Probing the (an)-isotropy of expansion with weak gravitational lensing

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Why investigating (an)-isotropy in expansion?

- To test the cosmological principle
  - We keep the homogeneity assumption
  - We release the isotropy assumption and want to constrain the anisotropy.

- To constrain the nature of Dark Energy

  We investigate a possible homogeneous anisotropic stress.

Anisotropic spaces in a two slides

Geometry: Spatial sections indexed by the time

$$g_{\mu\nu} = -\mathrm{d}t_\mu \otimes \mathrm{d}t_\nu + h_{ij}(t) {e^i}_\mu \otimes {e^j}_\nu$$

- $h_{ij}(t)$ tells how each spatial slice is glued to the previous one

- $[e_i, e_j] = {C^k}_{ij} e_k$ gives the structure of the spatial slices.

  Classification of anisotropic spaces from these constants.

$$^3\bar{R}_{ij}{}^{kl} = -\frac{1}{2}{C^p}_{ij}{C_p}^{kl} + \frac{1}{2}{C_p}^l{}_i {C^{pk}}_j + {C_p}^l{}_j {C_i}^{kp} + {C_p}^l{}_j {C^k}_i{}^p + C_{ijp}$$

Second slide

- $h_{ij}(t) = a(t)^2\delta_{ij}$

  Friedmann Lemaître (FL) case (for some cases).

  If $e^i = \mathrm{d}x^i,\ C^k{}_{ij} = 0$ Flat FL.

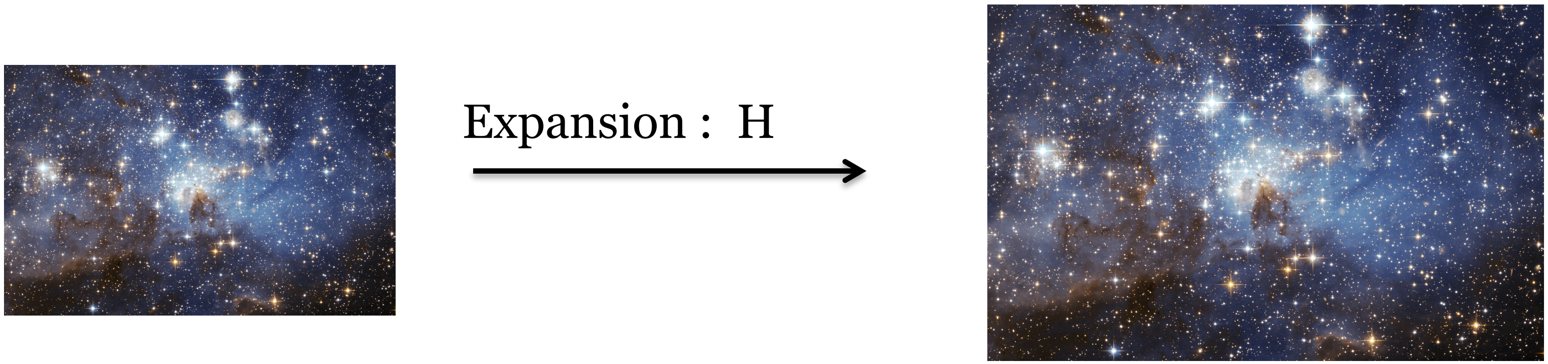
  The expansion describes everything $H = \frac{\dot{a}}{a}$

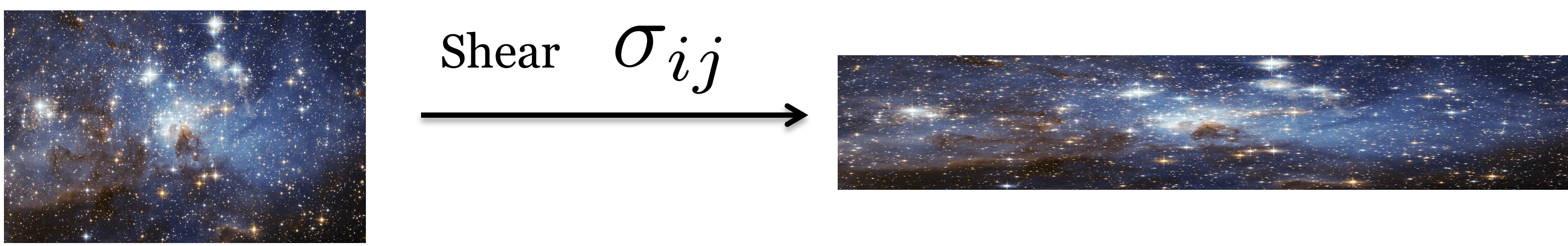
- $h_{ij} = a^2(t)[\mathrm{e}^{2\beta}]_{ij} = a^2(t)(\delta_{ij} + 2\beta_{ij} + \dots)$

  Matrix β_{ij} is traceless and encodes volume preserving deformations.

  Deformation rate is the shear $\boxed{\sigma_{ij} \equiv \dot{\beta}_{ij}}$

Illustration of shear and expansion





Friedmann equation

$$H^2 = \frac{8\pi G\rho}{3} + \frac{\sigma_{ij}\sigma^{ij}}{6} + \propto \frac{1}{a^2}\left[C^i{}_{jk}C_i{}^{jk} + \ldots\right]$$

C^i_{jk} Contributes as $1/a^2$

$\sigma^2 = \sigma_{ij}\sigma^{ij}$ Contributes as $1/a^6$

-Shear effects relevant at early times (BBN, CMB)

-Constants of structure effects relevant *at late time see e.g. Bianchi VIIh papers.*

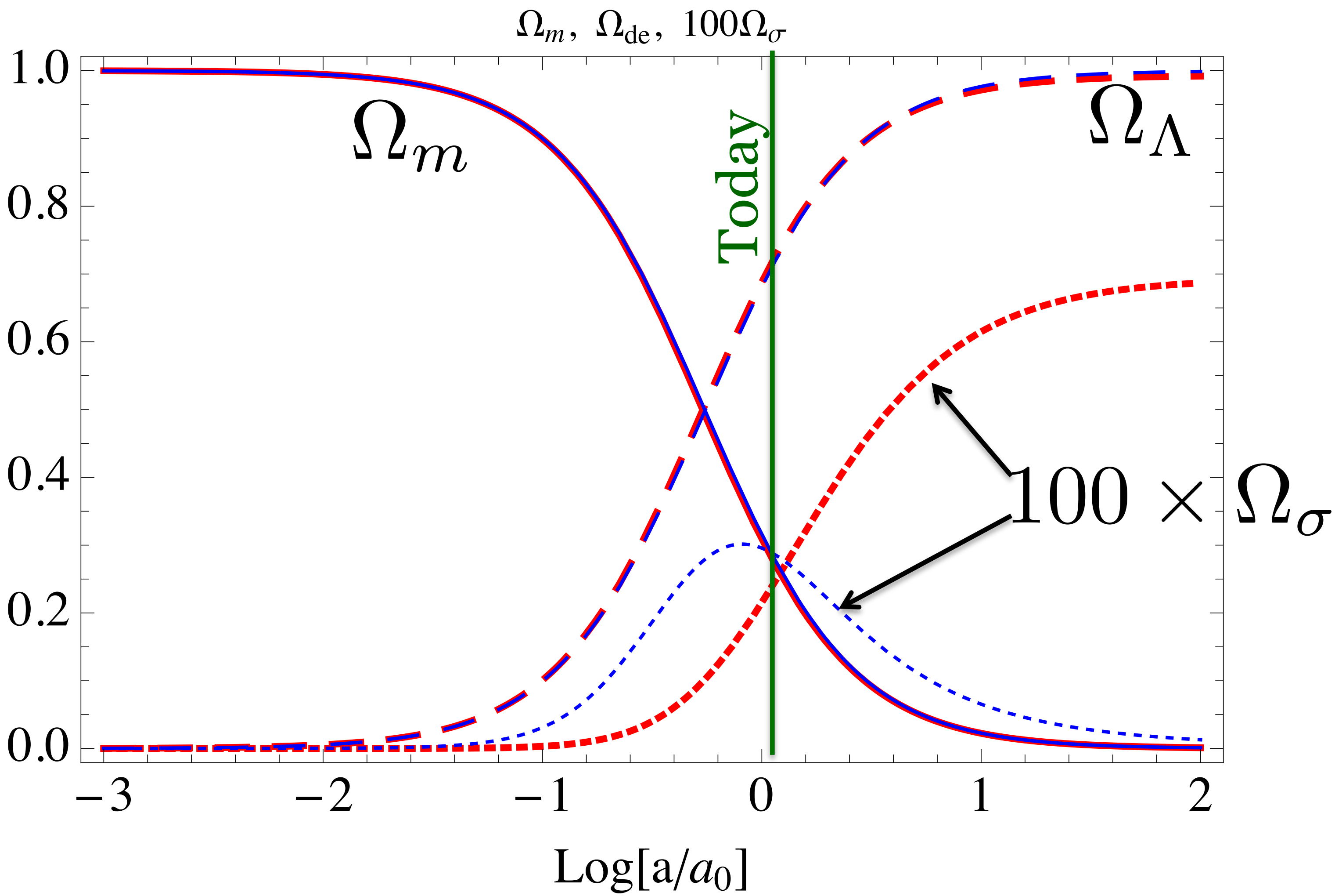
Bianchi I

$$\sigma_{ij} \neq 0$$

$$C^k{}_{ij} = 0$$

Unless there is anisotropic stress

$$\sigma'_{ij} + 3H\sigma_{ij} = \Pi_{ij} \neq 0$$

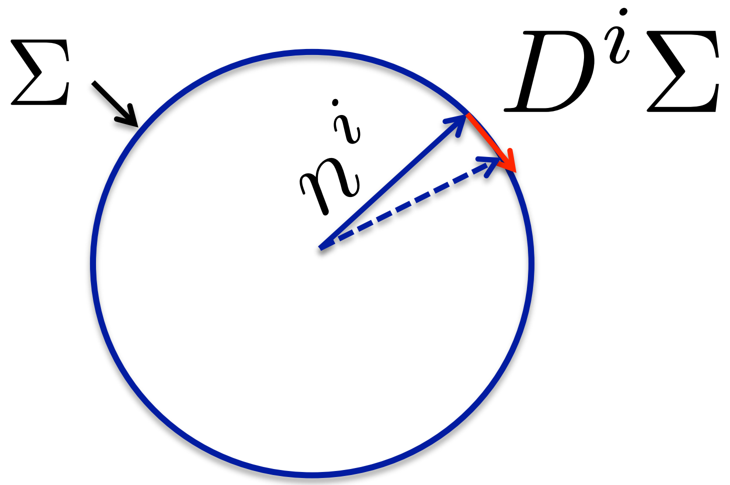


Observation? Effects on geodesic

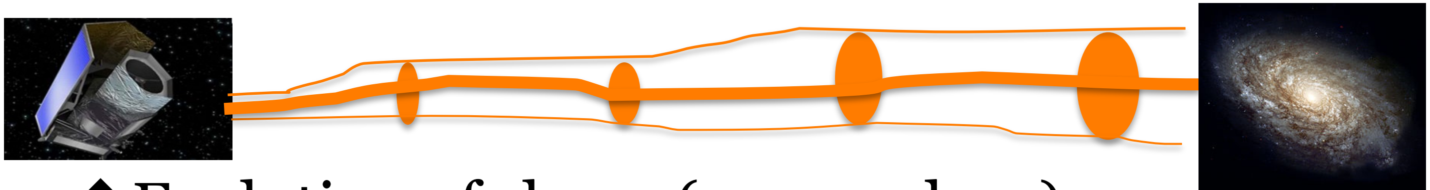
- Evolution of Energy $$\dot{E} + HE + 2\Sigma E = 0$$

- Evolution of direction $$\dot{n}^i = -D^i\Sigma$$

Lensing Potential $$\Sigma = \frac{1}{2}\sigma_{ij}n^in^j$$



Effect on geodesic deviation (weak lensing)



◆ Evolution of shape (e.g. a galaxy)

$$\frac{\mathrm{d}^2\mathcal{D}^a_b}{\mathrm{d}\lambda^2} = \mathcal{R}^a_c \mathcal{D}^c_b$$

$\mathcal{D}^c_b$ → Deformation Matrix

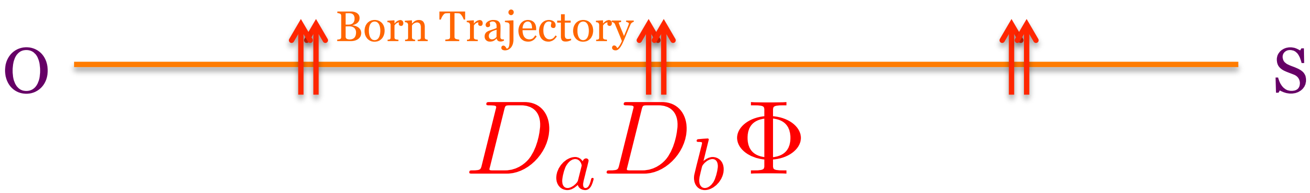
$\mathrm{d}\lambda^2$ → Derivative on central geodesic

$\mathcal{R}^a_c$ → Source related to curvature

◆ FL case : $\mathcal{R}_{ab} = 2 D_a D_b \Phi$

D_a → Gradient orthogonal to line of sight

Φ → Gravitational potential (metric perturbation)



Effect of anisotropic background

◆ Anisotropy as a perturbation

Physical Space $\xrightarrow{\;\Phi\;}$ Anisotropic Background $\xrightarrow{\;\frac{\sigma}{H},\ \beta_{ij}\;}$ FL Background

◆ Double expansion scheme.

◆ Effect of anisotropy appears at order $\frac{\sigma}{H}\Phi$

Harmonic space

$$\mathcal{D}_{ab} \equiv \kappa I_{ab} + V \epsilon_{ab} + \gamma_{ab}$$

$$\gamma_{ab} = D_a D_b E + \epsilon_{ac} D^c D_b B$$

Depends on the observing direction $\mathbf{n}^o$

$$E(\mathbf{n}^o) = \sum_{\ell\, m} E_{\ell m} Y_{\ell m}(\mathbf{n}^o)$$

$$\kappa(\mathbf{n}^o) = \sum_{\ell\, m} \kappa_{\ell m} Y_{\ell m}(\mathbf{n}^o)$$

$$B(\mathbf{n}^o) = \sum_{\ell\, m} B_{\ell m} Y_{\ell m}(\mathbf{n}^o)$$

Harmonic dictionary

$$D_a \rightarrow \ell$$

Dominant effect

Standard FL lensing

$$\gamma_{ab}^{\rm FL} \propto 2D_aD_b\Phi$$ ➡ Only E modes

Anisotropic case. Dominant effect is

$$\gamma_{ab}^{\rm Anis} \propto D_c\alpha D^c\left(2D_aD_b\Phi\right)$$

α is related to $\beta_{ij}n^in^j$

Anisotropy acts like a deflecting potential on the central geodesic

E-B mode structure

- **Analogy with CMB lensing:** $Q \pm \mathrm{i}U \to \gamma^{\pm}$
  B modes generated by lensing.

$$\tilde{T}(\mathbf{n}) = T(\mathbf{n}) + D_a\alpha(\mathbf{n})D^aT(\mathbf{n})$$

$$\widetilde{Q \pm \mathrm{i}U} = Q \pm \mathrm{i}U + D_a\alpha D^a(Q \pm \mathrm{i}U)$$

$$Q \pm \mathrm{i}U = \sum_{\ell m} E_{\ell m} \pm \mathrm{i}B_{\ell m}Y^{\pm 2}_{\ell m}$$

- **Difference with CMB lensing:**
  The lensing potential is a pure quadrupole.
  It is NOT-statistical
  The 5 degrees of freedom are those of β_{ij}

$$\alpha(\mathbf{n}^o) = \sum_m \alpha_{2m}Y_{2m}(\mathbf{n}^o)$$

E-B mode structure

- Analogy with local boost (peculiar velocity)

$$\tilde{T} = (1+\alpha)T - D_a \alpha D^a T \qquad \alpha = \mathbf{v}\cdot\mathbf{n}$$

Energy effect is $\alpha T \to Y_{\ell_1 m_1} Y_{\ell_2 m_2}$

Aberration effect is $D_a \alpha D^a T \to D_a Y_{\ell_1 m_1} D^a Y_{\ell_2 m_2}$

- However α is a pure non-statistical dipole (l=1).

- For an anisotropic expansion we look for a quadrupolar modulation which is of aberration type. We look into the shear so the analogy is with aberrated CMB polarization.

Ylm-ology

$$Y_{\ell_1 m_1} Y_{\ell_2 m_2} = C^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} C^{000}_{\ell_1 \ell_2 \ell_3} Y_{\ell_3 m_3}$$

$$D_a Y_{\ell_1 m_1} D^a Y_{\ell_2 m_2} = f(\ell_1, \ell_2, \ell_3) Y_{\ell_1 m_1} Y_{\ell_2 m_2}$$

Local boost :

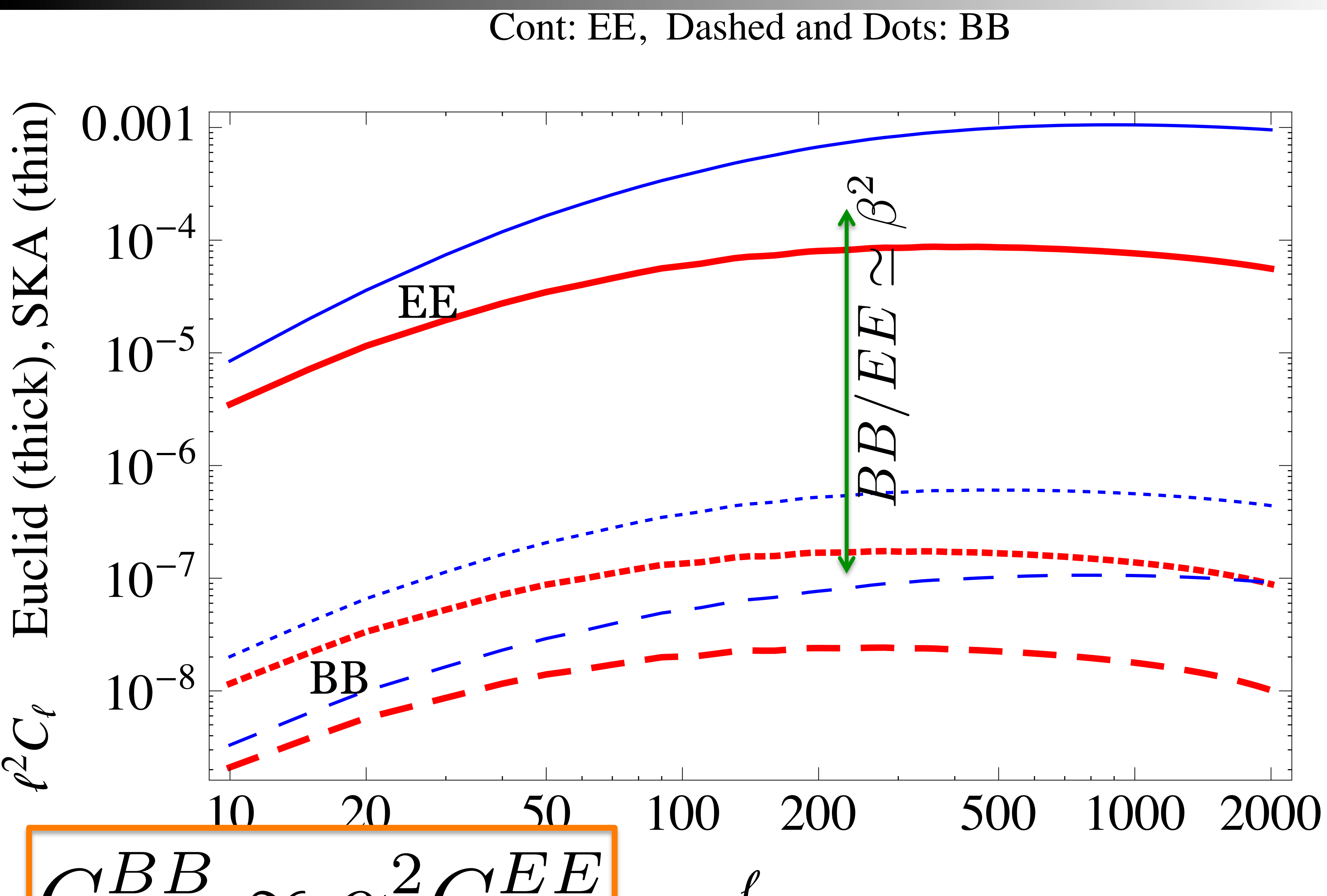
$$\ell_1 = 1 \qquad \ell_3 = \ell_2 \pm 1$$

Anisotropic expansion

$$\ell_1 = 2 \qquad \ell_2 - 2 \leq \ell_3 \leq \ell_2 + 2$$

Determines the structure of the correlations

BB correlations



$$C_\ell^{BB} \propto \alpha^2 C_\ell^{EE}$$

Off-diagonal correlations. EE and EB.

The anisotropy generates also off-diagonal correlations

$$\langle E_{\ell m} E^{\star}_{\ell+2 m+M} \rangle \qquad \propto \alpha_{2M} C_\ell^{EE}$$

$$\langle B_{\ell m} E^{\star}_{\ell+1 m+M} \rangle$$

$$M = -2, -1, 0, 1, 2$$

Interesting for two reasons

- Of order β compared to EE correlations.
- Enable to get the components α_{2M} of the deflecting potential and thus the β_{ij} .

Conclusion.

- BB correlations in weak lensing can probe a late anisotropic era
- The sensitivity in β^2 is given by the upper bounds on C_ℓ^{BB}
- Off diagonal correlations are more powerful (first order in β)
- Off diagonal correlations allow for consistency checks.
- The estimators can be based on those used to measure our peculiar velocity from CMB off-diagonal components
  « epure si muove » **Planck 2013 results. XXVII. 1303.5087**
  and all references therein