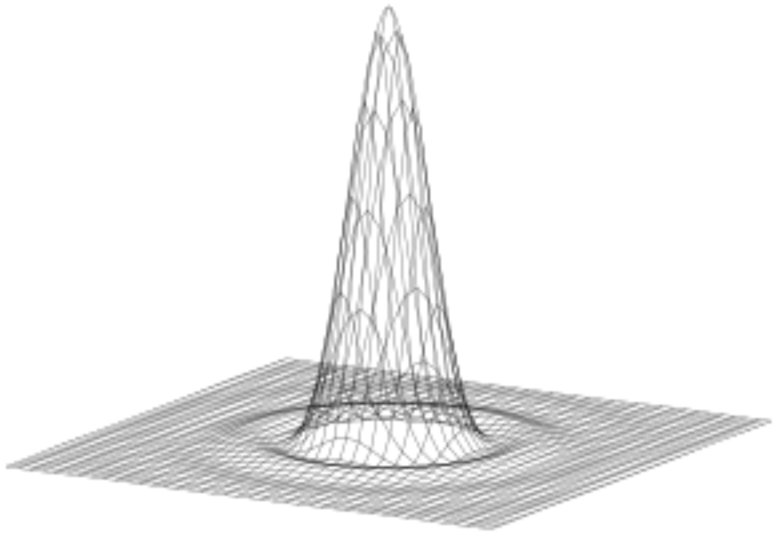
Astrophysics Division
CosmoStat Lab

Sparse matrix factorization for PSFs field estimation

PSF Meeting, Saclay
September 27

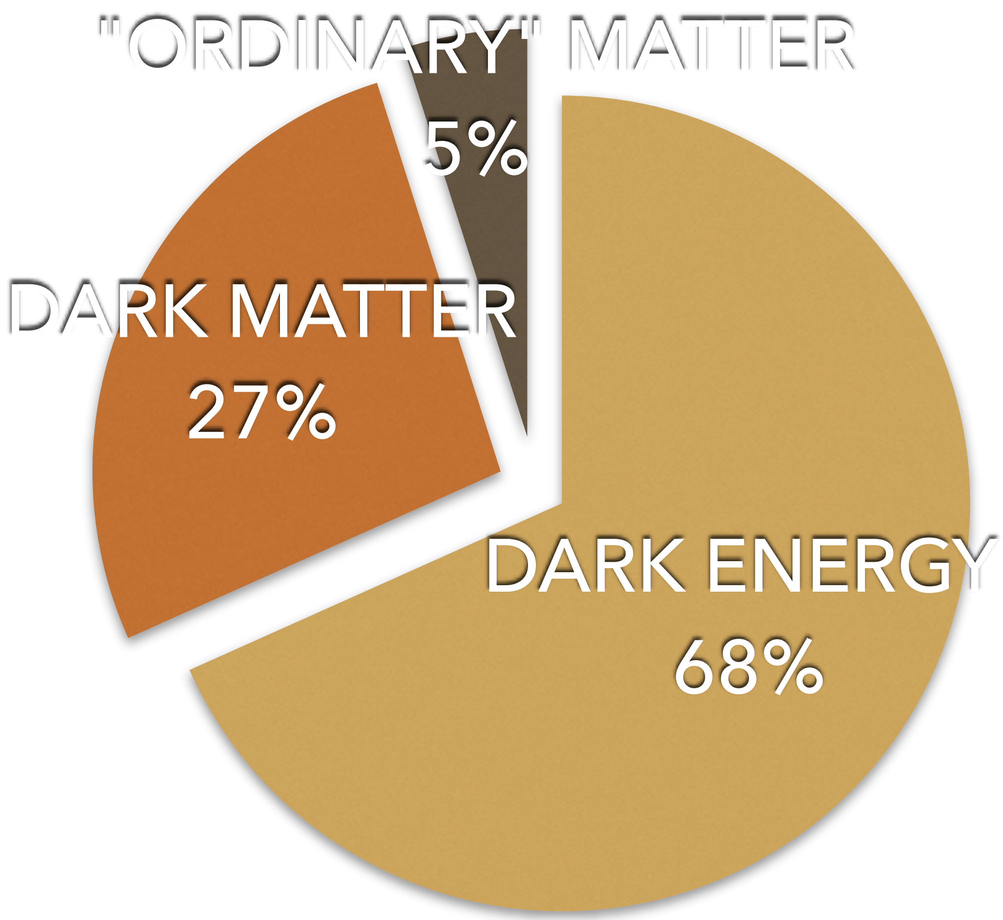
Fred Ngolè



Outline

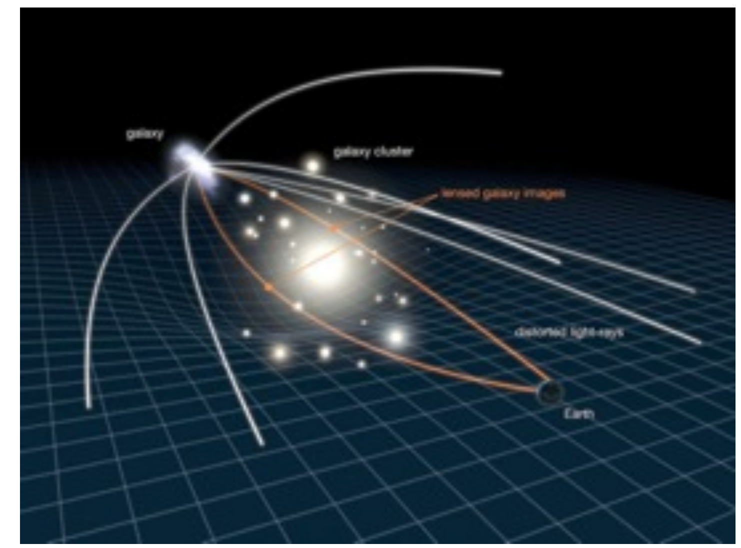
- Context
- Monochromatic PSFs joint super-resolution
- Conclusion

Context



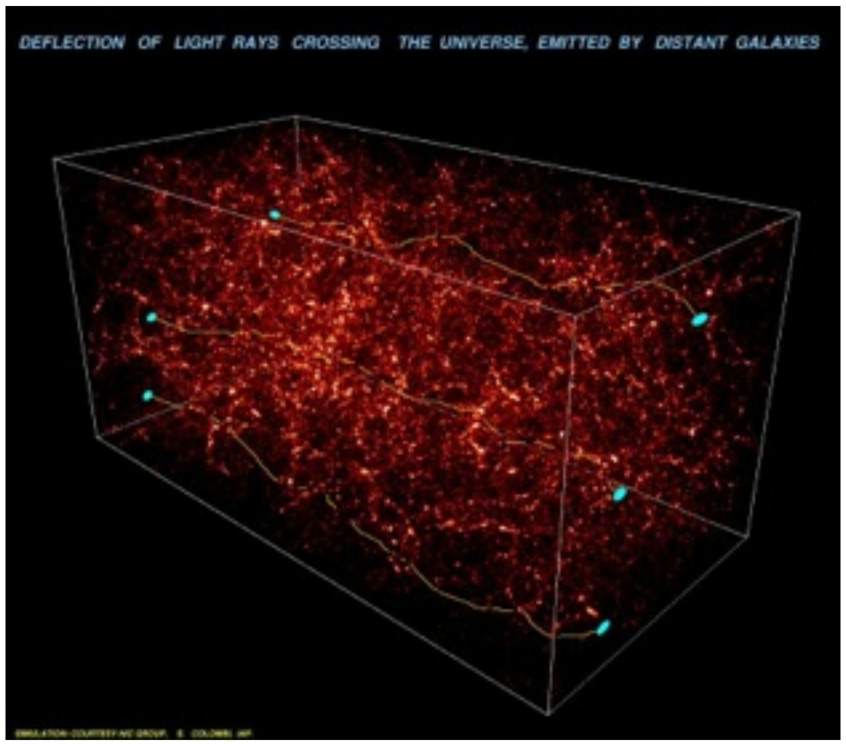
Dark Universe

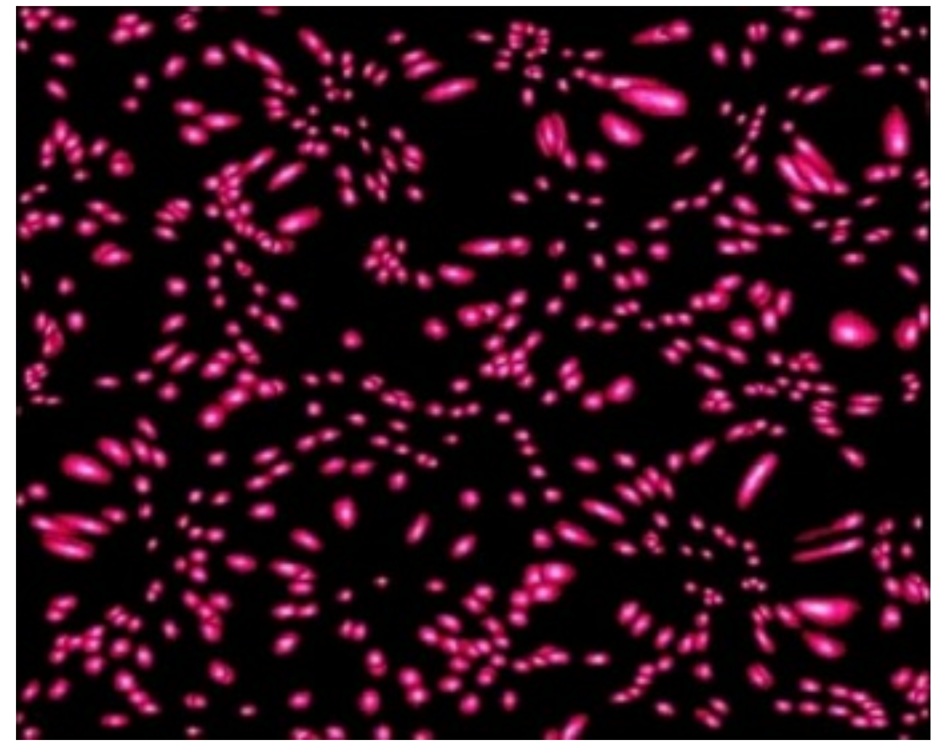
GR



Gravitational lensing

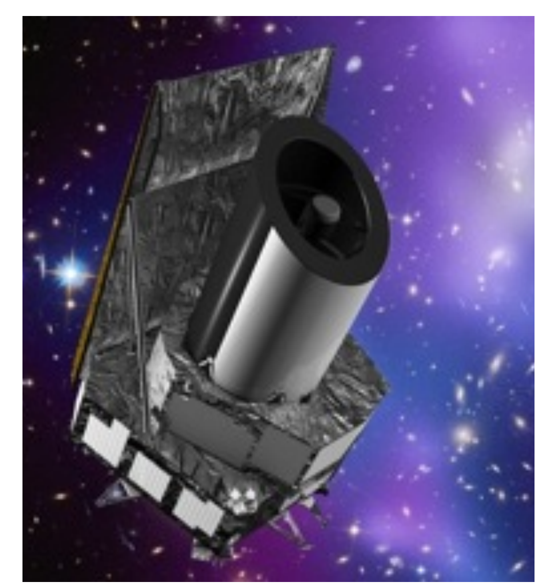
Context





Weak gravitational lensing
($\leq$ 10% of distortion)

➡ Data from the ESA Euclid mission
(to be launched in 2020)



Context

Weak lensing measurement bottleneck: the PSF

Context

Model-free PSFs estimation accounting for:

- noise and aliasing
- spatial and temporal variations
- wavelength dependency.

Monochromatic PSFs joint super-resolution

Observation model

$$\mathbf{y}_k = \mathbf{M}_k \mathbf{x}_k + \mathbf{n}_k,\ k = 1 \cdots p$$

$\mathbf{y}_k$: k^{th} low resolution image
$\mathbf{M}_k$: shift and downsampling operator
$\mathbf{x}_k$: k^{th} well resolved image
$\mathbf{n}_k$: gaussian noise

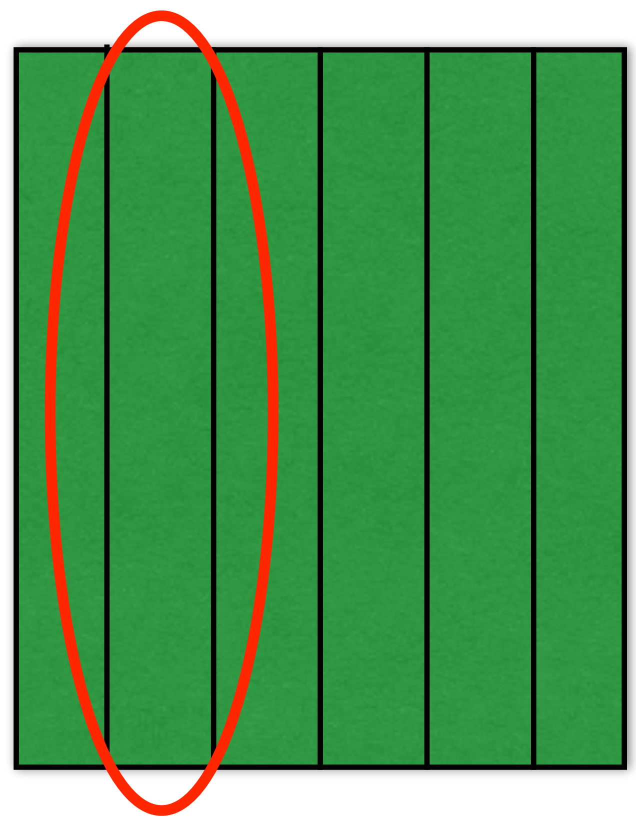
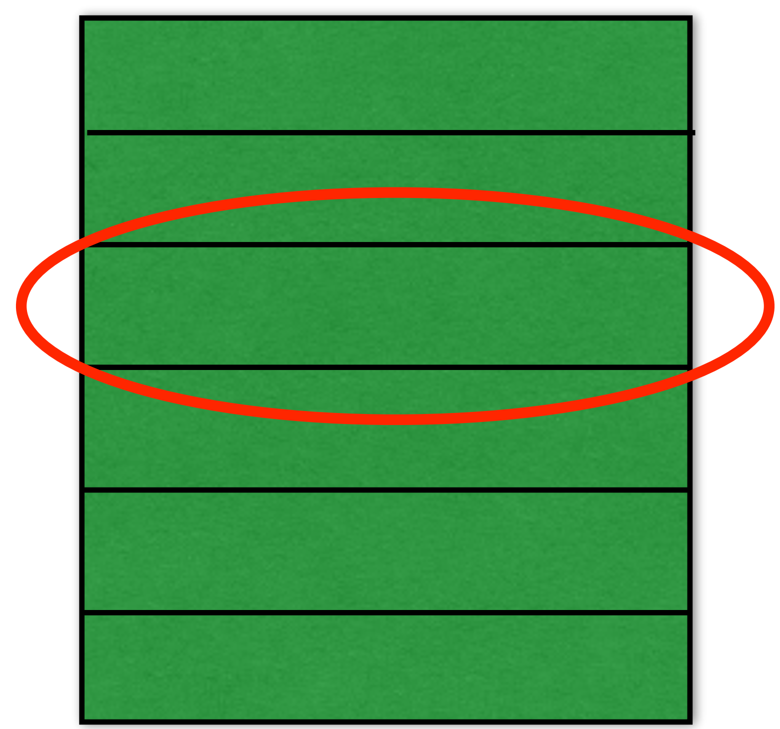
Monochromatic PSFs joint super-resolution

Constraints

- Piece-wise smoothness
- Low dimensionality
- Spatial regularity
- Positivity

Monochromatic PSFs joint super-resolution

Constraints

$\mathbf{X} = [\mathbf{x}_1, ..., \mathbf{x}_p] =$  $=$ 

$\mathbf{\Phi X}$ columns sparse

Pixel domain features dictionary

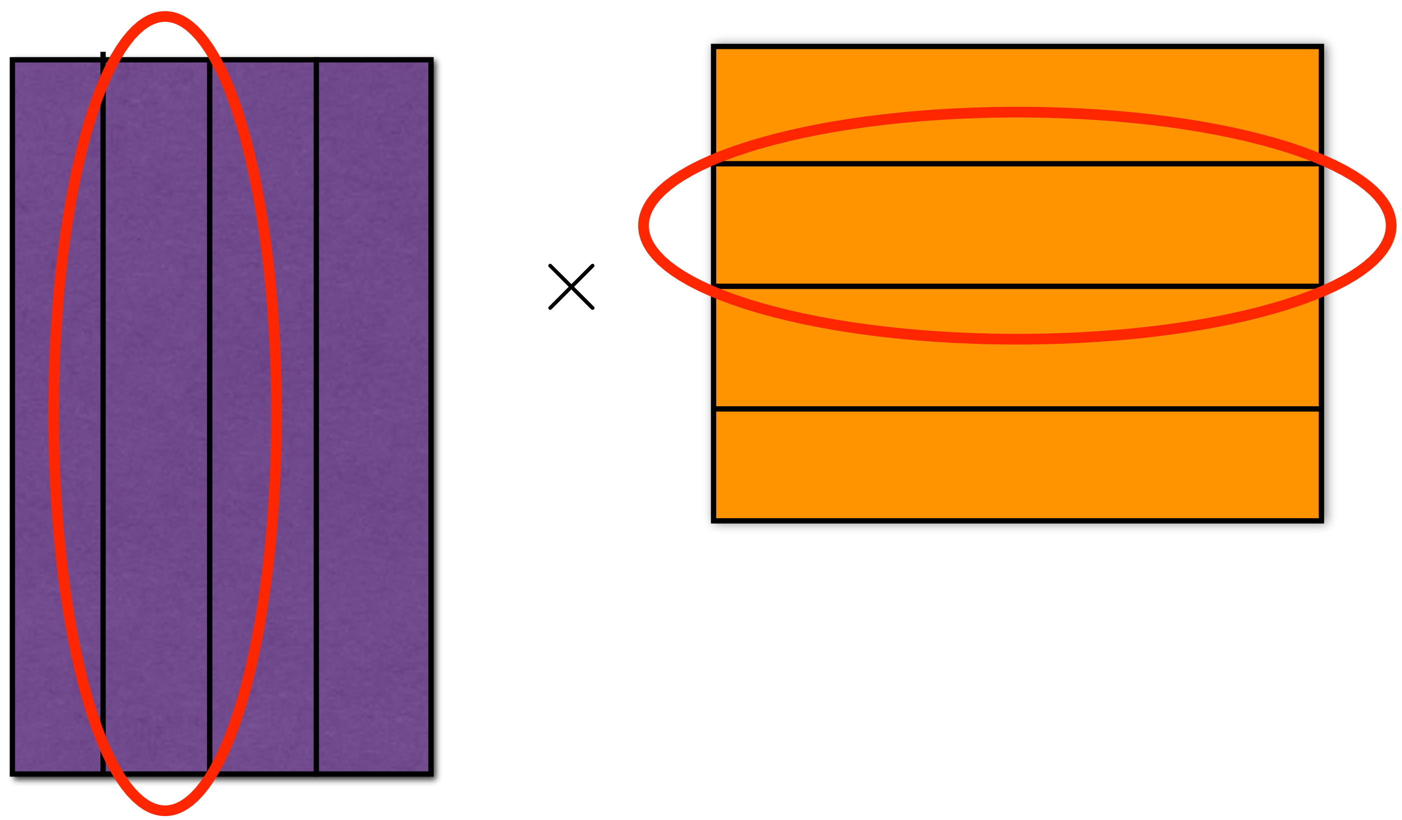
$\mathbf{X}^T = \mathbf{VW}$ with $\mathbf{W}$ columns sparse

Spatial frequencies dictionary

Monochromatic PSFs joint super-resolution

Constraints

$\mathbf{X} = \mathbf{S}\mathbf{A} =$



$\mathbf{\Phi S}$ columns sparse $\mathbf{A}^T = \mathbf{V}\boldsymbol{\alpha}^T$ with $\boldsymbol{\alpha}^T$ columns sparse

$\mathbf{S}$

Monochromatic PSFs joint super-resolution

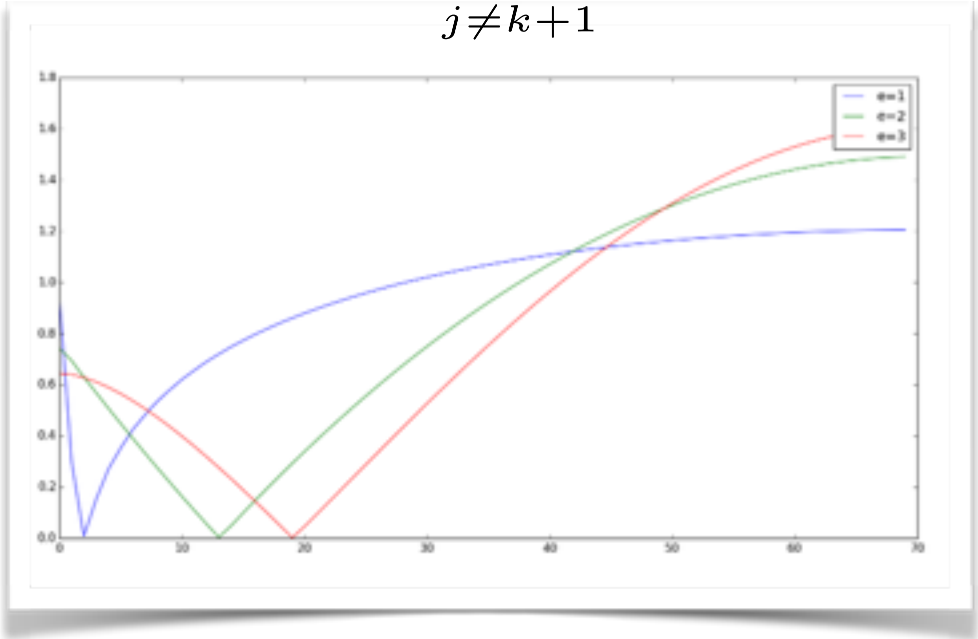
Spatial frequency dictionary

Observed stars spatial locations: $(\mathbf{u}_i)_{1 \leq i \leq p}$

1D case, regular spacing, p=2k+1

$$\psi_i = \psi_{p-i+1} = -1/|\mathbf{u}_i - \mathbf{u}_{k+1}|^e \qquad a > 0, e > 0$$

$$\psi_i = \sum_{\substack{j=1 \\ j \neq k+1}}^{p} a/|\mathbf{u}_j - \mathbf{u}_{k+1}|^e$$



"Notch" filters: varying notch frequency
depending on e and a

$$\Psi_{e,a}(\mathbf{v}) = \|\mathbf{v} \star \boldsymbol{\psi}_{e,a}\|_2^2$$

$$\boldsymbol{\Psi} : M_{rp}(\mathbb{R}) \mapsto \mathbb{R}^+ \ , \mathbf{A} \rightarrow \sum_{i=1}^{r} \Psi_{e_i,a}(\mathbf{A}[i,:])$$

$$0 < e_1 < e_2 < \cdots < e_r$$

Monochromatic PSFs joint super-resolution

Spatial frequency dictionary

$$\widehat{\Psi}_{e,a} : \mathbb{R}^p \mapsto \mathbb{R}^+ \ , \mathbf{v} \rightarrow \sum_{k=1}^{p} \Big(\sum_{\substack{i=1 \\ i \neq k}}^{p} \frac{a v_k - v_i}{\|\mathbf{u}_k - \mathbf{u}_i\|_2^e}\Big)^2$$

$$\widehat{\Psi}_{e,a}(\mathbf{v}) = \|\mathbf{P}_{e,a}\mathbf{v}\|_2^2$$

$$\mathbf{P}_{e,a}[i,j] = -\frac{1}{\|\mathbf{u}_i - \mathbf{u}_j\|_2^e} \text{ if } i \neq j \qquad \mathbf{P}_{e,a}[i,i] = \sum_{\substack{j=1 \\ j \neq i}}^{p} \frac{a}{\|\mathbf{u}_i - \mathbf{u}_j\|_2^e}$$

$$\widehat{\Psi}_{e,a}(\mathbf{v}) = \mathbf{v}^T \mathbf{Q}_{e,a} \mathbf{v}, \quad \mathbf{Q}_{e,a} = \mathbf{P}_{e,a}^T \mathbf{P}_{e,a} = \mathbf{V}_{e,a} \mathbf{D}_{e,a} \mathbf{V}_{e,a}^T$$

$$\mathbf{\Psi}(\mathbf{A}) = \sum_{i=1}^{r} \sum_{j=1}^{p} \mathbf{d}_{e_i,a}[j] \langle \mathbf{v}, \mathbf{V}_{e_i,a}[:,j] \rangle^2$$

Monochromatic PSFs joint super-resolution

Spatial frequency dictionary

$$(e_i, a_i)_{1 \leq i \leq r} \qquad \mathbf{V} = [\mathbf{V}_{e_1,a_1}, \dots, \mathbf{V}_{e_r,a_r}]$$

$\mathbf{P}_{e,1}$: laplacian matrix of a graph

Monochromatic PSFs joint super-resolution

Optimization problem

$$\Omega_1 = \{\boldsymbol{\alpha} \in M_{r,N}(\mathbb{R}) / \|\boldsymbol{\alpha}[l,:]\|_0 \leq \eta_l,\ l = 1 \ldots r\}$$

$$\Omega_2 = \{(\mathbf{S}, \boldsymbol{\alpha}) \in M_{nr}(\mathbb{R}) \times M_{r,N}(\mathbb{R}) / \mathbf{S}\boldsymbol{\alpha}\mathbf{V}^T \geq_{M_{np}(\mathbb{R})} 0\}$$

$$\min_{\boldsymbol{\alpha}, \mathbf{S}} \frac{1}{2} \|\mathbf{Y} - \mathcal{F}(\mathbf{S}\boldsymbol{\alpha}\mathbf{V}^T)\|_F^2 + \sum_{i=1}^{r} \|\mathbf{w}_i \odot \boldsymbol{\Phi}_s \odot \mathbf{s}_i\|_1 + \iota_{\Omega_1}(\boldsymbol{\alpha}) + \iota_{\Omega_2}(\mathbf{S}, \boldsymbol{\alpha})$$

Monochromatic PSFs joint super-resolution

Algorithm 1 Resolved components analysis (RCA)

Parameters estimation and initialization:
Harmonic constraint parameters $(e_i, a_i)_{1\leq i\leq r} \to \mathbf{V}, \mathbf{A}_0$
Noise level, $\mathbf{A}_0 \to \mathbf{W}_{0,0}$
Alternate minimization
for $k = 0$ to $k_{\max}$ **do**
    for $j = 0$ to $j_{\max}$ **do**
        $\mathbf{S}_k = \underset{\mathbf{S}}{\operatorname{argmin}} \frac{1}{2}\|\mathbf{Y} - \mathcal{F}(\mathbf{S}\mathbf{A}_k)\|_F^2 + \sum_{i=1}^{r} \|\mathbf{W}_{k,j}[:, i] \odot \mathbf{\Phi}_s \mathbf{S}[:, i]\|_1$ s.t. $\mathbf{S}\mathbf{A}_k \geq 0$
        update: $\mathbf{W}_{k,0}, \mathbf{S}_k \to \text{update}(\mathbf{W}_{k,j+1})$
    end for
    $\boldsymbol{\alpha}_{k+1} = \underset{\boldsymbol{\alpha}}{\operatorname{argmin}} \frac{1}{2}\|\mathbf{Y} - \mathcal{F}(\mathbf{S}_k \boldsymbol{\alpha} \mathbf{V}^T)\|_F^2$ s.t. $\|\boldsymbol{\alpha}[l, :]\|_0 \leq \eta_l$
    update: Noise level, $\boldsymbol{\alpha}_{k+1} \to \mathbf{W}_{k+1,0}$
    $\mathbf{A}_{k+1} = \boldsymbol{\alpha}_{k+1}\mathbf{V}^T$
    $\mathbf{A}_{k+1}[i, :] = \mathbf{A}_{k+1}[i, :]/\|\mathbf{A}_{k+1}[i, :]\|_2$, for $i = 1 \ldots r$
end for
Return: $\mathbf{S}_{k_{\max}}$, $\mathbf{A}_{k_{\max}}$.

Monochromatic PSFs joint super-resolution

Numerical results

Data

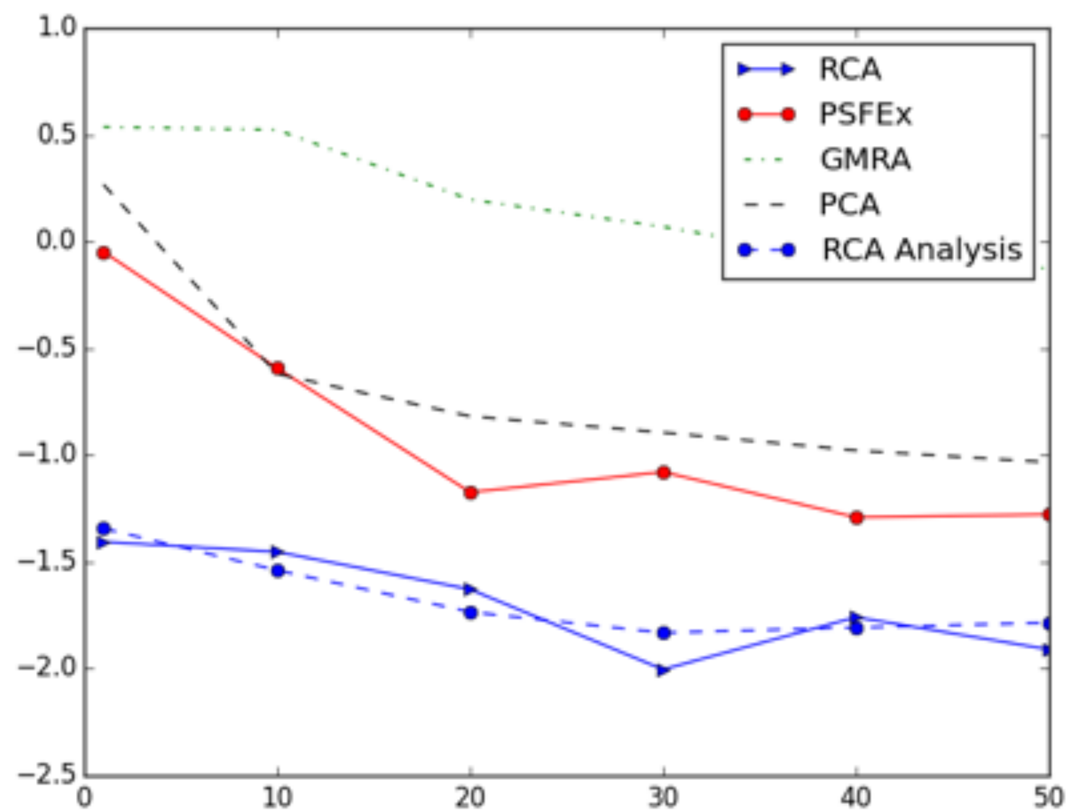
- 500 42x42 simulated Euclid telescope optical PSFs
- Realistic effects included such polishing and alignement defects
- Upsampling factor of 2 in the SR case

Monochromatic PSFs joint super-resolution

Numerical results

Dimension reduction

Ellipticity vector error

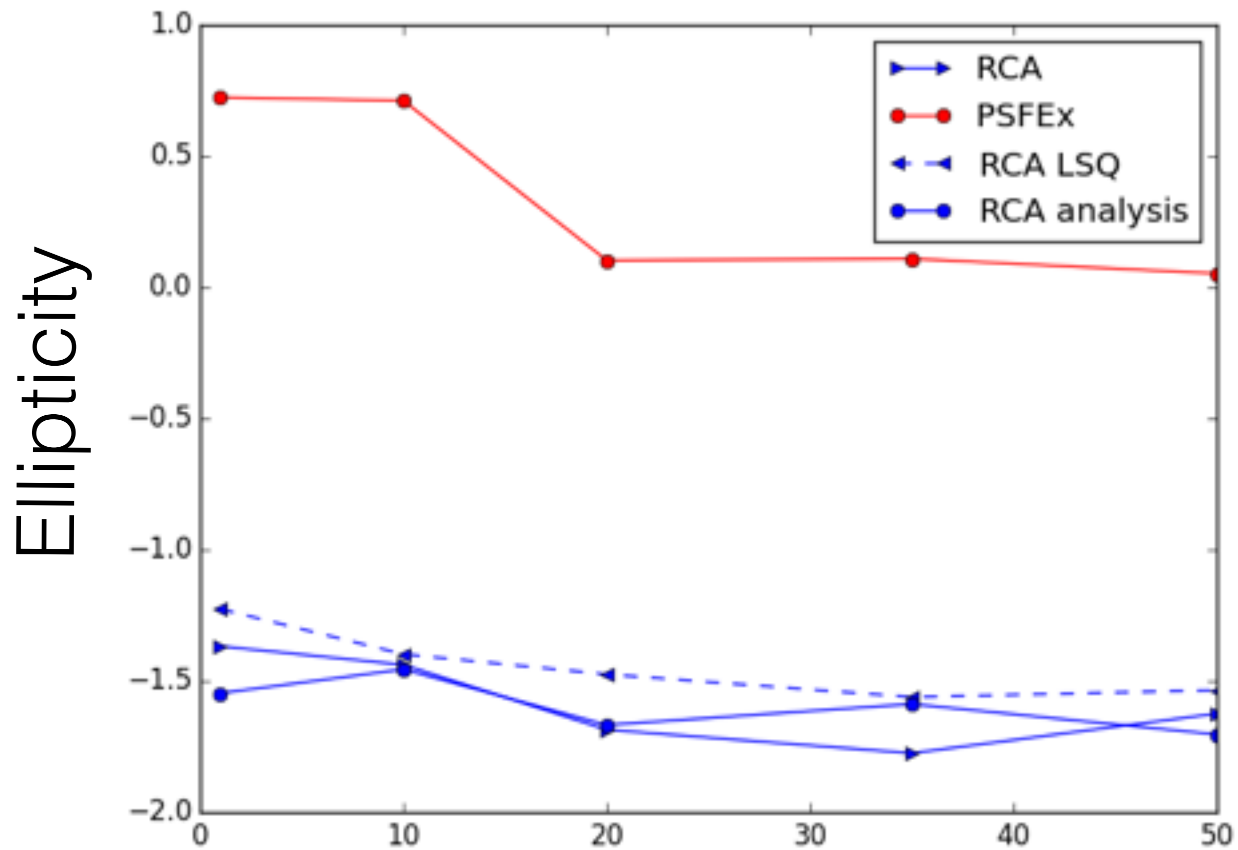


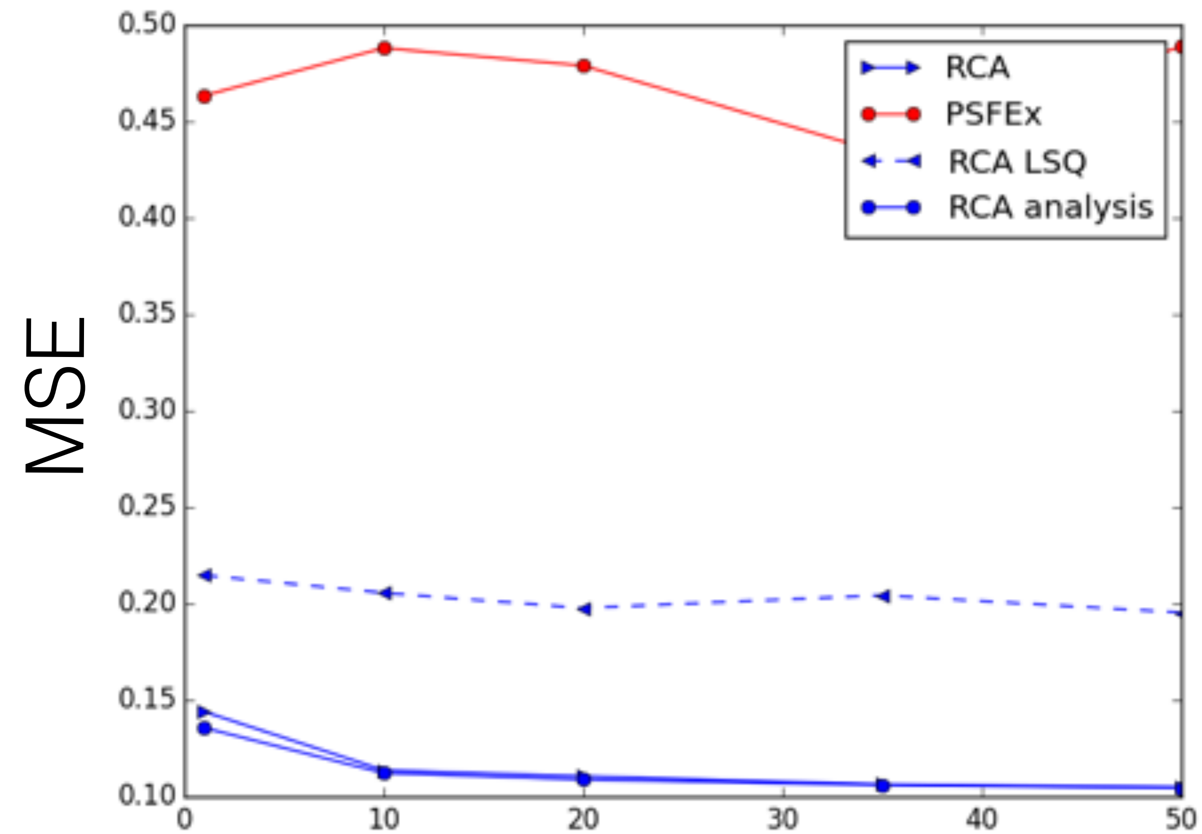
SNR

Monochromatic PSFs joint super-resolution

Numerical results

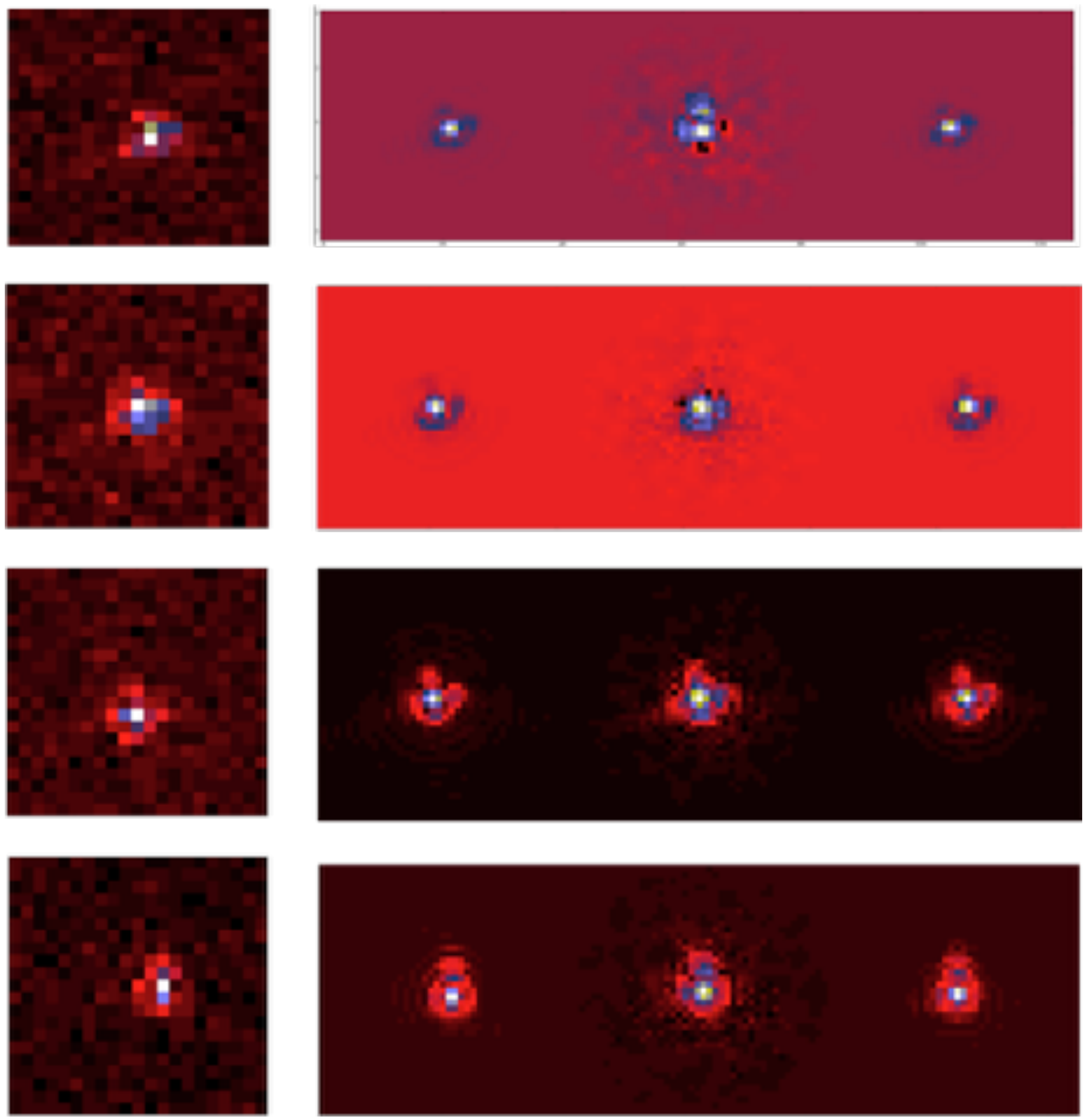
Dimension reduction and super-resolution





SNR

Monochromatic PSFs joint super-resolution



Conclusion

- Noise robust dimension reduction and super-resolution method taking advantage of the PSFs field spatial regularity
- Good accuracy on both PSFs shape and pixels values

Bibliography

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- Condat, Laurent. "*A primal–dual splitting method for convex optimization involving Lipschitzian, proximable and linear composite terms.*" Journal of Optimization Theory and Applications 158.2 (2013): 460-479..

Thank you for your attention!