

## Part I day 2. Reminder: Overview

### Part I day 1: Principles of gravitational lensing

Brief history of gravitational lensing  
 Light deflection in an inhomogeneous Universe  
 Convergence, shear, and ellipticity  
 Projected power spectrum  
 Real-space shear correlations

### Part I day 2: Measurement of weak lensing

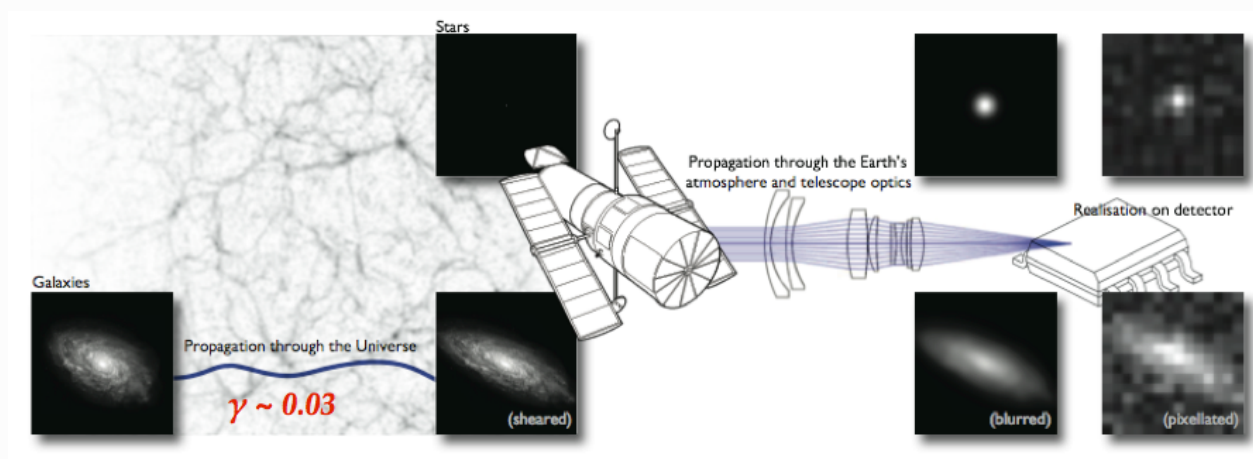
Galaxy shape measurement  
 PSF correction  
 Photometric redshifts  
 Estimating shear statistics

### Part I day 3: Surveys and cosmology

Cosmological modelling  
 Results from past and ongoing surveys (CFHTLenS, KiDS, DES)  
 Euclid

### Part I day 3+: Extra stuff

## The shape measurement challenge

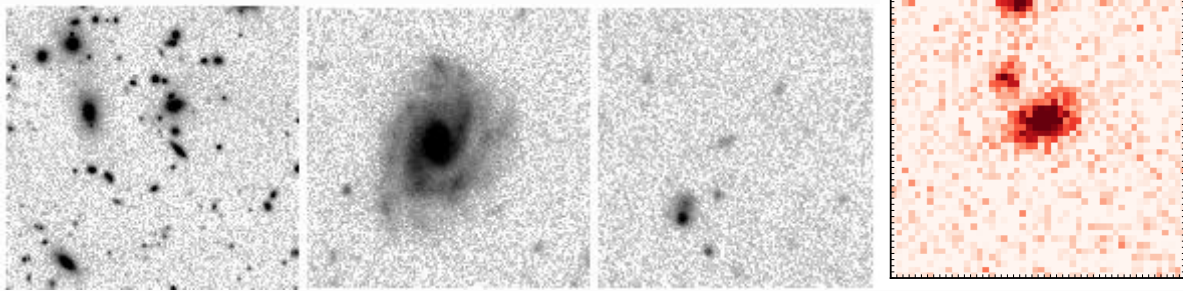


Bridle et al. 2008, great08 handbook

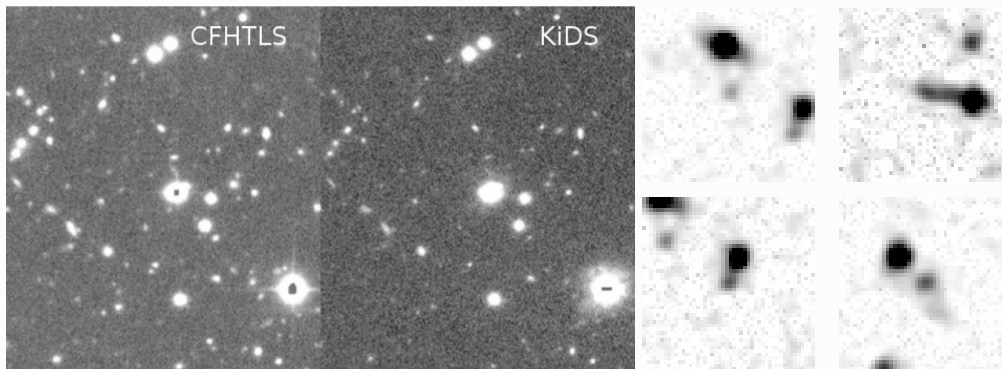
- Cosmological shear  $\gamma \ll \epsilon$  intrinsic ellipticity
- Galaxy images corrupted by PSF
- Measured shapes are biased

## The shape measurement challenge

How do we measure "ellipticity" for irregular, faint, noisy objects?



[Y. Mellier/CFHT(?)] — (Jarvis et al. 2016)

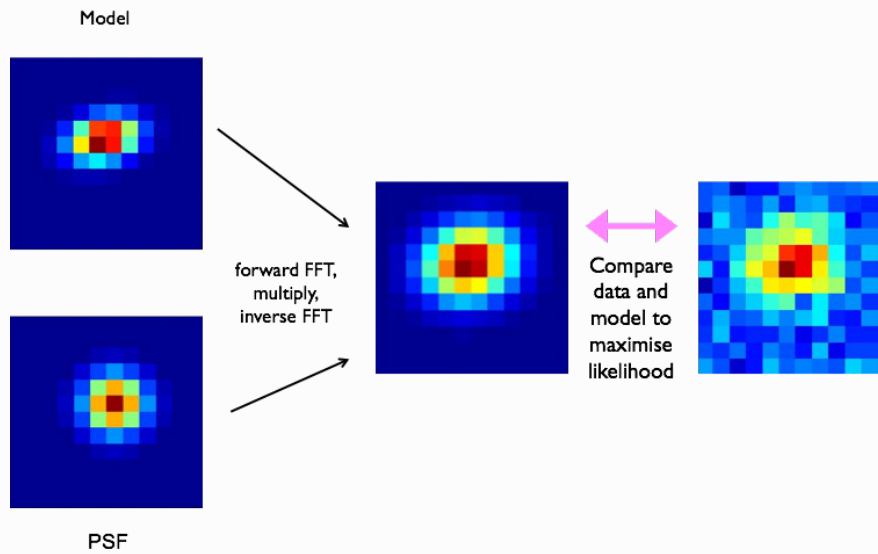


[CFHTLenS/KiDS image — CFHTLenS postage stamps]

## Shape measurement methods

- Parametric: model fitting.  
(Kuijken 1999), *lensfit* (Miller et al. 2007), *gfit* (Gentile et al. 2012), *im3shape* (Zuntz et al. 2013) and many more.
- Non-parametric: direct estimation.
  - Perturbative: weighted moments.  
*KSB* — (Kaiser et al. 1995) + many improvements  
*DEIMOS* — (Melchior et al. 2011) (PSF correction in moment space)  
*HOLICs* — (Okura & Futamase 2009) — Higher-order moments
  - Non-perturbative: Decomposition into basis functions.  
*shapelets* — (Refregier 2003) + many improvements

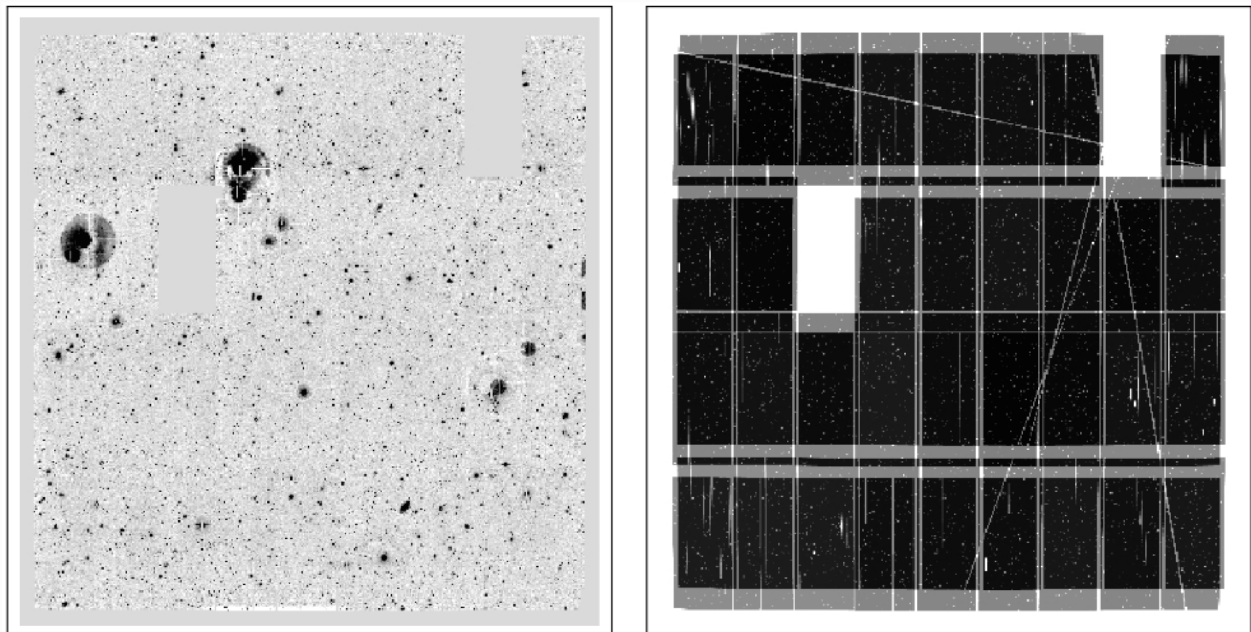
## Model fitting methods



### Forward model-fitting (example *lensfit*)

- Convolution of model with PSF instead of deconvolution of image
- Combine multiple exposures (in Bayesian way, multiply posterior density), avoiding co-adding of (dithered) images

## Dithering



Left: Co-add of two  $r$ -band exposures of CFHTLenS.

Right: Weight map.

## Moment-based methods I

### Moments and ellipticity

How are moments connected to ellipticity?

Q: Simple case: qualitatively, what are the 0<sup>th</sup>, 1<sup>st</sup>, 2<sup>nd</sup> moments of a 1D distribution? Of a 2D distribution?

Quadrupole moment of **weighted** light distribution  $I(\boldsymbol{\theta})$ :

$$Q_{ij} = \frac{\int d^2\theta q[I(\boldsymbol{\theta})] (\theta_i - \bar{\theta}_i)(\theta_j - \bar{\theta}_j)}{\int d^2\theta q[I(\boldsymbol{\theta})]}, \quad i, j = 1, 2$$

$q$  : **weight function**

$$\bar{\boldsymbol{\theta}} = \frac{\int d^2\theta q_I[I(\boldsymbol{\theta})] \boldsymbol{\theta}}{\int d^2\theta q_I[I(\boldsymbol{\theta})]} : \quad \text{barycenter (first moment!)}$$

### Ellipticity

$$\varepsilon = \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}$$

Circular object  $Q_{11} = Q_{22}, Q_{12} = Q_{21} = 0$

## Moment-based methods II

### KSB PSF correction

Perturbative ansatz for PSF effects

$$\varepsilon^{\text{obs}} = \varepsilon^{\text{s}} + P^{\text{sm}} \varepsilon^* + P^{\text{sh}} \gamma$$

[c.f.  $\varepsilon^{\text{obs}} = \varepsilon^{\text{s}} + \gamma$  from before]

$P^{\text{sm}}$	smear polarisability, (linear) response of to ellipticity to PSF anisotropy
$e^*$	PSF anisotropy
$P^{\text{sh}}$	shear polarisability, isotropic seeing correction
$\gamma$	shear

$P^{\text{sm}}, P^{\text{sh}}$  are functions ( $2 \times 2$  tensors) of galaxy brightness distribution.

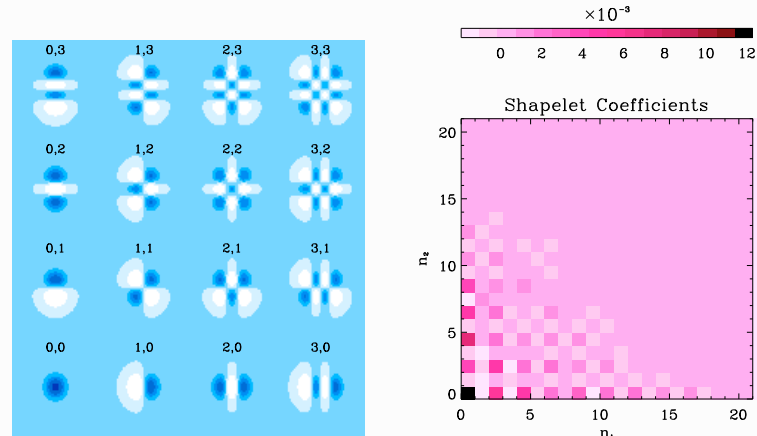
**Problematic:** Strongly anisotropic PSF, error estimation, combining multiple exposures.

## Non-perturbative methods

### Shapelets

(Refregier 2003, Massey & Refregier 2005, Kuijken 2006)

- Decompose galaxies and stars into basis functions.



- PSF correction, convergence and shear acts on shapelet coefficients, deconvolution feasible
- **Problems:** series truncation, basis functions not representative, need to set size parameter

## Further methods and techniques

- Machine-Learning, e.g. LUT by supervised learning, (Tewes et al. 2012)
- Self-calibration
- Further Bayesian methods
  - Hierarchical Multi-level Bayesian Inference (MBI), (Schneider et al. 2014). Joint posterior of shear, galaxy properties, PSF, nuisance parameters given pixel data.
  - (Bernstein & Armstrong 2014). Does not measure ellipticity of individual galaxies, direct posterior estimation of shear for population. Needs prior from deep images.

## Shear measurement biases I

### Origins

- **Noise bias**

In general, ellipticity is non-linear in pixel data (e.g. normalization by flux). Pixel noise  $\rightarrow$  biased estimators.

- **Model bias**

Assumption about galaxy light distribution is in general wrong.

- Model-fitting method: wrong model
- Perturbative methods (*KSB*, *DEIMOS*, *HOLICS*): weight function not appropriate
- Non-perturbative methods (*shapelets*): truncated expansion, bad eigenfunction representation
- Color gradients
- Non-elliptical isophotes

- **Other**

- Imperfect PSF correction
- Detector effects (CTI — charge transfer inefficiency)

## Shear measurement biases II

- Selection effects (probab. of detection/successful  $\varepsilon$  measurement depends on  $\varepsilon$  and PSF)

### Characterisation

Bias can be multiplicative ( $\mathbf{m}$ ) and additive ( $\mathbf{c}$ ):

$$\gamma_i^{\text{obs}} = (1 + m_i) \gamma_i^{\text{true}} + c_i; \quad i = 1, 2.$$

Biases  $\mathbf{m}$ ,  $\mathbf{c}$  are typically complicated functions of galaxy properties (e.g. size, magnitude, ellipticity), redshift, PSF, .... They can be scale-dependent.

Current methods:  $|m| = 1\% - 10\%$ ,  $|c| = 10^{-3} - 10^{-2}$ .

Challenges such as STEP1, STEP2, great08, great10, great3 quantified these biases with blind simulations.

### Calibration

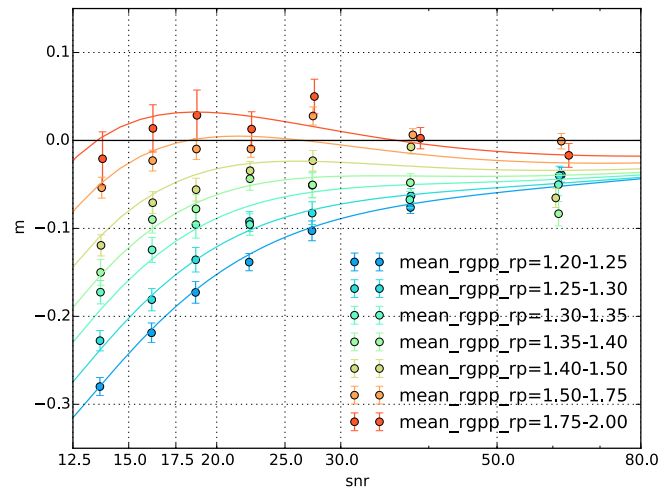
Usually biases are calibrated using simulated or emulated data, or self-calibration.

Current surveys produce their own image simulations with properties of galaxy sample and PSF matching to data.



## Shear measurement biases III

Functional dependence of  $m$  on observables must not be too complicated (e.g. not smooth, many variables, large parameter space), or else measurement is *not calibratable*!



(Jarvis et al. 2016)

### Requirements

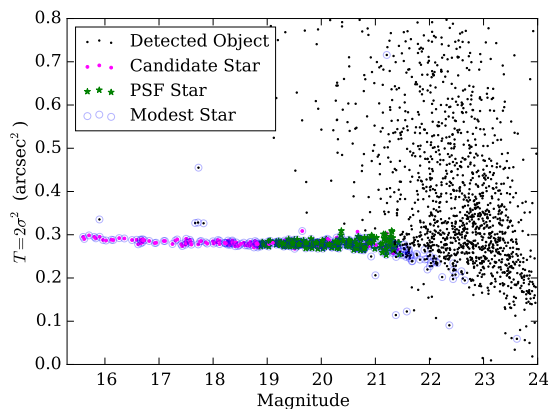
Normalisation  $\sigma_8 \propto m$ !

Necessary knowledge of residual biases  $\Delta|m|, \Delta|c|$  (after calibration):

Current surveys 1%.

Future large missions (Euclid, LSST, ...)  $10^{-4} = 0.1\%$ !

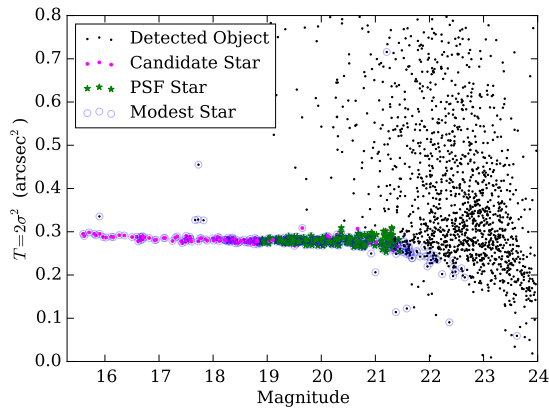
## PSF correction



(Jarvis et al. 2016)

- Select clean sample of stars
- Measure star shapes
- Create PSF model and interpolate (pixel values, ellipticity, PCA coefficients, ...) to galaxy positions. Space-based observations: global PSF model from many exposures possible
- Correct for PSF: galaxy image deconvolution or other (e.g. linearized) correction, or convolve model

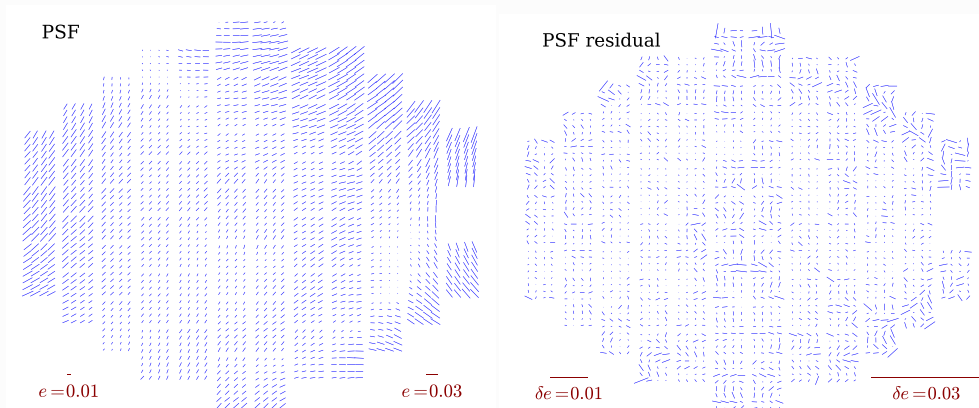
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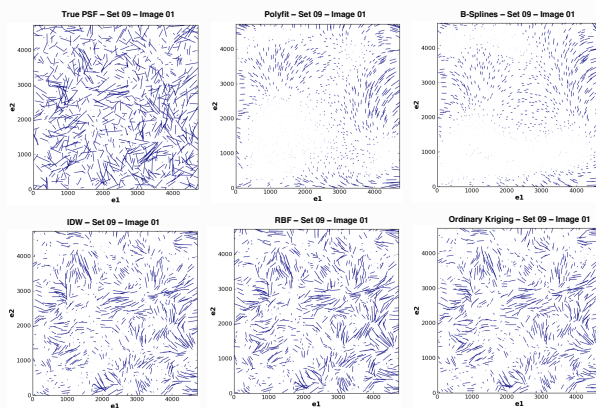


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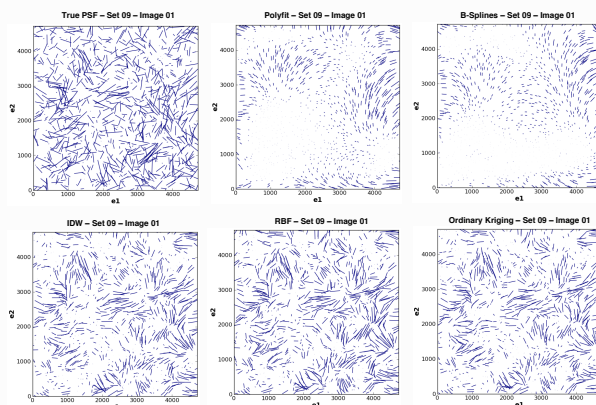
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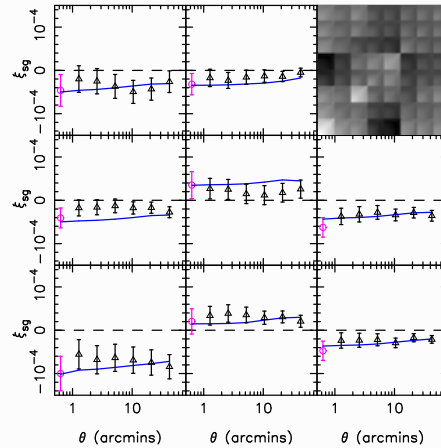
## Quantifying PSF residuals I

Null test:  $\xi_{\text{sys}}$  correlation between star and galaxy shapes expected to vanish, unless PSF correction (using stars to correct galaxy shapes) is not perfect.

$$\xi_{\text{sys}} = \langle \varepsilon^* \varepsilon \rangle$$

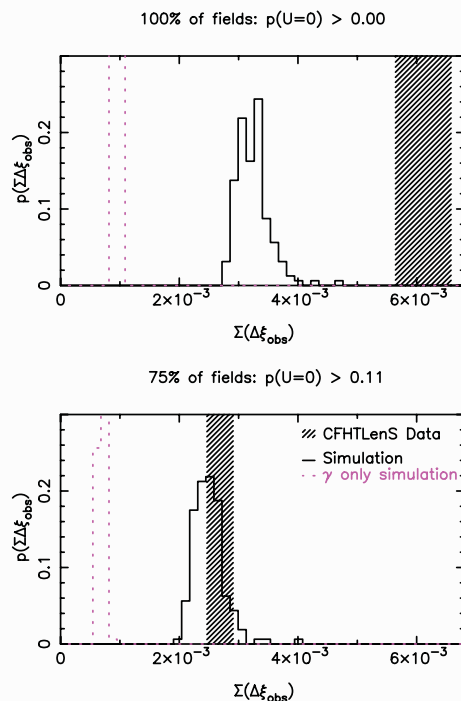
This measures residual PSF pattern leakage onto galaxy field.

**Caveat:** LSS can show chance alignments with PSF pattern. Sample or *cosmic variance* has to be accounted for  $\rightarrow$   $N$ -body simulations!



(Heymans et al. 2012)

## Quantifying PSF residuals II

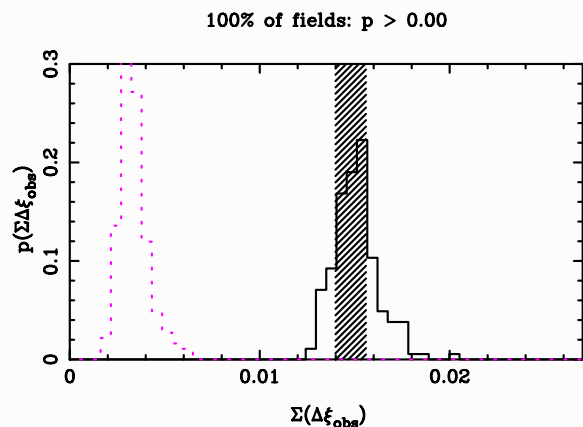


[Heymans et al. 2012, CFHTLenS]

Histogram of probability  $p$  that  $\Sigma \xi_{\text{obs}} \sim \Sigma |\xi_{\text{sys}}|$  is not zero (sum over all pointings), from simulations.

Shaded region = data.

Magenta: simulations without LSS.



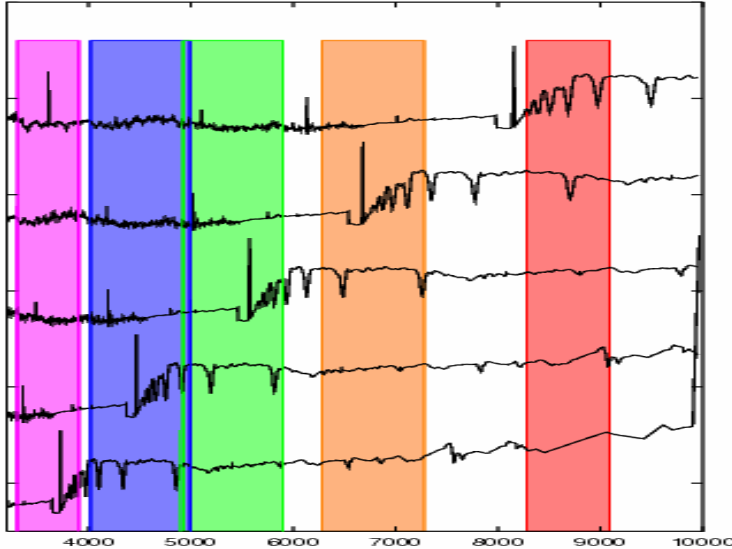
[Hildebrandt et al. 2016, KiDS-450]

## Redshift estimation I

Redshifted galaxy spectra have different colors.

Photometric redshifts = very low resolution spectra.

#bands between 3 (RCS) and 30 (COSMOS). Typical are 4-5 optical filters ( $g, r, i, y, z$ ), maybe with UV ( $u$ ) and IR ( $I, J, K$ ).



**4000 Å-break** strongest feature  
 → ellipticals (old stellar population) best, spirals ok, irregular/star-burst (emission lines) more unreliable.

[from Y. Mellier]

## Redshift estimation II

### Properties

- **Redshift desert**  $z \approx 1.5 - 2.5$ , neither 4000 Å-break nor Ly-break in visible range, very hard to access from ground.
- Confusion between low- $z$  dwarf ellipticals and high- $z$  galaxies. Confusion between Balmer and Lyman break. **Catastrophic outliers**, typically a few to a few 10
- Need UV band and IR for high redshifts! **But:** UV very inefficient, IR absorbed by atmosphere, have to go to space.
- Need spectroscopic galaxy sample for comparison, calibration, or cross-correlation. In general  $N_{\text{spec}} \ll N_{\text{WL}}$ .
- Typical accuracy of photo- $z$ 's  $\sigma/(1+z) \sim 0.05$  (depending on filters).

## Redshift estimation III

### Redshift accuracy and cosmology

To interpret weak lensing correlations in cosmological context, the redshift distribution needs to be known accurately!

To first order:

$$P_{\kappa}(\ell \sim 1000) \propto \Omega_{\text{de}}^{-3.5} \sigma_8^{2.9} \bar{z}^{1.6} |w|^{0.31} \quad (\text{Huterer et al. 2006})$$

### Methods

- Template fitting.  
Redshifted synthetic or observed templates of various types are fitted to flux in observed bands.  
Examples *LePhare* (Ilbert et al. 2006), *BPZ* (Benítez 2000), *HyperZ* (Bolzonella et al. 2000).  
Spectroscopic sample for calibration, priors.
- Machine-learning.  
Learn data using training set (of spectroscopic sample).  
Examples: *ANNz* (Collister & Lahav 2004).

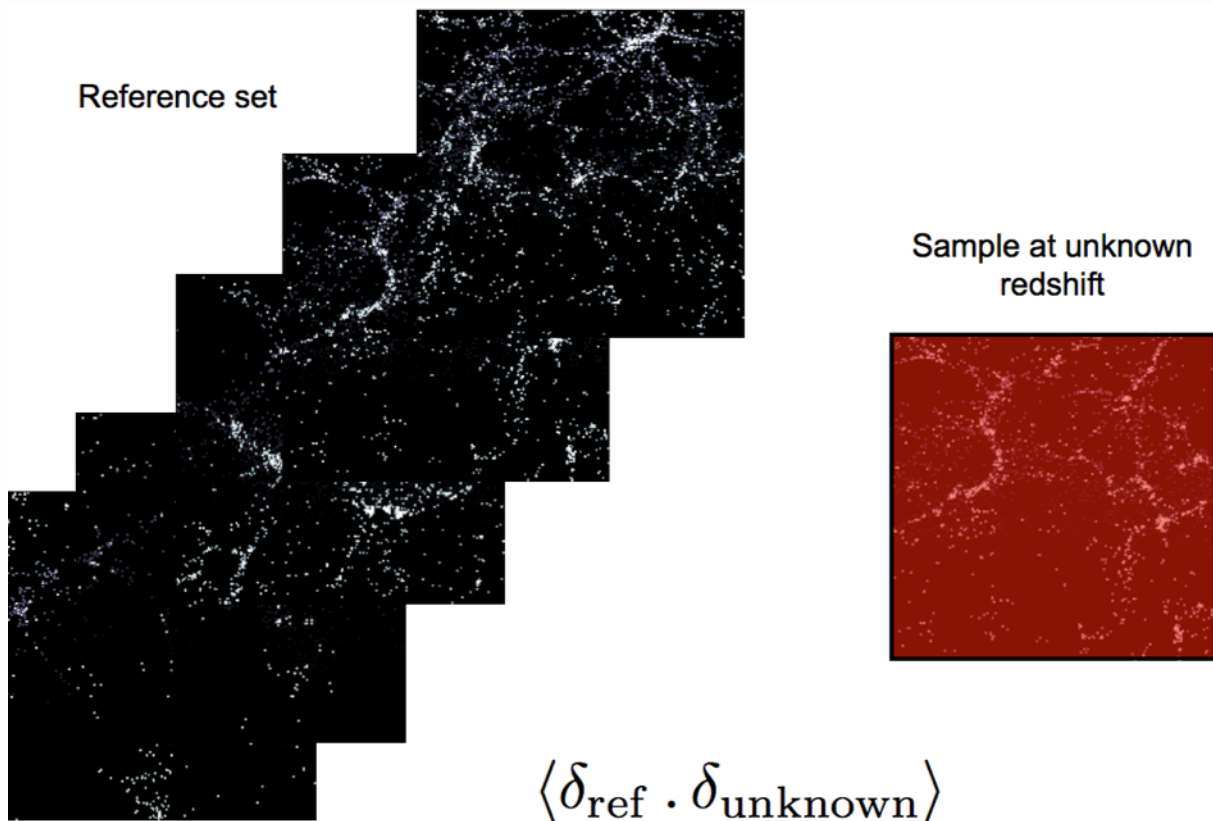
## Redshift estimation IV

- Matching photometric properties to spectroscopic sample (?) (direct calibration).
- Spatial cross-correlation with spectroscopic survey (clustering redshifts)

**Spectroscopic sample** has to be representative in some properties, depending on the method:

- Template fitting: Same magnitude limit as photometric sample
- Neural networks: Cover redshift range, properties (colors)
- Matching: Cover (color) parameter space
- Clustering: Cover redshift range, sky overlap

## Clustering redshifts (slide from Vivien Scottez)



## Estimator of second-order functions I

Remember the shear two-point correlation function (2PCF)?

$$\xi_{\pm}(\vartheta) = \langle \gamma_t \gamma_t \rangle(\vartheta) \pm \langle \gamma_{\times} \gamma_{\times} \rangle(\vartheta)$$

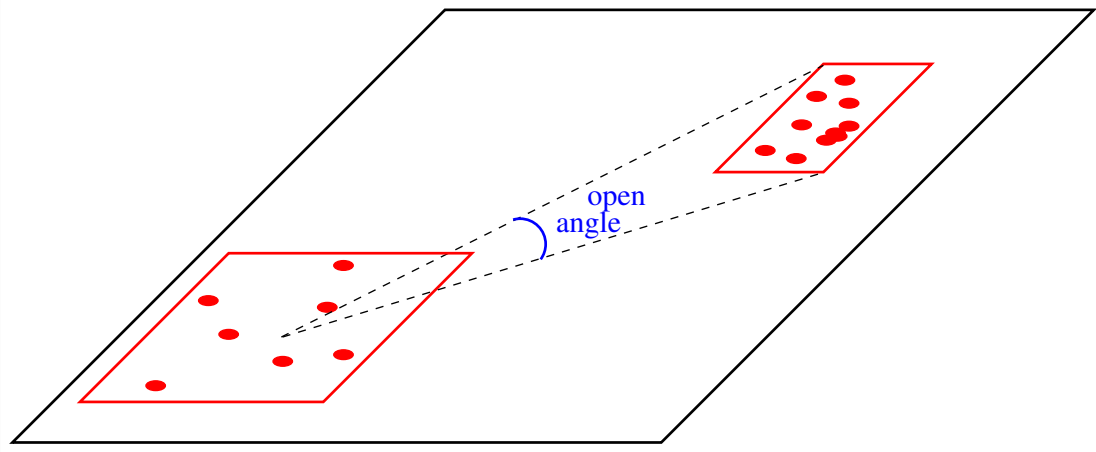
Unbiased estimator of  $\xi_{\pm}$  just involves sums over galaxy pairs:

$$\hat{\xi}_{\pm}(\theta) = \frac{\sum_{ij} w_i w_j (\varepsilon_{t,i} \varepsilon_{t,j} \pm \varepsilon_{\times,i} \varepsilon_{\times,j})}{\sum_{ij} w_i w_j}.$$

Sum over galaxy pairs with angular distance within bin of  $\theta$ .

- Unbiased estimator (for bin size  $\rightarrow 0$ , and in absence of intrinsic alignment)
- No need for random catalogue, or mask geometry, since  $\xi = 0$  in absence of lensing.
- No need to pixellise data, can use brute-force or tree codes/linked lists (adaptive pixellisation, effective smoothing)

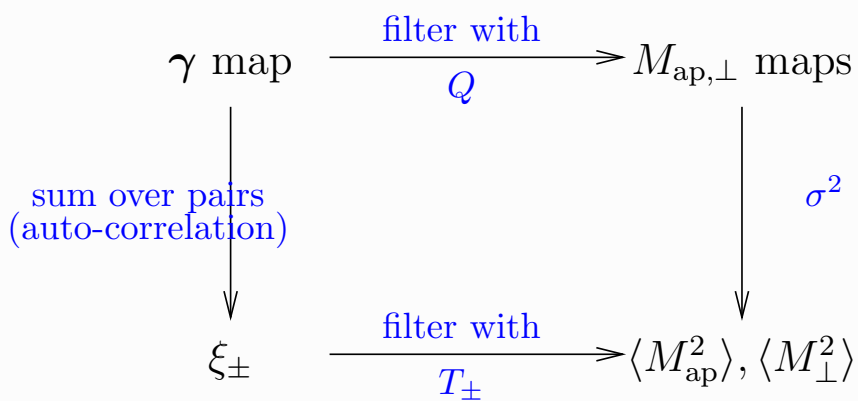
## Estimator of second-order functions II



Tree code: correlating two ‘nodes’ (2D regions).

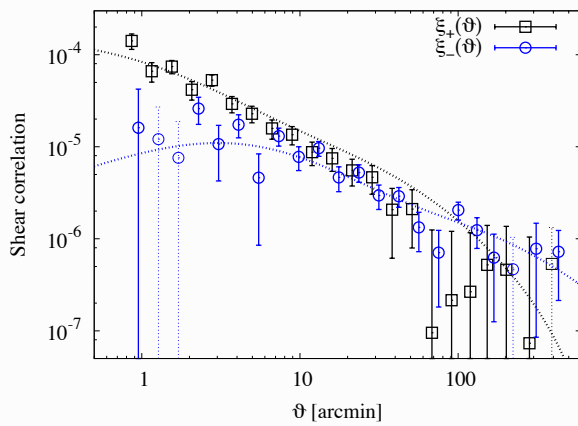
## Estimator of second-order functions III

From the 2PCF estimator, the aperture-mass dispersion and other second-order functions can be derived:

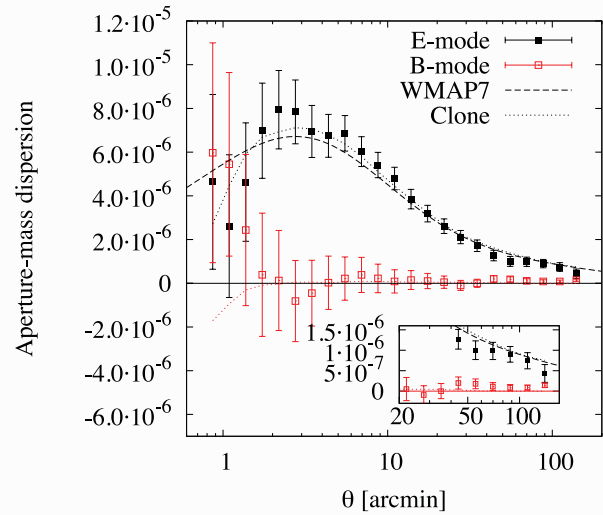




# Estimator of second-order functions IV



(Kilbinger et al. 2013)



End of day 2.