print(p_delta_name)
    dat = ascii.read(p_delta_name)
    if iz == 0:
        k = dat['col1']
    pk = dat['col2']
    this_p_delta = \
        interpolate.InterpolatedUnivariateSpline(k, pk)
    p_delta.append(this_p_delta)
```

If `astropy` is not installed, use instead:

```
dat = np.loadtxt(p_delta_name)
k = dat[:,0]
pk = dat[:,1]
```

Code up Limber equation in python III

Writing down Limber equation for a simple case

Now, let's look at the Limber equation, as shown in the lecture before:

$$
\begin{aligned}
P_\kappa(\ell) &= \int \mathrm{d}\chi \, G^2(\chi) P_\delta \left(k = \frac{\ell}{\chi} \right) \\
G(\chi) &= \frac{3}{2} \left(\frac{H_0}{c} \right)^2 \frac{\Omega_\mathrm{m}}{a(\chi)} \int_\chi^{\chi_\mathrm{lim}} \mathrm{d}\chi' \, p(\chi') \frac{\chi' - \chi}{\chi'}
\end{aligned}
$$

First, to simplify, let's assume all galaxies are at a single redshift z_0, or comoving distance χ_0.
The pdf becomes a Dirac delta function, $p(\chi') = \delta_\mathrm{D}(\chi' - \chi_0)$.
Solve the integral in G.

Code up Limber equation in python IV

Next, since CLASS outputs P_δ as function of redshift and not comoving distance, we have to change variables from χ to z.

Start with the FLRW metric and the equation for geodesics, $\mathrm{d}s = 0$ (see TD by M. Kunz):

$$\mathrm{d}s^2 = 0 = c^2 \mathrm{d}t^2 - a^2 \mathrm{d}\chi^2$$

and write $\mathrm{d}\chi$ as function of $\mathrm{d}z$.

You will need the Hubble expansion rate $H(z)$. Assume a flat ΛCDM model.

Finally, write down P_κ as integral over z over the density power spectrum. Check the units of the involved quantities.

Code up Limber equation in python V

Numerical integration of the Limber equation

Discretise the above integral. In `python`, write a function that performs this discrete sum, evaluating P_δ at the redshifts that are output by CLASS.
To get the comoving distance as function of redshift, use the code from the general cosmolgy TD, or a package such as `astropy`:

```
def chi(z, Omega_m, h):
    """Return comoving distance in units of Mpc/h
    """

    from astropy import cosmology
    cosmo  = cosmology.FlatLambdaCDM(H0=100*h, Om0=Omega_m)
    # Multiply with h to go from Mpc to Mpc/h
    chi    = cosmo.comoving_distance(z).value * h

    return chi
```

(Note that CLASS output units are k $[h/\mathrm{Mpc}]$ and $P_\delta(k)$ $[(\mathrm{Mpc}/h)^3]$, so for consistency we want to deal with χ in units of $[\mathrm{Mpc}/h]$.)

Code up Limber equation in python VI

Plot and compare

Make a plot of P_κ, using the function in `limber.py` or writing your own.

Compare with Fig. 8 from (?). They measured P_κ on CFHTLenS from the two redshift bins from (?), with mean redshifts 0.7 and 1.05, respectively. Use one of those values as the single redshift z_0.

Code up Limber equation in python VII

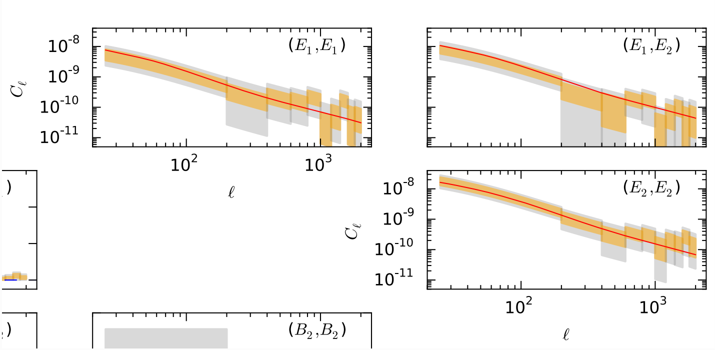


Figure 8 from (?), E-mode power spectra from CFHTLenS.

Bonus: Use CLASS to produce the P_κ using the exact (non-Limber) expression, or other codes (e.g. `nicaea`, `CLASS`, `CosmoSIS`). Compare.

Weak-lensing statistics on a CFHTLenS field I

This exercise will show you how to estimate second-order shear statistics (2PCF, aperture-mass dispersion, band-power spectrum) and their errors, and how to compare these estimates with theoretical predictions.

1. Download shear catalogue

Note: The catalogue size is 158 Mb, so this might take a while. So do this step well before the start of the TD, or use the downloaded catalogue on the common disk.

Go to `http://cfhtlens.org` → Fellow astronomers → **Quick link**: Access the CFHTLenS Shear and Photometric Redshift catalogues.

This brings you to the catalogue query page on CADC `http://www.cadc-ccda.hia-iha.nrc-cnrc.gc.ca/en/community/CFHTLens/query.html`.

We will download the shear data from the W1 field (but feel free to use another field — check the coordinates in (?)). The following steps are advised (for some of these you have to edit the string in the query field):

- Un-select id

Weak-lensing statistics on a CFHTLenS field II

- Select ALPHA_J2000, DELTA_J2000, e1, e2, weight. These are the x- and y-coordinates, the two ellipticity components, and the galaxy weight.
- Choose the ranges ALPHA_J2000≥ 25, ALPHA_J2000≤ 45, DELTA_J2000≥ -20, and DELTA_J2000≤ 0. This selects coordinates in the W1 field (you can double-check in (?)).
- Choose weight> 0, the code `athena` that computes the 2PCF does not like objects with zero weight.
- Choose the range ≥ 0.0 and ≤ 0.0, but *do not* select fitclass. This flag is zero for galaxies, one for stars, and negative for other detections. We only want galaxies, but do not need this flag in our catalogue.
- If you like you can do a test by clicking on "submit query" to see the first 10 objects. If you are happy with the result, choose "Asynchronous" as submission method, "Tab Separated Values", and delete "top 10" from the query field (we don't only want 10 objects)

Weak-lensing statistics on a CFHTLenS field III

The text in the query field should now look something like the following:

```
SELECT
ALPHA_J2000, DELTA_J2000, e1, -1*e2, weight
FROM
cfht.clens
WHERE
ALPHA_J2000>=25
AND ALPHA_J2000<=45
AND DELTA_J2000>=-20
AND DELTA_J2000<=0
AND fitclass>=0.0
AND fitclass<=0.0
AND weight>0
```

I recommend to flip the ε_2-coordinate, by placing a minus sign in front of e2 in the second line. The original coordinates have North and East defined such that (x, y) have a left-handed orientation. The ε_2 flip accounts for that (why?).

Weak-lensing statistics on a CFHTLenS field IV

- Submit query and wait. The processing of the query can take a few minutes up to several hours! After it is done the web page will show a link, from where you can download the catalogue.

Once the catalogue is downloaded, check whether it contains five columns that make sense. You can for example make a scatter plot of α and δ to see whether the selected galaxy coordinates are as desired.
Before proceeding, remove the first (header) line.

2. Use `athena` to get the 2PCF ξ_+ and ξ_-.
If `athena` is not installed, download version 1.7 from `www.cosmostat/athena.html`, and compile.
First, create a config file. The easiest is to copy the example file from `/path/to/athena/test/test_xi/config_tree` and modify it to set the following entries:

- GALCAT1 to the name of the catalogue you downloaded, GALCAT2 either the same or “-”.
- SCOORD_INPUT to “deg”

Weak-lensing statistics on a CFHTLenS field V

- THMIN, THMAX to whatever you like; Note that the shear correlation function is very noisy on scales smaller than 0.1 arcmin due to a very small number of galaxy pairs at such small distances; on scales larger than a few degrees it is more or less consistent with zero for the survey area in consideration.
- RADEC to 1
- OATH: the smaller, the more precise but also the slower the calculation. For testing you can put it to 0.2; for serious calculations it should be 0.05 or smaller.

We will perform two runs: (A) to compute and plot the 2PCF in a few coarse bins; (B) to compute the 2PCF in many narrow bins that then will be integrated to get aperture-mass and band-power spectrum.
The settings in the config file for the two cases:

	A	B
NTH	~ 20	~ 1000 or more
BINTYPE	LOG	LIN (recommended) or LOG

Weak-lensing statistics on a CFHTLenS field VI

Run **athena** with the correct config file for case A or B. You can make sure that the output from another case is not overwritten by running them in different subdirectories, or by using specific suffixes with option (`--out_suf`).

```
> /path/to/athena/bin/athena -c config_tree_A --out_suf _A
```

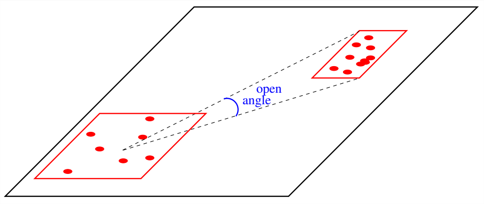
The run will take around 10-20 minutes.

`athena` implements the pairwise galaxy sum estimator of the 2PCF, see Part I:

$$\hat{\xi}_\pm(\theta) = \frac{\sum_{ij} w_i w_j \left(\varepsilon_{\mathrm{t},i}\varepsilon_{\mathrm{t},j} \pm \varepsilon_{\times,i}\varepsilon_{\times,j}\right)}{\sum_{ij} w_i w_j}$$

with a tree code algorithm.

Weak-lensing statistics on a CFHTLenS field VII



[The official Euclid OU-LE3 PF WL-2PCF software is the C++ version of **`athena`** (written by Bertrand Morin, Florent Sureau).]

3. From run (A) plot ξ_+, ξ_-.

The file `xi<suffix>` contains as columns:
angular bin center ϑ , ξ_+, ξ_-, $\xi_\times$, weight w, raw Poisson error $\sqrt{D}$, corrected Poisson error $\sqrt{D_{\rm cor}}$.
Plot ξ_+, ξ_- with error bars $\sqrt{D_{\rm cor}}$ versus θ.
The file `xi.resample<suffix>` contain mean and rms of the resampled ξ_+ and ξ_-, in our case (default config file) from the Jackknife method. Plot the ξ_+ and ξ_- with resampled error bars, and compare mean and error bars.

Weak-lensing statistics on a CFHTLenS field VIII

4. Use `pallas.py` to get derived second-order statistics: aperture-mass dispersion and band-power spectrum

```
> /path/to/athena/bin/pallas.py -i xi<suffix>
```

The resulting important output files are:

- `output_map2_poly.txt`, columns: smoothing scale/circle radius θ, $\langle M_{\rm ap}^2\rangle(\theta)$, $\langle M_\times^2\rangle(\theta)$, $\langle M_{\rm ap} M_\times\rangle(\theta)$.
- `output_pkappa_band.txt`, columns: 2D Fourier mode bin center ℓ, $P_\kappa^{\rm E}(\ell)$, $P_\kappa^{\rm B}(\ell)$, $P_\kappa^{\rm EB}(\ell)$, lower bin limit $\ell_{\rm lo}$, upper bin limit $\ell_{\rm hi}$.

Plot E-, B-, and mixed EB-modes of both quantities in separate plots.
Note: If you find B-mode amplitudes comparible to the E-mode, you might have done the ε_2-flip inorrecly.

Weak-lensing statistics on a CFHTLenS field IX

5. Use the Limber code from cycle I, or `nicaea`, or `CLASS` to create theoretical prediction of the power spectrum.

Use $\bar{z} = 0.75$ (?) for the mean redshift. Add the resulting convergence power spectrum P_κ to the previous plot, by plotting on the y-axis $\ell(\ell+1)/(2\pi)P_\kappa(\ell)$. Note that the shear catalogue is not calibrated. The calibration for the multiplicative shear bias m is around 6% on average, that makes around 12% in amplitude for the 2PCF.

Additional bonus exercises

1. Download the catalogue again with additional fields. Note that if you want to re-run `athena`, you have to create a copy of the catalogue without those additional fields, since for ascii catalogues only 5 input columns are accepted.
   Extra catalogue on USB sticks.

Weak-lensing statistics on a CFHTLenS field X

- Redshift distribution. Select Z_B, photometric redshift. From this, create a histogram that you can use as redshift distribution $n(z)$ for the theoretical prediction, instead of placing all galaxies at one single redshift.
  Extra-bonus if you make a weighted histogram using the weights w.
  Special extra bonus if you download the full pdf information, `PZ_full`, and create the $n(z)$ from the sum of weighted pdf's.
- Shear calibration. Select `SNratio`, signal-to-noise ratio, and `scalelength` for galaxy size.
  - Additive shear bias c: The correction for c can be done for each galaxy. Use eq. (19) of (?) for c_2; note that `scalelength` is in pixels, with one pixel being 0.187 arc seconds. On average, c_2 should be of order 0.002.
    The 1-component of the additive bias, c_1, was measured to be consistent with zero, and no calibration is required.
    Subtract c_2 from ε_2 for each galaxy. This should be done before the ε_2 flip. Note that a constant additive bias shows up in the 2PCF, but not the aperture-mass dispersion. (Why?)

Weak-lensing statistics on a CFHTLenS field XI

- Multiplicative shear bias m: Correction for m should not be done on individual galaxies. This might introduce correlations between their weights w and m, and could up-weigh badly measured galaxies if they have a large $|m|$. Instead, we need to compute a total calibration correction for the entire galaxy sample. For the 2PCF, this is the expression (16) in (?), or (14) in (?),

  The 2PCF is then globally calibrated by

$$\xi_\pm^{\rm cal}(\vartheta) = \frac{\xi_\pm(\vartheta)}{1 + K(\vartheta)}.$$

  We can use **athena** to compute the two-point correlation function of m, $1 + K(\vartheta)$. Since m is a scalar and not a spin-2 quantity like ellipticity, we can to the following trick: In the original shear catalogue, we replace ε_1 with $1 + m$, and ε_2 with 0. The output $\xi_+ = \langle \varepsilon_1 \varepsilon_1 \rangle + \langle \varepsilon_2 \varepsilon_2 \rangle$ of athena then results in $\langle (1 + m)(1 + m) \rangle$, which corresponds to $1 + K$. (ξ_- will not be a meaningful output.)
  Use (17) of (?) to compute m for each galaxy. In this equation, $\log = \log_{10}$, and α is in inverse pixel. Do the replacement in the catalogue as descibed above. The modified catalogue should now contain the 5 columns ALPHA_J2000, DELTA_J2000, $1 + m$, 0, w. and run **athena** with the modified catalogue.
  Plot the calibrated 2PCF $\xi^{\rm cal}$ and compare to the previous result.

Weak-lensing statistics on a CFHTLenS field XII

2. Code up the Hankel transform to obtain ξ_+ and ξ_- from the theorical model,

$$\xi_+(\vartheta) = \frac{1}{2\pi} \int_0^\infty \mathrm{d}\ell \, \ell \mathrm{J}_0(\ell\vartheta) P_\kappa(\ell)$$
$$\xi_-(\vartheta) = \frac{1}{2\pi} \int_0^\infty \mathrm{d}\ell \, \ell \mathrm{J}_4(\ell\vartheta) P_\kappa(\ell),$$

   Plot together with the data.

3. Plot theoretical power spectrum for different values of σ_8.

4. Error bars for $\langle M_{\rm ap} \rangle$ and $P_\kappa(\ell)$.
   Re-run `athena` with the options `--out_ALL_xip_resample XIP_NAME --out_ALL_xim_resample XIM_name`. This outputs all resampled realisations of ξ_+ and ξ_- into the files `XIP_NAME` and `XIM_name`, respectively.
   There are two options to proceed:

Weak-lensing statistics on a CFHTLenS field XIII

4.1 Bring these files into the format of `athena` output file `xi`. Use dummy values for columns `xi_x, w, sqrt_D, sqrt_Dcor, n_pair` (for example copy the ones from `xi`. Create a different new `xi` file for each of the NRESAMPLE resample realisation.
Run `pallas.py` with each of the resample `xi` files. This should provide NRESAMPLE output files; make sure they have unique names or are stored in different sub-directories.
The errors bars on $\langle M_{\rm ap} \rangle$ and $P_\kappa(\ell)$ are then simply the rms between the different realizations (the errors on $\xi_\pm$ have been properly propagated to the derive quantities).

4.2 Reading resampled input and computing resample errors bars for $\langle M_{\rm ap} \rangle$ and $P_\kappa(\ell)$ is already implemented for FITS format, in a new version of pallas.py. See function `read_xi_resample`.
Download this new version and implement resampling for ASCII format. Compare to option 1.

Weak-lensing statistics on a CFHTLenS field XIV

5. Extend the computation of the jackknife variance (see previous point) to the co-variance.
   Code up a simple Gaussian likelihood function with the inverse of this covariance.
   Compute the likelihood for various values of σ_8, and make a plot.
   Special extra super bonus: Use this likelihood in a sampler, e.g. `MontePython`. Do an MCMC and plot parameter constraints.

Convergence and shear I

1. Calculate the effects of κ and γ on a circular image, using the linearized lens equation,

$$I(\boldsymbol{\theta}) = I^{\rm s}(\boldsymbol{\beta}(\boldsymbol{\theta})) \approx I^{\rm s}(\boldsymbol{\beta}(\boldsymbol{\theta}_0) + \mathcal{A}(\boldsymbol{\theta} - \boldsymbol{\theta}_0)),$$

with the Jacobi matrix

$$A = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}.$$

   1.1 Convergence
   Set shear to zero.
   Parametrize a circular isophote (line of constant surface brighness I) in image coordinates $\boldsymbol{\theta}$. Set $\boldsymbol{\beta}(\boldsymbol{\theta}_0) = 0$ (this is just an offset), and compute the source coordinates. Show that a positive (negative) κ results in a magnified (demagnified) image compared to the source.

   1.2 Shear
   For simplicity, set $\gamma_2 = 0$, and $\kappa \neq 0$. Repeat the calculation from above and show that the transformed image is an ellipse.

Convergence and shear II

2. Show that the power spectra of the convergence and the shear are equal. Use the expressions of κ and γ in terms of the lensing potential, using

$$A_{ij} = \delta_{ij} - \partial_i \partial_j \psi.$$

2.1 Write these relations in Fourier space, and express $\hat{\gamma}$ as a function of $\hat{\kappa}$.

$$\hat{\kappa}(\boldsymbol{\ell}) = -\frac{1}{2}\left(\ell_1^2 + \ell_2^2\right)\hat{\psi}(\boldsymbol{\ell});$$
$$\hat{\gamma}(\boldsymbol{\ell}) = -\frac{1}{2}\left(\ell_1^2 - \ell_2^2\right)\hat{\psi}(\boldsymbol{\ell});$$
$$\hat{\gamma}(\boldsymbol{\ell}) = -\ell_1 \ell_2 \hat{\psi}(\boldsymbol{\ell}).$$

Therefore,

$$\hat{\gamma}(\boldsymbol{\ell}) = \frac{(\ell_1 + \mathrm{i}\ell_2)^2}{\ell^2} = \mathrm{e}^{2\mathrm{i}\beta}\hat{\kappa}(\boldsymbol{\ell}).$$

2.2 Now show that $P_\kappa = P_\gamma$

Tangential shear and projected overdensity I

Exercise:

Show that the average tangential shear around a point at an angular radius θ is equal to the projected mass overdensity within θ, minus a boundary term.

$$\langle \gamma_{\rm t} \rangle (\theta) = \bar\kappa(\le \theta) - \langle \kappa \rangle (\theta).$$

The projected mass overdensity $\bar\kappa(\le \theta)$ averaged over the disk with radius θ, D_θ, is given by (**?**, **?**)

$$\bar\kappa(\le \theta) := \frac{1}{\pi \theta^2} \int_{D_\theta : |\boldsymbol{\theta}'| < \theta} {\rm d}^2\theta' \kappa(\boldsymbol{\theta}').$$

Tangential shear and projected overdensity II

1. First, use the Poisson equation to relate the convergence to the lensing potential ψ. Apply Gauss' law to replace the 'volume' integral over the disk D_θ by a 'surface' integral over the boundary of the disk, ∂D_θ, which is the circle at radius θ.
   Poisson equation: $\kappa = \frac{1}{2}\Delta\psi = \frac{1}{2}\mathrm{div}\,\mathrm{grad}\,\psi$. Applying Gauss' law:
   $$\bar\kappa(\le\theta) = \frac{1}{2\pi\theta^2}\int_{\partial D_\theta} \mathrm{d}\boldsymbol{\lambda}\cdot\boldsymbol{\nabla}\psi(\theta,\varphi).$$

2. Replace the integration over the line element along the circle with an integral over the polar angle φ, accounting for the circle length $2\pi\theta$. Convince yourself that the gradient of the potential ψ is projected to the radial direction $\hat{\boldsymbol{e}}_\theta$ normal to the circle; the tangential derivative is projected out by the scalar product.
   $$\bar\kappa(\le\theta) = \frac{1}{2\pi\theta}\int_0^{2\pi} \mathrm{d}\varphi\,\partial_\theta\psi(\theta,\varphi).$$
   where the variable transformation $(2\pi\theta)^{-1}\mathrm{d}\boldsymbol{\lambda} \to (2\pi)^{-1}\mathrm{d}\varphi$ was used.

Tangential shear and projected overdensity III

3. To further evalute the lensing potential, we need its second derivatives. Multiply the last result with θ, and take the derivative with respect to θ. The term $\partial_\theta \partial_\theta \psi$ can be expressed in terms of convergence and tangential shear using the relations derived earlier in the lecture. Do this in a local Cartesian coordinate system $(\hat{\boldsymbol{e}}_\theta, \hat{\boldsymbol{e}}_\varphi)$. What is the interpretation of the second shear component in this system when seen from the canonical coordinate system?
   Define circularly averaged quantities

$$\langle a \rangle(\theta) := \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\varphi \, a(\theta, \varphi).$$

   and express $\partial[\theta \bar{\kappa}(\leq \theta)]/\partial\theta$ in terms of circularly averaged convergence and tangential shear.

$$\frac{\partial[\theta \bar{\kappa}(\leq \theta)]}{\partial \theta} = \int_0^{2\pi} \frac{\mathrm{d}\varphi}{2\pi} \, \partial_\theta \partial_\theta \psi(\theta, \varphi) = \langle \kappa \rangle (\theta) - \langle \gamma_\mathrm{t} \rangle (\theta). \qquad (1)$$

Tangential shear and projected overdensity IV

4. Write $\bar\kappa(\le \theta)$ of eq. (26) as function of $\langle\kappa\rangle$.

$$\bar\kappa(\le \theta) = \frac{2}{\theta^2} \int_0^\theta \mathrm{d}\theta' \, \theta' \langle\kappa\rangle(\theta').$$

Multiply by θ and take the derivative with respect to θ, as with the equation before.

$$\frac{\partial \theta \bar\kappa(\le \theta)}{\partial \theta} = 2\langle\kappa\rangle(\theta) - \bar\kappa(\le \theta).$$

Equate this with the previous expression to get the final result.

$$\langle\gamma_\mathrm{t}\rangle(\theta) = \bar\kappa(\le \theta) - \langle\kappa\rangle(\theta).$$

Tangential shear and projected overdensity V

5. In addition (for relation between aperture-mass filters U and Q): Express $\partial\bar{\kappa}(\theta)$ as function of $\langle\gamma_{\rm t}\rangle(\theta)$.
   From eq. (1):

$$\frac{\partial[\theta\bar{\kappa}(\leq\theta)]}{\partial\theta} = \langle\kappa\rangle(\theta) - \langle\gamma_{\rm t}\rangle(\theta) = \bar{\kappa}(\leq\theta) + \theta\frac{\partial\bar{\kappa}(\leq\theta)}{\partial\theta}$$

   Therefore

$$\theta\frac{\partial\bar{\kappa}(\leq\theta)}{\partial\theta} = \langle\kappa\rangle(\theta) - \bar{\kappa}(\leq\theta) - \langle\gamma_{\rm t}\rangle(\theta) = -2\langle\gamma_{\rm t}\rangle(\theta).$$

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