False-alarm regulation for target detection in Hyperspectral Imaging

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***Abstract*—Classical target detection schemes are usually obtained deriving the likelihood ratio under Gaussian hypothesis and replacing the unknown background parameters by their estimates. In this paper, the adaptive versions of the classical Matched Filter and the Normalized Matched Filter are analyzed for the case when the mean vector of the background is unknown and has to be estimated jointly with the covariance matrix, as it is the case in hyperspectral imaging. More precisely, theoretical closed form expressions for false-alarm regulation are derived and these results are extended to non-Gaussian cases using robust estimation procedures. Finally, simulations validate the theoretical contribution.**

I. INTRODUCTION

Target detection tasks arise in many different military and civilian applications and have been widely investigated in several signal processing domains such as radar, sonar, communications, etc. In many applications, signals are assumed to be Gaussian with zero mean or with a known mean vector that can be removed. In such context, Statistical Detection Theory [1] has led to several well-known algorithms, for instance the Matched Filter (MF) and its adaptive versions, the Kelly Detector [2] or the Adaptive Normalized Matched Filter [3].

This paper addresses the problem of target detection in Hyperspectral Imaging (HSI). HSI extends from the fact that for any given material, the amount of radiation emitted varies with wavelength. HSI sensors measure the radiance of the materials within each pixel area at a very large number of contiguous spectral bands and provide image data containing both spatial and spectral informations. Hyperspectral target detection and anomaly detection may be used to locate targets that generally cannot be detected using only spatial resolution [4]. In this case, data represent reflectance values, and hence, both covariance matrix and mean vector have to be estimated to model the background in the detection process.

This work deals with the classical Adaptive Matched Filter (AMF) and the Adaptive Normalized Matched Filter (ANMF). Both detectors have been derived under Gaussian assumptions and benefit from great popularity in HSI target detection literature, see e.g. [5], [6]. To evaluate the detector performance, the classical process, according to the Neyman-Pearson criterion is first to regulate the false-alarm, by setting is a detection threshold for a given probability of false-alarm (PFA). Then, the probability of detection is evaluated for different Signal-to-Noise Ratios (SNR). Therefore, keeping the false-alarm rate constant (CFAR) is essential to set a proper detection threshold. The aim is to build a CFAR detector which provides detection thresholds that are relatively immune to noise and background variation, and allow target detection with a constant false-alarm rate. The theoretical analysis of CFAR methods for adaptive detectors is a challenging problem since in adaptive schemes, the statistical distribution of the detectors is not always available in a closed-form expression. The main contribution of this article is the exact derivation of the distribution of the proposed detection schemes under null hypothesis where the mean of the background is unknown an has to be estimated. Through Gaussian assumption, closed-form expressions for the false-alarm regulation are obtained, i.e. they allow to theoretically set the detection threshold for a given PFA.

Since Gaussian assumption is not always valid for real hyperspectral data, alternative robust estimation techniques are proposed in this work. They can be used jointly with the ANMF detection test to obtain good detection performance. Due to the lack of space, only the Fixed Point (FP) estimators (also called Tyler's estimators [7]) are used as an alternative to the Sample Covariance Matrix (SCM) and the Sample Mean Vector (SMV). The resulting detection performance are analyzed and are shown to be similar than those obtained with the SCM-SMV under Gaussian assumptions while they overcome under non-Gaussian contexts.

This paper is organized as follows. Section II introduces the required background on classical detection techniques, while Section III provides the main contribution by deriving the exact "PFA-threshold" relationship for both the AMF and the ANMF under Gaussian assumption. Then, in Section IV, simulations validate the theoretical analysis while conclusions and perspectives are given in Section V.

II. BACKGROUND

A. Complex Normal distributions

A m-dimensional vector $\mathbf{x} = \mathbf{u} + j\mathbf{v}$ has a complex normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma} = E[(\mathbf{x} - \boldsymbol{\mu})(\mathbf{x} - \boldsymbol{\mu})^H]$, denoted $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, if $\mathbf{z} = (\mathbf{u}^T, \mathbf{v}^Y)^T \in \mathbb{R}^{2m}$

has a normal distribution [8]. If rank$(\mathbf{\Sigma}) = m$, the probability density function exists and is of the form

$$f_{\mathbf{x}}(\mathbf{x}) = \pi^{-m}|\mathbf{\Sigma}|^{-1}\exp\{-(\mathbf{x}-\boldsymbol{\mu})^H\mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\}.$$

The resulting Maximum Likelihood Estimates (MLE) are the well-known SCM and SMV defined as:

$$\hat{\boldsymbol{\mu}}_{SMV} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i \quad \hat{\mathbf{\Sigma}}_{SCM} = \frac{1}{N}\sum_{i=1}^{N}(\mathbf{x}_i-\hat{\boldsymbol{\mu}})(\mathbf{x}_i-\hat{\boldsymbol{\mu}})^H$$

where the $\mathbf{x}_i$ are independent and identically distributed (IID) $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{\Sigma})$.

B. Target Detection Schemes

The problem of the detecting a known signal $\mathbf{s}$ corrupted by an additive noise $\mathbf{b}$ in a m-dimensional complex vector $\mathbf{x}$ can be stated as a the following binary hypothesis test:

$$\begin{cases} H_0 : \mathbf{x} = \mathbf{b} & \mathbf{x}_i = \mathbf{b}_i\,,\ i = 1,\ldots,N \\ H_1 : \mathbf{x} = \mathbf{s}+\mathbf{b} & \mathbf{x}_i = \mathbf{b}_i\,,\ i = 1,\ldots,N, \end{cases}$$

where the $\mathbf{x}_i$ are the so-called secondary data (signal-free) used to estimate the noise parameters.

1) Adaptive Matched Filter: the MF detector is the optimal linear filter for maximizing the SNR in the presence of additive Gaussian noise with known parameters [1] and takes the form:

$$\Lambda_{MF} = \frac{|\mathbf{p}^H\,\mathbf{\Sigma}^{-1}\,(\mathbf{x}-\boldsymbol{\mu})|^2}{(\mathbf{p}^H\,\mathbf{\Sigma}^{-1}\,\mathbf{p})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda\,.$$

Note that it differs from the classical MF by the term $\boldsymbol{\mu}$, the background mean, but without any consequence since $\mathbf{x}-\boldsymbol{\mu} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$. Moreover, the "PFA-threshold" relationship is given by:

$$PFA_{MF} = \exp(-\lambda).$$

The AMF, denoted $\Lambda^{(N)}_{AMF\,\hat{\mathbf{\Sigma}}}$ to underline the dependency with N, is usually built replacing the covariance matrix $\mathbf{\Sigma}$ by its estimate $\hat{\mathbf{\Sigma}}$ obtained from the N secondary data. The mean vector is generally supposed to be known. Thus, the adaptive version becomes:

$$\Lambda^{(N)}_{AMF\,\hat{\mathbf{\Sigma}}} = \frac{|\mathbf{p}^H\,\hat{\mathbf{\Sigma}}^{-1}\,(\mathbf{x}-\boldsymbol{\mu})|^2}{(\mathbf{p}^H\,\hat{\mathbf{\Sigma}}^{-1}\,\mathbf{p})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda\,.$$

Then, the theoretical "PFA-threshold" relationship is given by [9]:

$$PFA_{AMF\,\hat{\mathbf{\Sigma}}} = {}_2F_1\left(N-m+1,\, N-m+2;\, N+1;\, -\frac{\lambda}{N}\right), \quad (1)$$

where ${}_2F_1(\cdot)$ is the hypergeometric function [10].

2) Adaptive Normalized Matched Filter: The Normalized Matched Filter (NMF) is obtained when considering that the covariance matrix is different under the two hypotheses. That is to say that the clutter has the same covariance structure but different variance. Therefore, a canonical detector, relying on a generalized likelihood ratio design approach, is the one implementing the following test [11] :

$$\Lambda_{NMF} = \frac{|\mathbf{p}^H\,\mathbf{\Sigma}^{-1}\,(\mathbf{x}-\boldsymbol{\mu})|^2}{(\mathbf{p}^H\,\mathbf{\Sigma}^{-1}\mathbf{p})\,\left((\mathbf{x}-\boldsymbol{\mu})^H\,\mathbf{\Sigma}^{-1}\,(\mathbf{x}-\boldsymbol{\mu})\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda,$$

for which one has [11] :

$$PFA_{NMF} = (1-\lambda)^{(m-1)}\,.$$

The ANMF is generally obtained when the unknown noise covariance matrix is replaced by an estimate [12]:

$$\Lambda^{(N)}_{ANMF\,\hat{\mathbf{\Sigma}}} = \frac{|\mathbf{p}^H\,\hat{\mathbf{\Sigma}}^{-1}\,(\mathbf{x}-\boldsymbol{\mu})|^2}{(\mathbf{p}^H\,\hat{\mathbf{\Sigma}}^{-1}\mathbf{p})\,\left((\mathbf{x}-\boldsymbol{\mu})^H\,\hat{\mathbf{\Sigma}}^{-1}\,(\mathbf{x}-\boldsymbol{\mu})\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda.$$

And the PFA follows [12]

$$PFA_{ANMF\,\hat{\mathbf{\Sigma}}} = (1-\lambda)^{a-1}\,{}_2F_1(a, a-1; b-1; \lambda)\,, \quad (2)$$

where $a = N-m+2$ and $b = N+2$.

III. Main Results

Let us now assume that the mean parameter is unknown as it is the case in HSI.

A. Preliminary results: Wishart distribution

Let $\mathbf{x}_1, ..., \mathbf{x}_N$ be an IID N-sample, where $\mathbf{x}_i \sim \mathcal{CN}(\boldsymbol{\mu}, \mathbf{\Sigma})$. Let us define $\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{\mu}}_{SMV}$ and $\hat{\mathbf{W}} = N\,\hat{\mathbf{\Sigma}}_{SCM}$ referred to as a Wishart matrix. Thus one has (see [13] for the real case):

- $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{W}}$ are independently distributed;
- $\hat{\boldsymbol{\mu}} \sim \mathcal{CN}(\boldsymbol{\mu}, \frac{1}{N}\mathbf{\Sigma})$;
- $\hat{\mathbf{W}} \sim \mathcal{CW}(N-1, \mathbf{\Sigma})$ is Whishart distributed with $N-1$ degrees of freedom

B. Adaptive Matched Filter Detector

When both covariance matrix and mean vector are unknown and have to be estimated from the secondary data, the AMF detector takes the following form:

$$\Lambda^{(N)}_{AMF\,\hat{\mathbf{\Sigma}},\hat{\boldsymbol{\mu}}} = \frac{|\mathbf{p}^H\,\hat{\mathbf{\Sigma}}^{-1}\,(\mathbf{x}-\hat{\boldsymbol{\mu}})|^2}{(\mathbf{p}^H\,\hat{\mathbf{\Sigma}}^{-1}\,\mathbf{p})} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

Proposition III.1 *The theoretical relationship between the PFA and the threshold is given by*

$$PFA_{AMF\,\hat{\mathbf{\Sigma}},\hat{\boldsymbol{\mu}}} = {}_2F_1\left(N-m,\, N-m+1;\, N;\, -\frac{\lambda'}{N-1}\right), \quad (3)$$

where $\lambda' = \frac{(N-1)^2}{N^2}\lambda$

Before turning into the proof, let us comment on this result. Interestingly, this detector also holds the CFAR property in the sense that its false-alarm expression depends only on the dimension m and on the number of secondary data N, but not on the noise parameters $\boldsymbol{\mu}$ and $\mathbf{\Sigma}$. Note that the only effect of estimating the mean is the loss of one degree of freedom and the modification of the threshold compared to eq. (1).

Proof: First, let us denote, for $\mathbf{x}_i \sim \mathcal{CN}(\boldsymbol{\mu}, \mathbf{\Sigma})$

$$\hat{\mathbf{W}}_{N-1} = \sum_{i=1}^{N}(\mathbf{x}_i-\hat{\boldsymbol{\mu}})(\mathbf{x}_i-\hat{\boldsymbol{\mu}})^H \sim \mathcal{CW}(N-1, \mathbf{\Sigma}),$$

Since $\hat{\boldsymbol{\mu}} \sim \mathcal{CN}(\boldsymbol{\mu}, \frac{1}{N}\mathbf{\Sigma})$, one has $\mathbf{x}-\hat{\boldsymbol{\mu}} \sim \mathcal{CN}(\mathbf{0}, \frac{N+1}{N}\mathbf{\Sigma})$. This can be equivalently rewritten as

$$\sqrt{N/(N+1)}(\mathbf{x}-\hat{\boldsymbol{\mu}}) \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}).$$

Now, let us set $\mathbf{y} = \sqrt{\frac{N}{N+1}}\,(\mathbf{x} - \hat{\boldsymbol{\mu}})$ with $\mathbf{y} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma})$.

When computing the SCM, one has

$$\hat{\boldsymbol{\Sigma}}_{SCM} = \frac{1}{N}\sum_{i=1}^{N}(\mathbf{x}_i - \hat{\boldsymbol{\mu}})(\mathbf{x}_i - \hat{\boldsymbol{\mu}})^H = \frac{1}{N}\hat{\mathbf{W}}_{N-1}.$$

As we jointly estimate the mean and the covariance matrix, we lose a degree of freedom compared with the only covariance matrix estimation problem.

Let us now consider the classical AMF test (i.e. $\boldsymbol{\mu}$ known) built from $N-1$ secondary data, rewritten with $\hat{\mathbf{W}}_{N-1}$ instead of the SCM estimate:

$$\Lambda^{(N-1)}_{AMF\,\hat{\boldsymbol{\Sigma}}} = (N-1)\,\frac{|\mathbf{p}^H\,\hat{\mathbf{W}}_{N-1}^{-1}\,\mathbf{y}|^2}{(\mathbf{p}^H\,\hat{\mathbf{W}}_{N-1}^{-1}\,\mathbf{p})},$$

where $\mathbf{y} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma})$ and whose "PFA-threshold" relationship is given by eq. (1) where N is replaced by $N-1$.

For the joint estimation problem, the AMF can be rewritten as:

$$\begin{aligned}\Lambda^{(N)}_{AMF\,\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} &= N\,\frac{|\mathbf{p}^H\,\hat{\mathbf{W}}_{N-1}^{-1}\,(\mathbf{x} - \hat{\boldsymbol{\mu}})|^2}{(\mathbf{p}^H\,\hat{\mathbf{W}}_{N-1}^{-1}\,\mathbf{p})}\\ &= N\,\frac{N+1}{N}\,\frac{|\mathbf{p}^H\,\hat{\mathbf{W}}_{N-1}^{-1}\,\mathbf{y}|^2}{(\mathbf{p}^H\,\hat{\mathbf{W}}_{N-1}^{-1}\,\mathbf{p})}\\ &= \frac{(N+1)}{(N-1)}\,\Lambda^{(N-1)}_{AMF\,\hat{\boldsymbol{\Sigma}}}\end{aligned}$$

where $(\mathbf{x} - \hat{\boldsymbol{\mu}})$ has been replaced by $\sqrt{N+1/N}\,\mathbf{y}$ with $\mathbf{y} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma})$, as previously.

Hence, one can determine the false-alarm relationship:

$$\begin{aligned}PFA_{AMF\,\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} &= \mathbb{P}\left(\Lambda^{(N)}_{AMF\,\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} > \lambda | H_0\right)\\ &= \mathbb{P}\left(\frac{(N+1)}{(N-1)}\Lambda^{(N-1)}_{AMF\,\hat{\boldsymbol{\Sigma}}} > \lambda | H_0\right)\\ &= \mathbb{P}(\Lambda^{(N-1)}_{AMF\,\hat{\boldsymbol{\Sigma}}} > \lambda' | H_0)\end{aligned}$$

where $\lambda' = \frac{(N-1)}{(N+1)}\lambda$, which leads to the conclusion. ■

C. Adaptive Normalized Matched Filter

Similarly, the ANMF for both mean vector and covariance matrix estimation becomes:

$$\Lambda_{ANMF\,\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = \frac{|\mathbf{p}^H\,\hat{\boldsymbol{\Sigma}}^{-1}\,(\mathbf{x} - \hat{\boldsymbol{\mu}})|^2}{(\mathbf{p}^H\,\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{p})\,\left((\mathbf{x} - \hat{\boldsymbol{\mu}})^H\,\hat{\boldsymbol{\Sigma}}^{-1}\,(\mathbf{x} - \hat{\boldsymbol{\mu}})\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda\,.$$

Proposition III.2 *The theoretical relationship between the PFA and the threshold is given by*

$$PFA_{ANMF\,\hat{\boldsymbol{\Sigma}},\hat{\boldsymbol{\mu}}} = (1-\lambda)^{a-1}\,{}_2F_1\left(a, a-1; b-1; \lambda\right)\,, \quad (4)$$

where $a = (N-1) - m + 2$ and $b = (N-1) + 2$

Proof: The proof is similar to the proof of Proposition III.1. The main difference is due to the normalization term $(\mathbf{x} - \hat{\boldsymbol{\mu}})^H\,\hat{\boldsymbol{\Sigma}}^{-1}\,(\mathbf{x} - \hat{\boldsymbol{\mu}})$. Indeed, the correction factor $(N+1)/N$ appears both at the numerator and at the denominator, and consequently, it disappears. The same argument is also true for the factor N that arises from the covariance matrix estimates, i.e. since the detector is homogeneous in terms of covariance matrix estimates, this scalar also disappears. Thus, the distribution of the ANMF with an estimate of the mean is exactly the same as in eq. (2) where N is replaced by $N-1$. ■

Remark that the derived relationships given by eqs. (3) and (4) are quite similar to those for which the mean is known. However, as illustrated in Fig. 1, there is an important difference for small values of N.

D. ANMF built with Robust Estimates

If the background do not fulfill the Gaussian hypotheses, the detector performance can be deteriorated increasing the false-alarm rate. To take into account the heterogeneity and non-Gaussianity for background modeling, a possible way is to use of the ANMF test built with robust estimates.

The Fixed Point (FP) estimators [7] satisfy the following implicit equations:

$$\hat{\boldsymbol{\mu}}_{FP} = \frac{\displaystyle\sum_{i=1}^{N}\frac{\mathbf{x}_i}{d_i^{1/2}}}{\displaystyle\sum_{i=1}^{N}\frac{1}{d_i^{1/2}}},\ \hat{\boldsymbol{\Sigma}}_{FP} = \frac{m}{N}\sum_{i=1}^{N}\frac{(\mathbf{x}_i - \hat{\boldsymbol{\mu}}_{FP})\,(\mathbf{x}_i - \hat{\boldsymbol{\mu}}_{FP})^H}{d_i}$$

where $d_i = (\mathbf{x}_i - \hat{\boldsymbol{\mu}}_{FP})^H\hat{\boldsymbol{\Sigma}}_{FP}^{-1}(\mathbf{x}_i - \hat{\boldsymbol{\mu}}_{FP})$.

The FP estimators have been widely investigated in statistics and signal processing literature. We refer to [14] for a detailed performance analysis. It can be shown that the $\hat{\boldsymbol{\Sigma}}_{FP}$ asymptotically behaves as the $\hat{\boldsymbol{\Sigma}}_{SCM}$ when the mean is unknown, i.e. for N sufficiently large, $\hat{\boldsymbol{\Sigma}}_{FP}$ behaves as a Wishart matrix with $\frac{m}{m+1}N$ degrees of freedom. Due to the lack of space, the proof of this result is postponed to a futher paper.

Thus, for N sufficiently large the "PFA-threshold" relationship is given by:

$$PFA_{ANMF-FP} = (1-\lambda)^{a-1}\,{}_2F_1(a, a-1; b-1; \lambda)\,, \quad (5)$$

with $a = \frac{m}{m+1}(N-1) - m + 2$ and $b = \frac{m}{m+1}(N-1) + 2$.

IV. Simulations

In this section, we validate the theoretical analysis on simulated data. The experiments were conducted on $m = 5$ dimensional Gaussian vectors, for N secondary data and the computations have been made through 10^6 Monte-Carlo trials.

Fig. 1 shows the false-alarm regulation for the MF, the AMF when only covariance matrix is unknown and the AMF for both covariance matrix and mean vector unknown. The perfect agreement of the green and yellow curves illustrates the results of Proposition III.1. Moreover, remark that when N increases both AMF get closer to each other, and they approach the known parameters case MF.

Fig. 2 presents the FA regulation for the ANMF under Gaussian assumption, for both the FP estimates and the SCM-SMV. This validates results of Proposition III.2 for the SCM-SMV while it shows that the correcting factor used for FP estimates (eq. (5)) allows to perfectly regulate the FA,

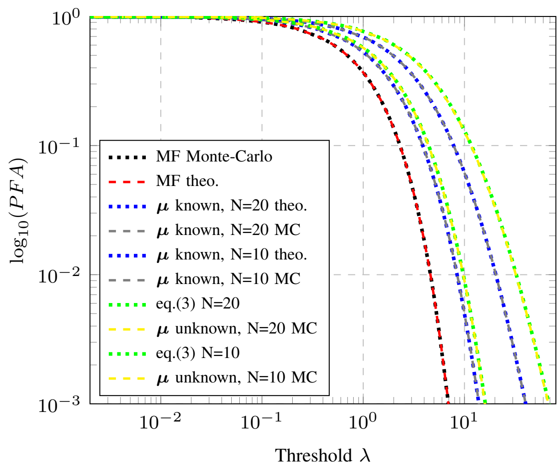

Fig. 1: PFA versus threshold for the AMF when
(1) $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are known (MF) (red and black curves)
(2) only $\boldsymbol{\mu}$ is known (gray and blue curves)
(3) Proposition III.1: both $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are unknown (yellow and green curves)

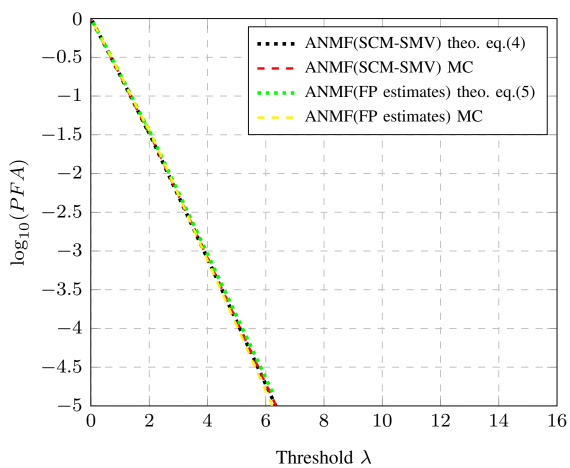

Fig. 2: PFA versus threshold for the ANMF under a Gaussian distribution for $m = 10$ and $N = 50$ when
(1) Proposition III.2: the SCM-SMV are used
(2) the FP estimates are used (yellow and green curves)

even in Gaussian context. Under a K-distribution, as shown on fig. 3, the theoretical "PFA-threshold" relationship is in perfect agreement with the Monte-Carlo simulations for the FP estimates while for the SCM-SMV, the regulation is not valid anymore (since the Gaussian assumption is not respected anymore). Notice that on both Gaussian and K-distribution contexts, the FA regulation for the FP estimates leads to the same results.

V. Conclusion

Two adaptive detection schemes, the AMF and the ANMF, have been analyzed in the case where both the covariance matrix and the mean are unknown and need to be estimated. In this context, theoretical closed-form expressions for false-alarm regulation have been derived user Gaussian assumptions

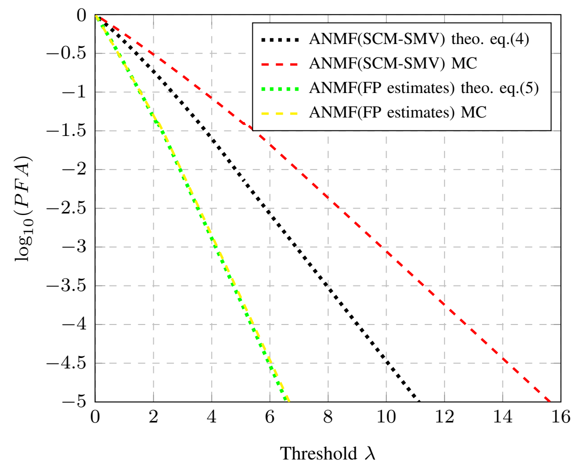

Fig. 3: PFA versus threshold for the ANMF under a K-distribution for $m = 10$ and $N = 50$ when
(1) the SCM-SMV are used (red and black curves)
(2) the FP estimates are used (yellow and green curves)

for the SCM-SMV estimates. Then, these results have been extended to the non-Gaussian case through the FP estimates. Finally, the theoretical analysis has been validated through simulations.

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