Investigating cosmological applications of the inhomogeneous Szekeres models

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A presentation for the
CosmoStat CosmoClub

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outline

Introduction and standard model
Szekeres models
Growth of cosmic structures
Distances in a Szekeres Swiss-cheese model
Summary
Future work
standard cosmological model

- **LCDM** = Lambda-Cold Dark Matter
- **expanding** universe that is accelerating today
- spatially **homogeneous** and **isotropic** (flat FLRW metric + perturbations)
- dynamics governed by **general relativity** (GR)
- basic model has 6 parameters
- Planck, Euclid, DES etc.  → precision era
Universe Pie Doughnut

<1960
Atoms 100%

2015
Atoms 5%
Dark Matter 27%
Dark Energy 68%
How do the lumpy structures in the universe affect our observations and our understanding of cosmology?

**AIM**

use Szekeres inhomogeneous cosmological models to pursue two primary lines of study

(1) growth rate of large-scale structures

(2) potential biases in distance measurements
Szekeres models

- family of **exact solutions** (metrics) of Einstein’s equations

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu} \]

(spacetime curvature = energy and momentum)

- **inhomogeneous** and **anisotropic** but contain FLRW metric as a special limit

- able to describe **exact deviations** (perturbations) of a smooth and homogeneous background

- able to model evolving structures while retaining the full **nonlinearity** of GR
more on Szekeres

- no symmetries in general (lacks Killing vector fields)
- dust source (pressureless matter)
- generalizes FLRW and LT (spherically symmetric)

The different formulations:
- P. Szekeres (1975)
- Goode & Wainwright (1982)
- C. Hellaby (1996)

Class I
- 5 degrees of freedom

Class II
- 4 degrees of freedom
simulation by A. Kravtsov and A. Klypin at NCSA
valid for dust and $|\delta| \ll 1$
Szekeres density evolution

Goode and Wainwright formulation

- metric

\[ ds^2 = -dt^2 + a^2 \left[ H^2 W^2 dr^2 + e^{2\nu} (dx^2 + dy^2) \right] \]

- spacetime coordinates

- scale factor \( a(t) \)

- evolution equation (Friedmann)

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{2M}{a^3} - \frac{k}{a^2} + \frac{\Lambda}{3} \]
• density

\[ \rho = \frac{6 M}{a^3} \left( 1 + \frac{F}{H} \right) \]

\[ = \rho_b \cdot (1 + \delta) \]

\[ = \text{background + exact deviation} \]

• evolution of density contrast

\[ \delta'' + \left( \frac{4 + 2 \Omega_\Lambda - \Omega_m}{2a} \right) \delta' - \frac{3 \Omega_m}{2a^2} \delta - \frac{2}{1 + \delta} \delta' \delta - \frac{3 \Omega_m}{2a^2} \delta^2 = 0 \]

FLRW part

additional nonlinear terms
matter-dominated era

Szekeres Class-II with an Einstein de Sitter background
Spherical Collapse Model in Einstein de Sitter model
FLRW - Einstein de Sitter model

• **increased growth rate** compared to spherical collapse and FLRW by up to 3-5 times

• similar results for spatially **curved models** (not shown)

• density **singularities avoided** due to growth rate suppression by L

• presence of **shear** and nonzero **tidal field** act as effective sources

• might Szekeres not need as much dark matter?
comparison to data

- growth factor

\[ f = \frac{d \ln \delta}{d \ln a} \quad \Rightarrow \quad f' + \left(2 + \frac{\dot{H}}{H^2}\right) f + f^2 - \frac{3}{2} \Omega_m = 0 \]

\[ f' + \left(1 + \Omega_\Lambda - \frac{1}{2} \Omega_m\right) f + \left(1 - \frac{2}{1 + \delta^{-1}}\right) f^2 - \frac{3}{2} (1 + \delta) \Omega_m = 0 \]

<table>
<thead>
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<th></th>
<th>Without priors</th>
<th>With priors</th>
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<tbody>
<tr>
<td></td>
<td>$\Omega^0_m$</td>
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<td><strong>SZEKERES</strong></td>
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<tr>
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<td>0.12</td>
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<td>0.56</td>
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<tr>
<td>Yes</td>
<td>0.26</td>
<td>0.69</td>
</tr>
</tbody>
</table>

- best-fit Szekeres model fits data as well as FLRW but with different parameters

- best-fit Szekeres has less matter and more spatial curvature

- nonzero curvature consistent with other studies of inhomogeneous models and averaging

- analyzing data in inhomogeneous model strongly impacts determination of cosmological parameters
Light propagation and distance measures in a Swiss-cheese model universe
the universal expansion

Supernova Cosmology Project (Union 2.1 Compilation): http://supernova.lbl.gov/Union/

Cluster Search (SCP)
Amanullah et al. (2010) (SCP)
Riess et al. (2007)
Tonry et al. (2003)

Miknaitis et al. (2007)
Astier et al. (2006)
Knop et al. (2003) (SCP)
Amanullah et al. (2008) (SCP)
Barris et al. (2004)
Perlmutter et al. (1999) (SCP)
Riess et al. (1998) + HZT
Holtzman et al. (2009)

Contreras et al. (2010)
Hicken et al. (2009)
Kowalski et al. (2008) (SCP)
Jha et al. (2006)
Riess et al. (1999)
Krisiu纳斯 et al. (2005)
Hamuy et al. (1996)
toward (more) realistic light propagation

\[ R_{\mu\nu\alpha\beta} = g_{\mu[\alpha} R_{\beta]\nu} - g_{\nu[\alpha} R_{\beta]\mu} - \frac{1}{3} R g_{\mu[\alpha} g_{\beta]\nu} + C_{\mu\nu\alpha\beta} \]

full (Riemann) curvature = \textbf{Ricci curvature} + \textbf{Weyl curvature}
Swiss-cheese model universe

LCMU

QUESTION

SZEKERES

GROWTH

LIGHT PROP.

SWISS-CHEESE

SUMMARY

FUTURE

LCMU
Swiss-cheese model universe

LCDM

SZEKERES
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Swiss-cheese model universe

- LCDM
- QUESTION
- SZEKERES
- GROWTH
- LIGHT PROP.
- SWISS-CHEESE
- SUMMARY
- FUTURE
The Swiss-cheese model universe:

- exact inhomogeneous universe with LCDM background dynamics
- Einstein’s equations continuous throughout spacetime
- lines of sight are primarily underdense
- overdense shells mimic filamentary cosmic web
- incorporates nonlinear GR effects
the Szekeres holes

- parameterized density profile with $\delta = -0.8$ within interior
- sizes of 30, 45, and 60 Mpc structures to mimic largest structures in the universe
- maximum: $\rho / \rho_{bg} \approx 12$  
  minimum: $\rho / \rho_{bg} \approx 1.3$

A. Peel, M. A. Troxel, and M. Ishak, Phys. Rev. D 90, 123536 (2014)
Sachs optical equations

\[ \frac{d\theta}{ds} + \theta^2 + |\sigma|^2 = -\frac{1}{2} R_{\mu\nu} k^\mu k^\nu \]

\[ \frac{d\sigma}{ds} + 2\theta \sigma = -\frac{1}{2} C_{\mu\nu\alpha\beta} m^\mu k^\nu m^\alpha k^\beta \]

expansion \( \theta \)  
(Ricci)

shear \( \sigma = \sigma_1 + i\sigma_2 \)  
(Weyl)
sanity check

reproduced results of spherically symmetric case

Peel, Troxel, Ishak, Phys. Rev. D 90, 123536 (2014)
various ways to traverse a hole

‘radial’ paths:

\[ \pi /2 \text{ direction} \]

\[ \pi \text{ direction} \]

\[ 3 \pi /2 \text{ direction} \]

arbitrary:
single hole ‘radial’ paths

\[ \Delta \mu = \mu_{\Lambda CDM} - \mu_{\text{Swiss-cheese}} \]

A. Peel, M. A. Troxel, and M. Ishak, Phys. Rev. D 90, 123536 (2014)
multiple randomized holes

A. Peel, M. A. Troxel, and M. Ishak, Phys. Rev. D 90, 123536 (2014)
statistical results

\[ \langle \Delta \mu \rangle = -3.56 \times 10^{-5} \quad \sigma_{\Delta \mu} = 5.81 \times 10^{-4} \]

\[ \langle \Delta \mu \rangle = -2.57 \times 10^{-5} \quad \sigma_{\Delta \mu} = 1.62 \times 10^{-3} \]

\[ \langle \Delta \mu \rangle = 3.69 \times 10^{-4} \quad \sigma_{\Delta \mu} = 2.74 \times 10^{-3} \]

\[ \langle \Delta \mu \rangle = 9.83 \times 10^{-4} \quad \sigma_{\Delta \mu} = 4.00 \times 10^{-3} \]

A. Peel, M. A. Troxel, and M. Ishak, Phys. Rev. D 90, 123536 (2014)
magnification expectation

![Graph showing probability distribution of magnification with $z_{\text{src}} = 1.00$.]
summary

- explored **two applications** of Szekeres models to precision cosmology
- found **larger growth rate** in MDE compared to FLRW and spherical collapse
- showed best-fit Szekeres model **fits growth data** as well as the standard model but with different parameters
- built a **Swiss-cheese universe** model with exact and nonlinearly evolving holes
- studied **effect of inhomogeneities** on distance measures
- saw $\langle \Delta \mu \rangle$ and $\sigma_{\Delta \mu}$ **too small** to induce systematic bias in Hubble diagram
future possibilities

• explore constant M notion in class II models

• build statistically homogeneous class II model (where $\langle \delta \rangle = 0$)

• compare results to higher-order perturbation theory

Improving the Swiss-cheese model:

• different hole density profiles

• more realistic hole sizes and distribution

• relax exact matching condition $\rightarrow$ different statistics?
Thank you!