Investigating cosmological applications of the inhomogeneous Szekeres models

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supervised by

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A presentation for the CosmoStat CosmoClub

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outline

Introduction and standard model

Szekeres models

Growth of cosmic structures

Distances in a Szekeres Swiss-cheese model

Summary

Future work

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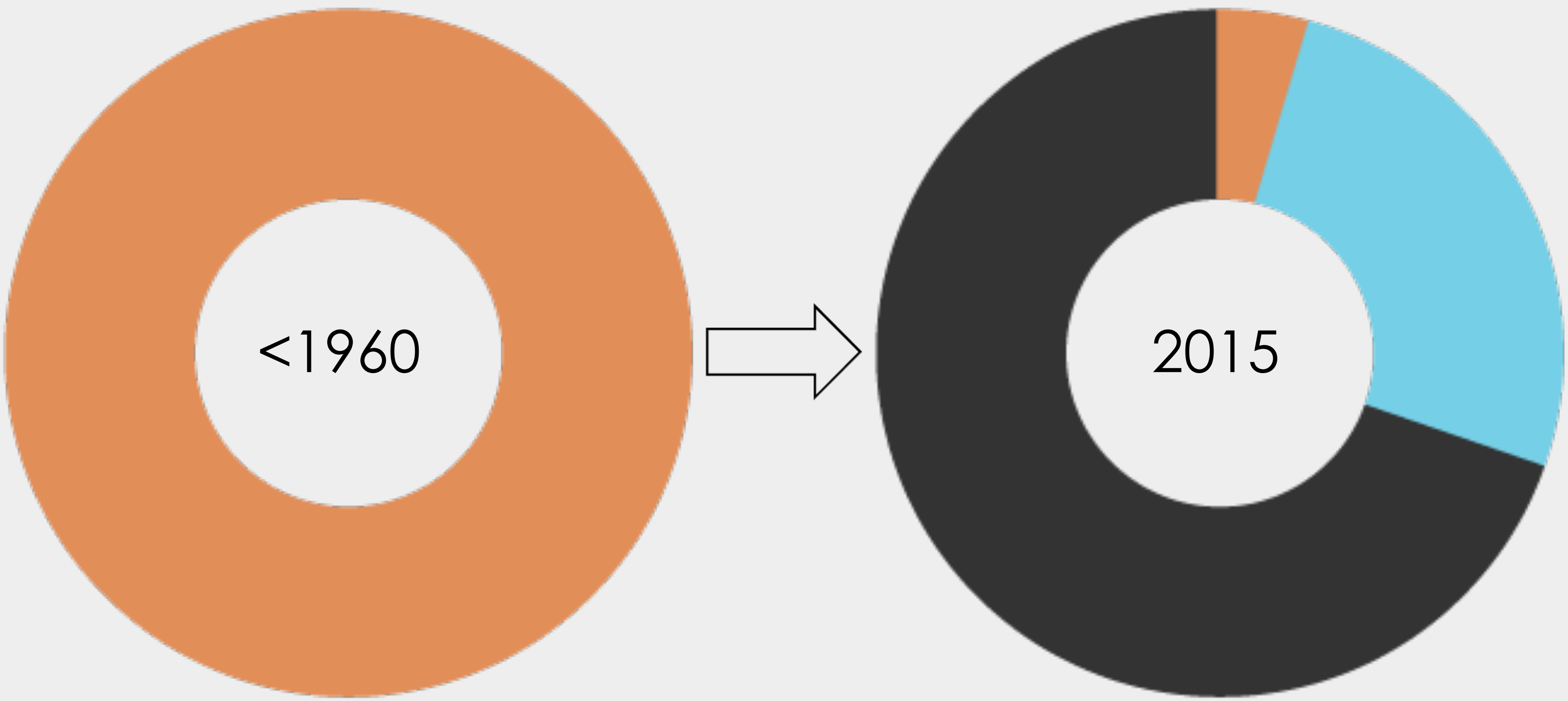
standard cosmological model

- **LCDM** = Lambda-Cold Dark Matter
- **expanding** universe that is accelerating today
- spatially **homogeneous** and **isotropic** (flat FLRW metric + perturbations)
- dynamics governed by **general relativity** (GR)
- basic model has 6 parameters
- *Planck, Euclid, DES etc.* → precision era

Universe ~~Pie~~ Doughnut



Atoms 100%

Atoms 5%

Dark Matter 27%

Dark Energy 68%

LCDM

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main question

How do the lumpy structures in the universe affect our observations and our understanding of cosmology?

AIM

use Szekeres inhomogeneous cosmological models to pursue two primary lines of study

(1) growth rate of large-scale structures

(2) potential biases in distance measurements

Szekeres models

- family of **exact solutions** (metrics) of Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

(spacetime curvature = energy and momentum)

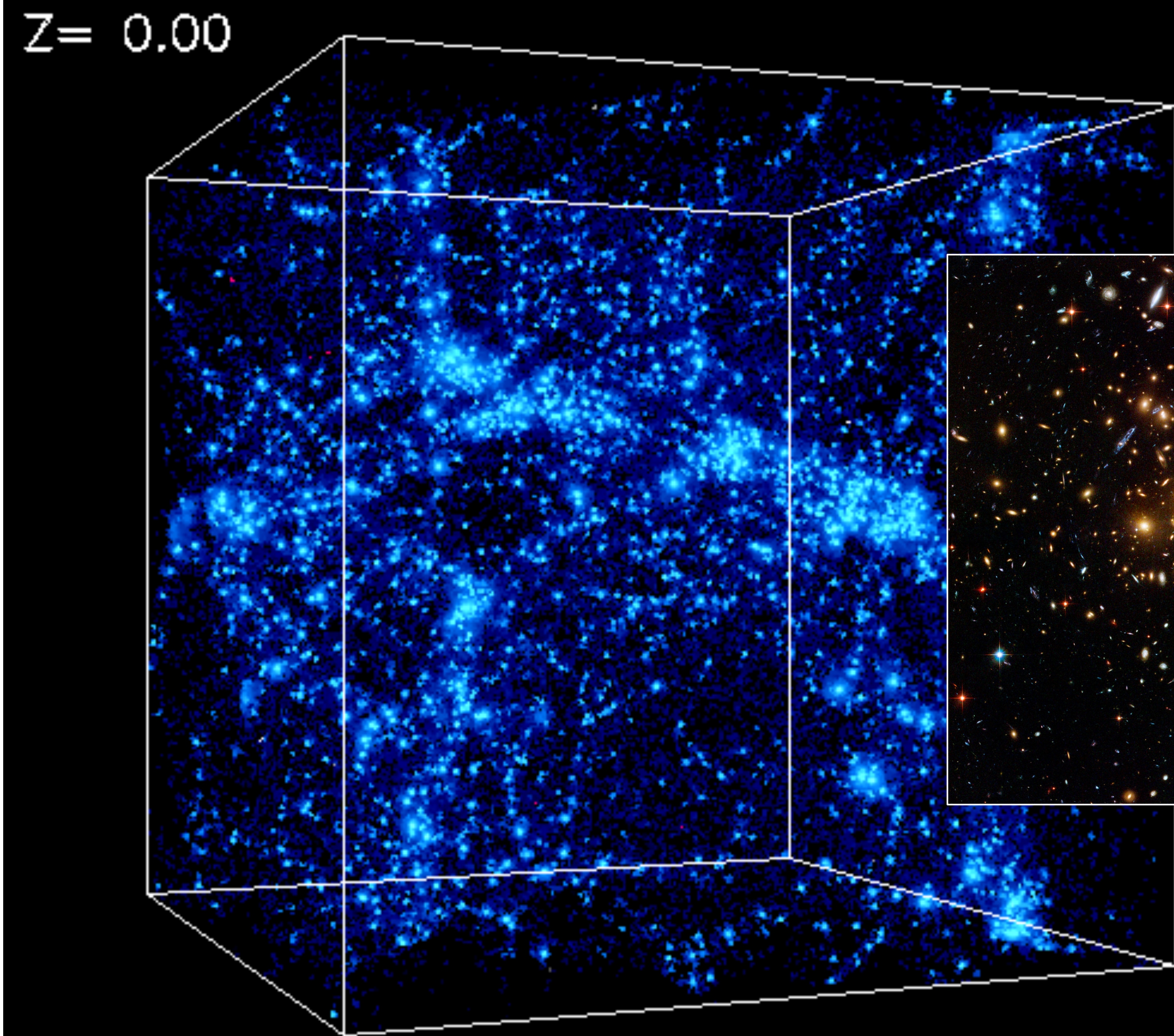
- **inhomogeneous** and **anisotropic** but contain FLRW metric as a special limit
- able to describe **exact deviations** (perturbations) of a smooth and homogeneous background
- able to model evolving structures while retaining the full **nonlinearity** of GR

more on Szekeres

different formulations:
- P. Szekeres (1975)
- Goode & Wainwright (1982)
- C. Hellaby (1996)

Class I	Class II
5 degrees of freedom	4 degrees of freedom

- no symmetries in general (lacks Killing vector fields)
- dust source (pressureless matter)
- generalizes FLRW and LT (spherically symmetric)



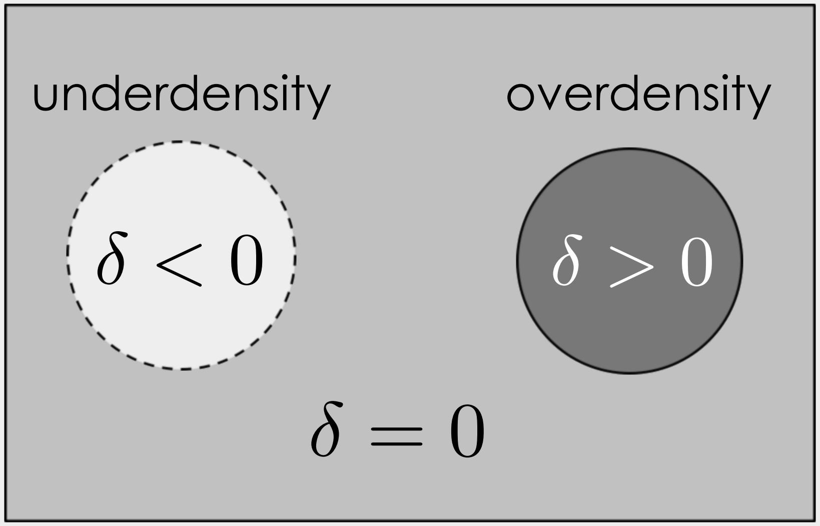
NASA/ESA

simulation by A. Kravtsov and A. Klypin at NCSA

describing density fluctuations

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

density contrast



linear evolution:

$$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} - 4\pi G \, \bar{\rho} \, \delta = 0$$

valid for *dust* and $|\delta| \ll 1$

Szekeres density evolution

Goode and Wainwright formulation

- metric

spacetime coordinates

$$ds^2 = -\mathrm{d}t^2 + a^2\left[H^2W^2\mathrm{d}r^2 + e^{2\nu}(\mathrm{d}x^2 + \mathrm{d}y^2)\right]$$

scale factor $a(t)$

- evolution equation (Friedmann)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{2M}{a^3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

- density

$$\rho = \frac{6M}{a^3}\left(1 + \frac{F}{H}\right)$$

$$= \rho_b \cdot (1 + \delta)$$

$=$ background + exact deviation

- evolution of density contrast

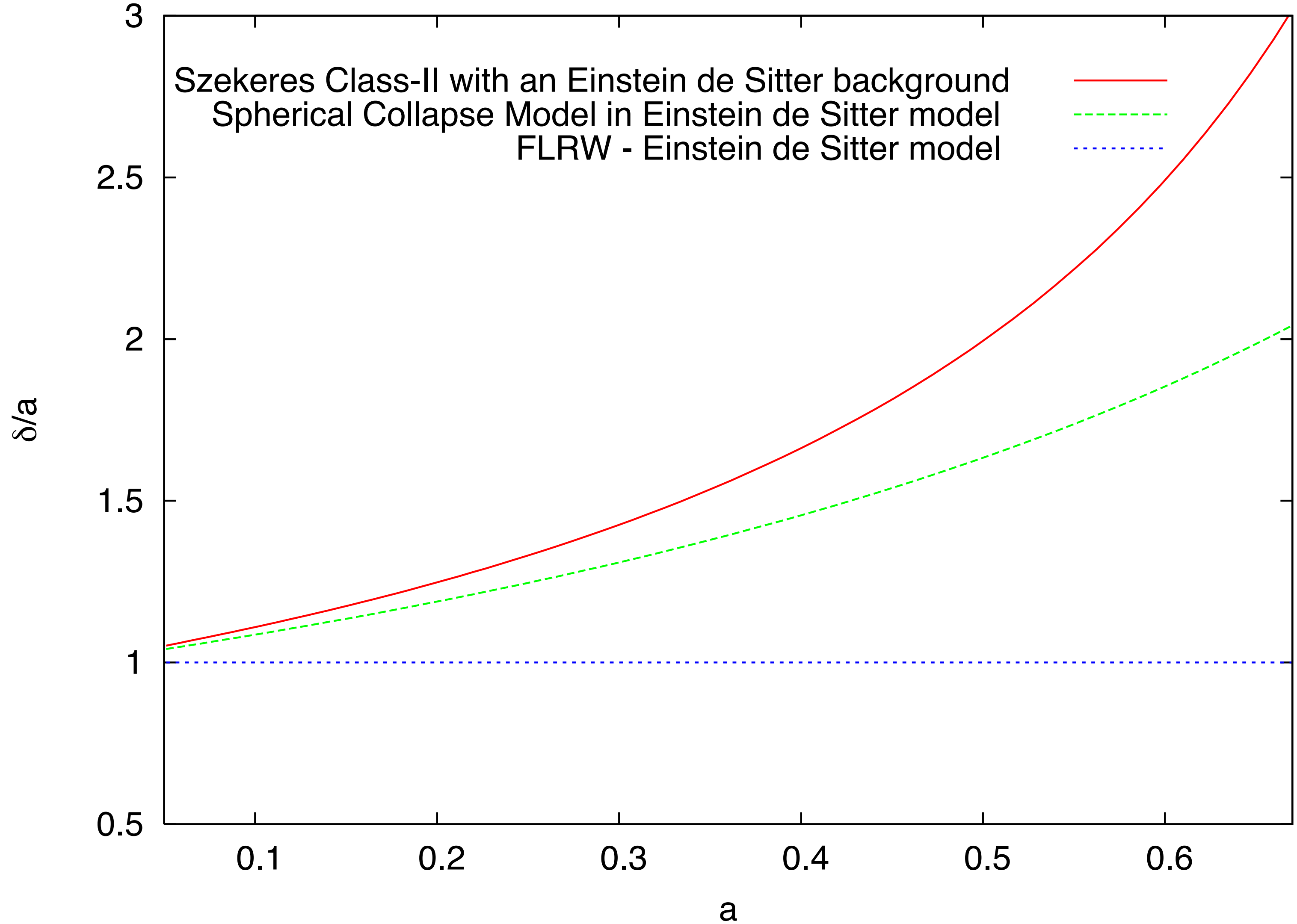
$$' = \frac{\partial}{\partial a}$$

$$\delta'' + \left(\frac{4 + 2\,\Omega_\Lambda - \Omega_m}{2a}\right)\delta' - \frac{3\,\Omega_m}{2\,a^2}\delta - \frac{2}{1+\delta}\delta'^2 - \frac{3\,\Omega_m}{2\,a^2}\delta^2 = 0$$

FLRW part

additional nonlinear terms

matter-dominated era



M. Ishak and A. Peel, Phys. Rev. D 85, 083502 (2012)

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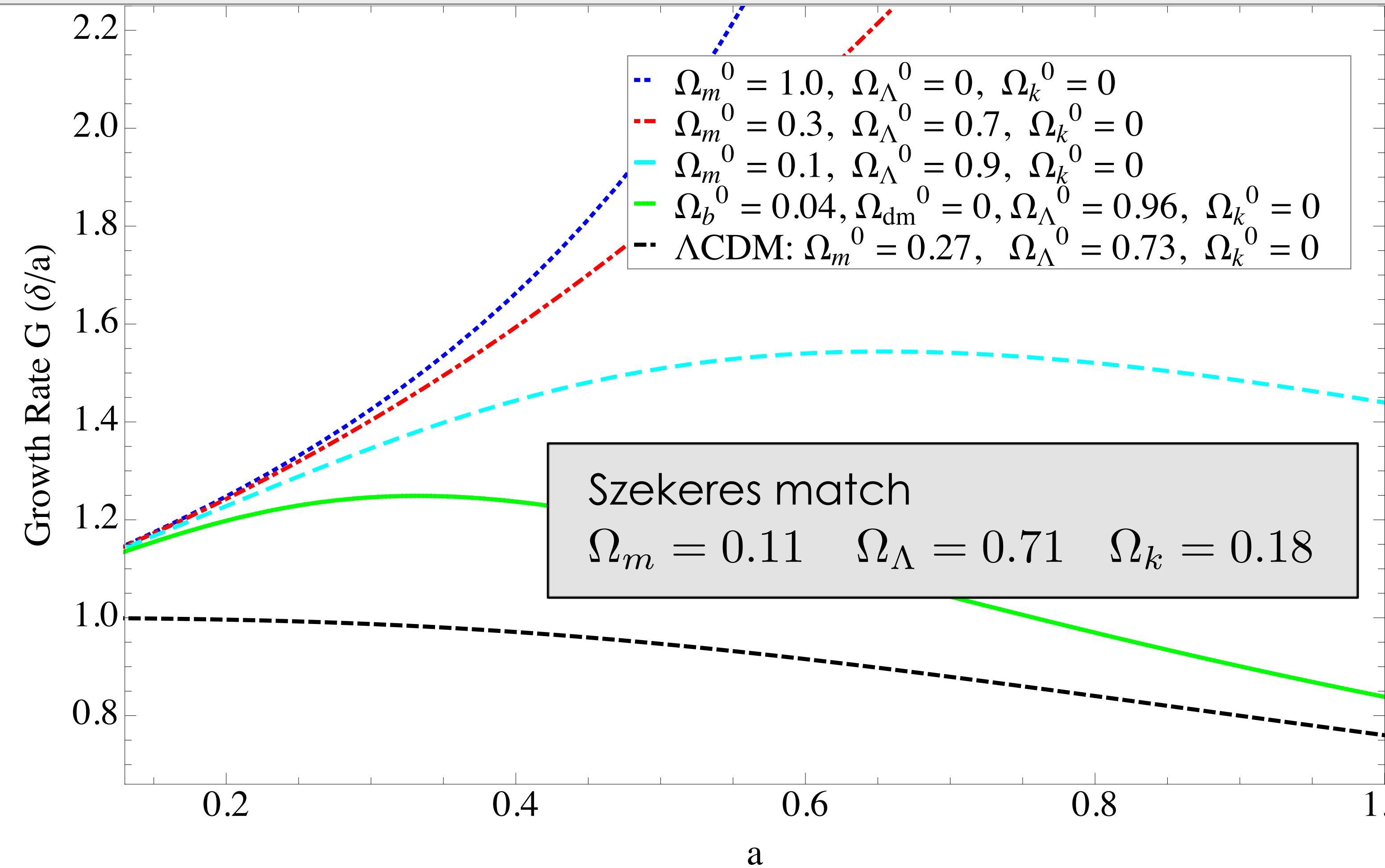
including cosmological const.

spatially flat models



A. Peel, M. Ishak, and M. A. Troxel, Phys. Rev. D 86, 123508 (2012)

results summary

- **increased growth rate** compared to spherical collapse and FLRW by up to 3-5 times
- similar results for spatially **curved models** (not shown)
- density **singularities avoided** due to growth rate suppression by L
- presence of **shear** and nonzero **tidal field** act as effective sources
- might Szekeres not need as much dark matter?

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comparison to data

- growth factor

$$f = \frac{\mathrm{d}\ln\delta}{\mathrm{d}\ln a}$$

FLRW

$$f' + \left(2 + \frac{\dot{\mathrm{H}}}{\mathrm{H}^2}\right) f + f^2 - \frac{3}{2}\Omega_m = 0$$

Szekeres

$$f' + \left(1 + \Omega_\Lambda - \frac{1}{2}\Omega_m\right) f + \left(1 - \frac{2}{1+\delta^{-1}}\right) f^2 - \frac{3}{2}(1+\delta)\,\Omega_m = 0$$

		Without priors				With priors			
	Lα?	Ω_m^0	Ω_Λ^0	Ω_k^0	(χ^2)	Ω_m^0	Ω_Λ^0	Ω_k^0	(χ^2)
Szekeres	No	0.12	0.59	0.29	(0.22)	0.05	0.98	-0.03	(0.62)
	Yes	0.11	0.69	0.20	(0.39)	0.05	0.98	-0.03	(0.63)
FLRW	No	0.29	0.56	0.15	(0.21)	0.27	0.73	0.00	(0.32)
	Yes	0.26	0.69	0.05	(0.39)	0.27	0.73	0.00	(0.43)

A. Peel, M. Ishak, and M. A. Troxel, Phys. Rev. D 86, 123508 (2012)

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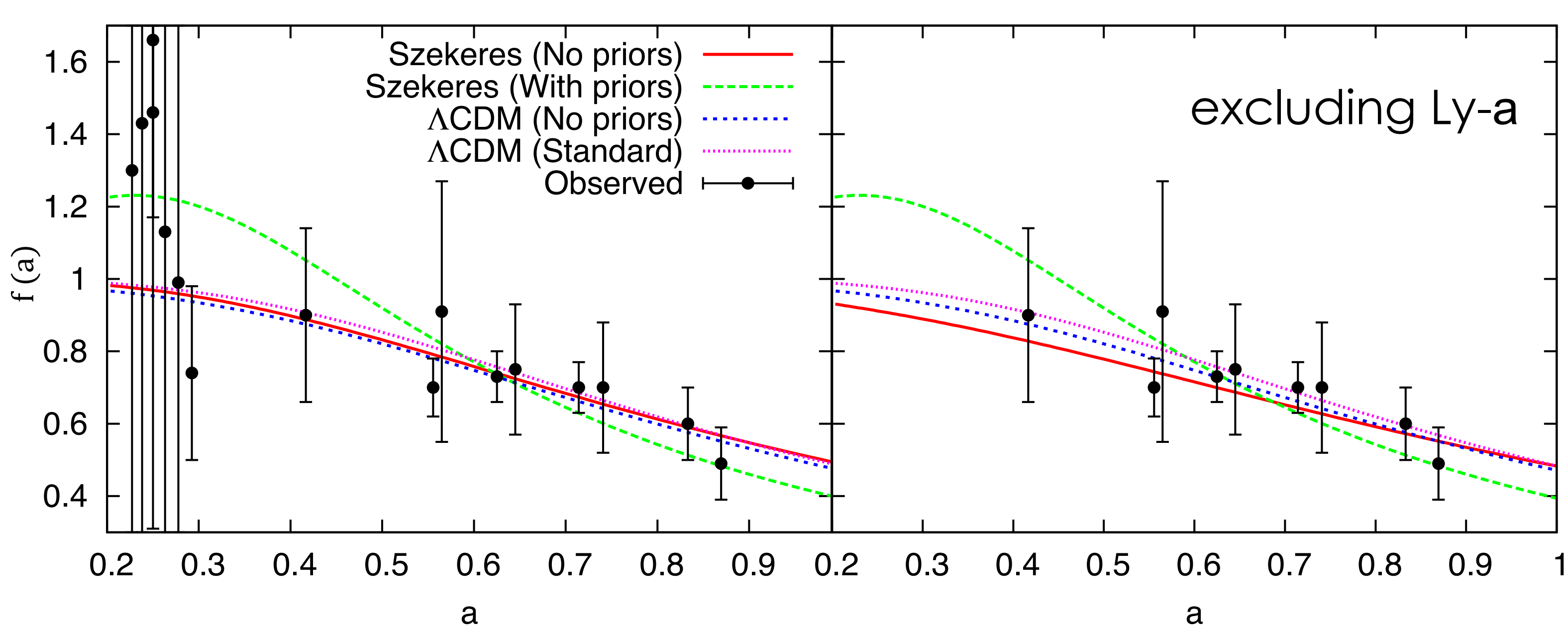

A. Peel, M. Ishak, and M. A. Troxel, Phys. Rev. D 86, 123508 (2012)

- best-fit Szekeres model fits data as well as FLRW but with different parameters
- best-fit Szekeres has less matter and more spatial curvature
- nonzero curvature consistent with other studies of inhomogeneous models and averaging
- analyzing data in inhomogeneous model strongly impacts determination of cosmological parameters

Light propagation and distance measures in a Swiss-cheese model universe

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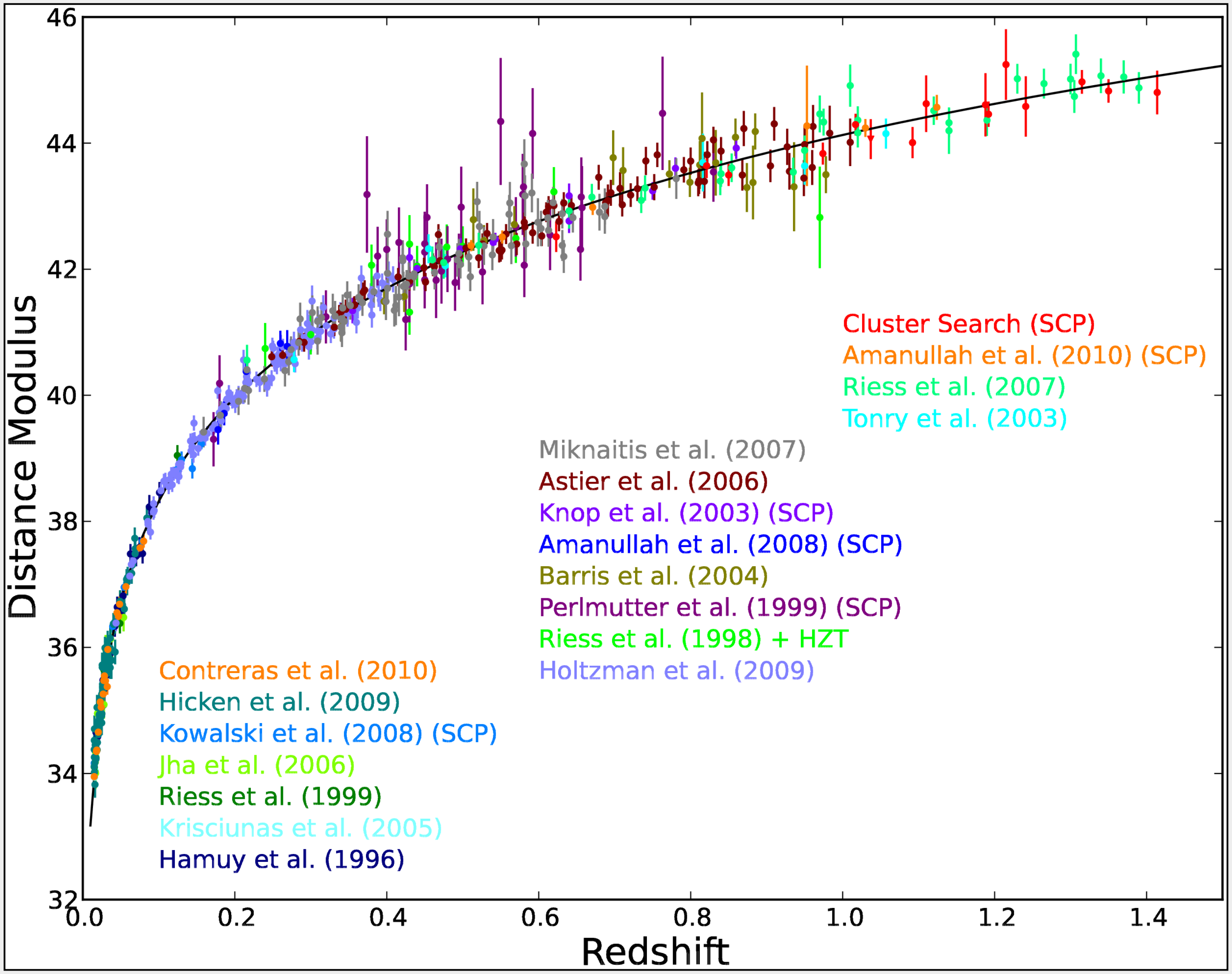
the universal expansion



Supernova Cosmology Project (Union 2.1 Compilation): http://supernova.lbl.gov/Union/

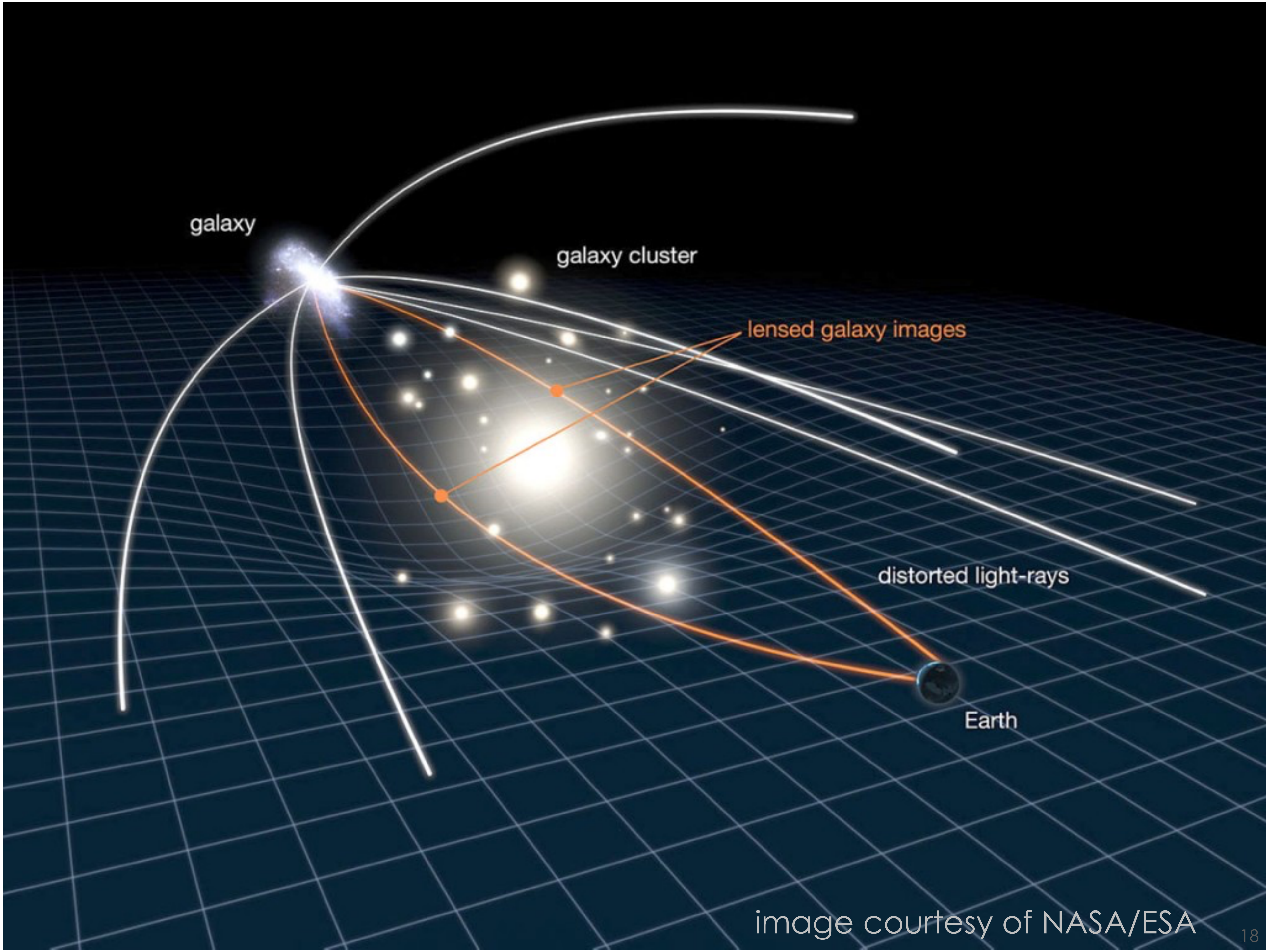
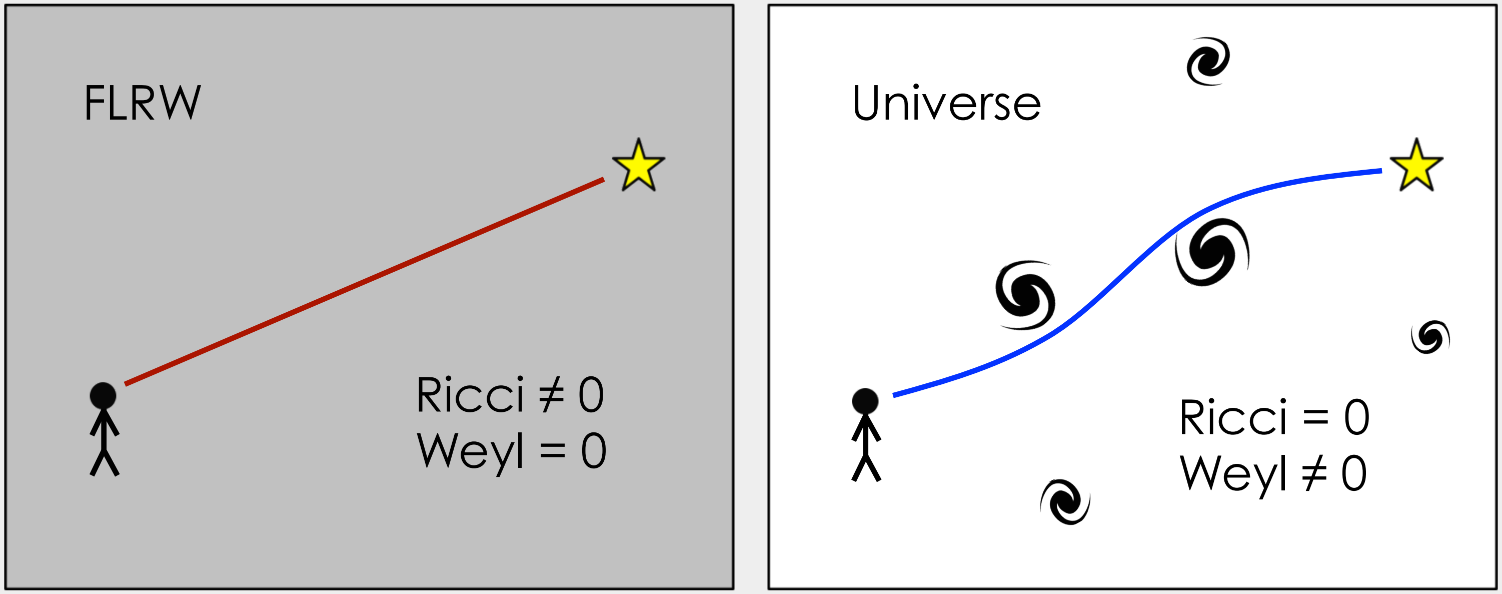


image courtesy of NASA/ESA

toward (more) realistic light propagation

$$R_{\mu\nu\alpha\beta} = g_{\mu[\alpha} R_{\beta]\nu} - g_{\nu[\alpha} R_{\beta]\mu} - \frac{1}{3} R\, g_{\mu[\alpha} g_{\beta]\nu} + C_{\mu\nu\alpha\beta}$$

full (Riemann) curvature = Ricci curvature + Weyl curvature



Swiss-cheese model universe

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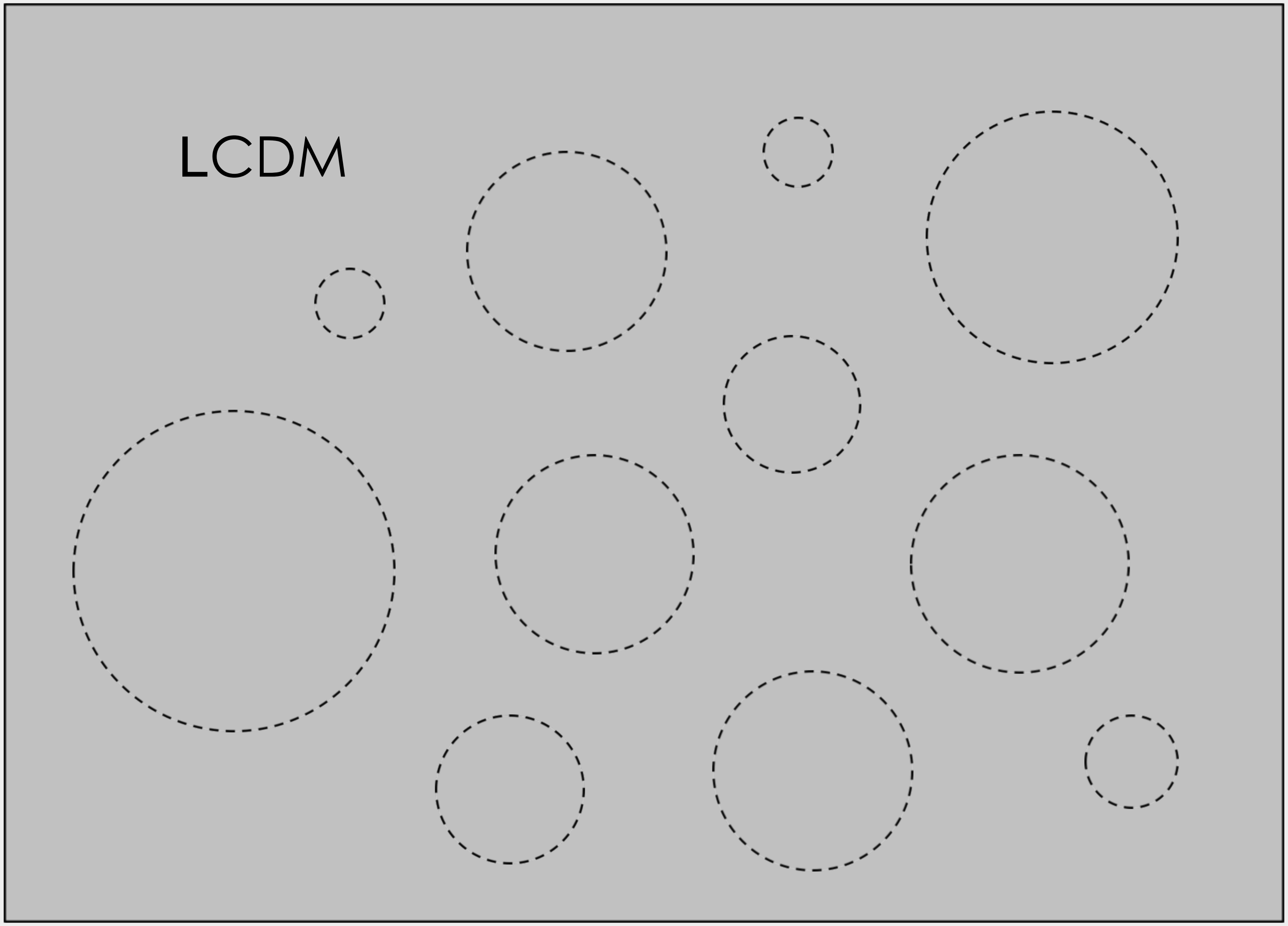
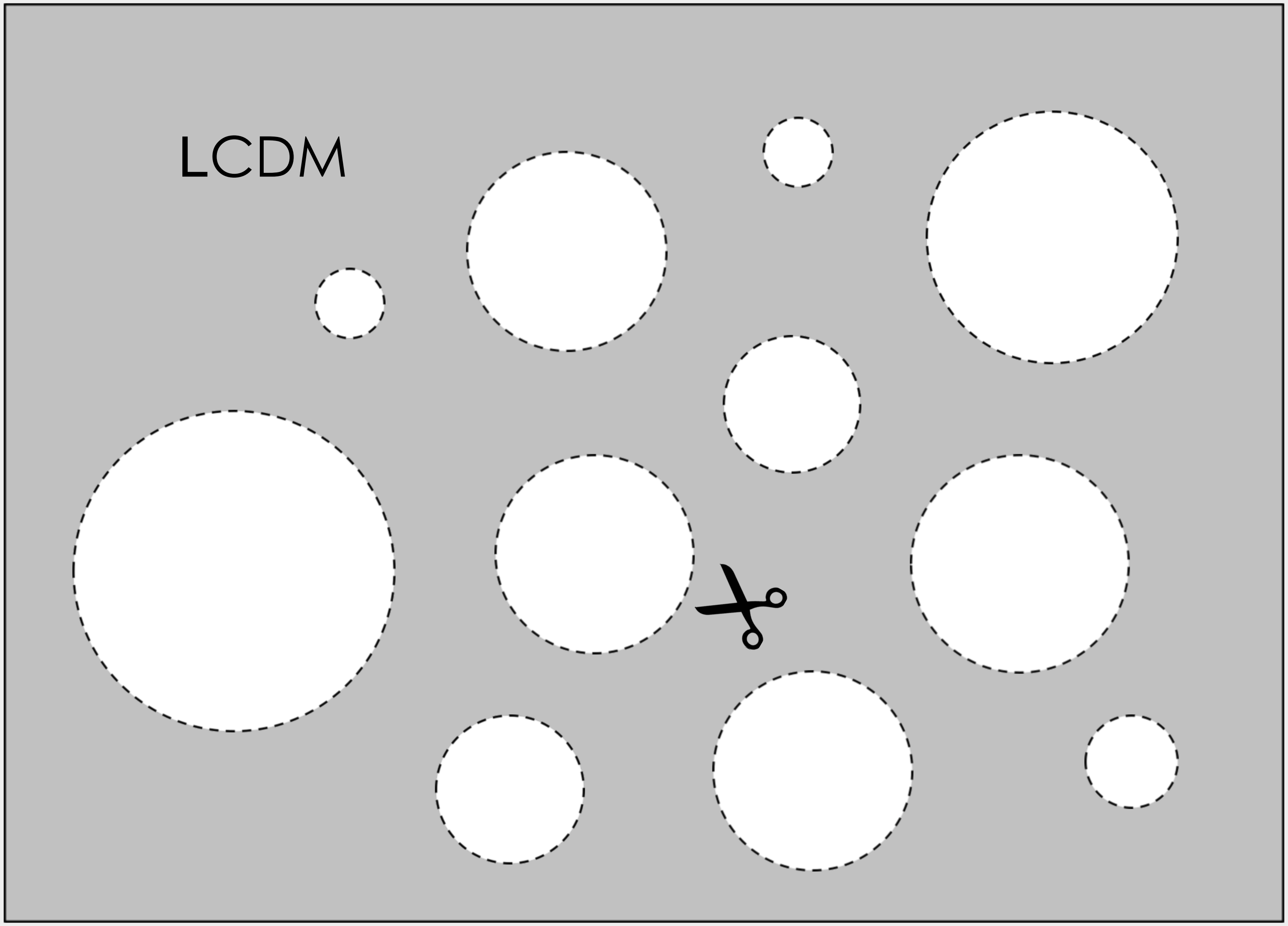
Swiss-cheese model universe



Swiss-cheese model universe



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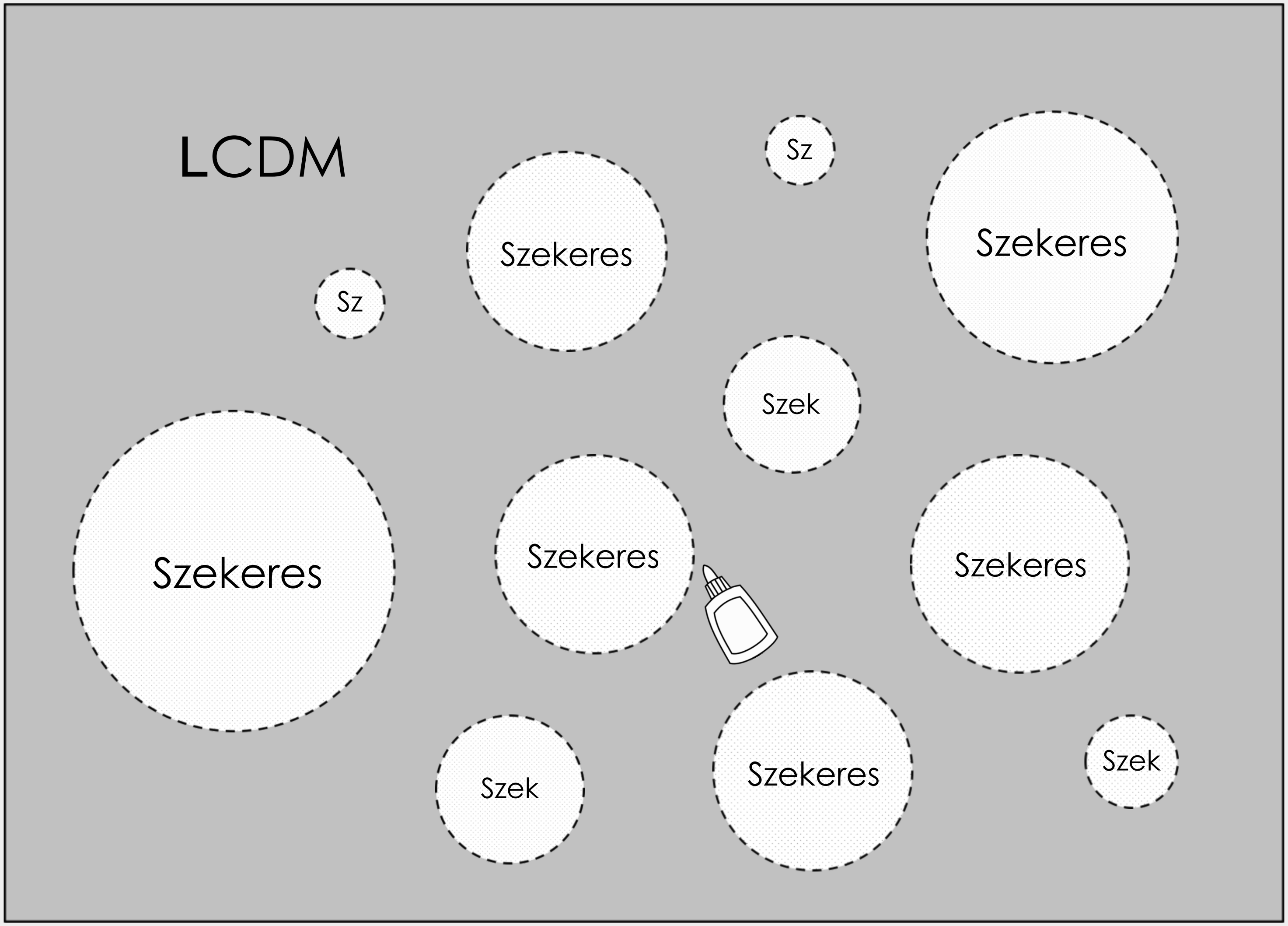
Swiss-cheese model universe



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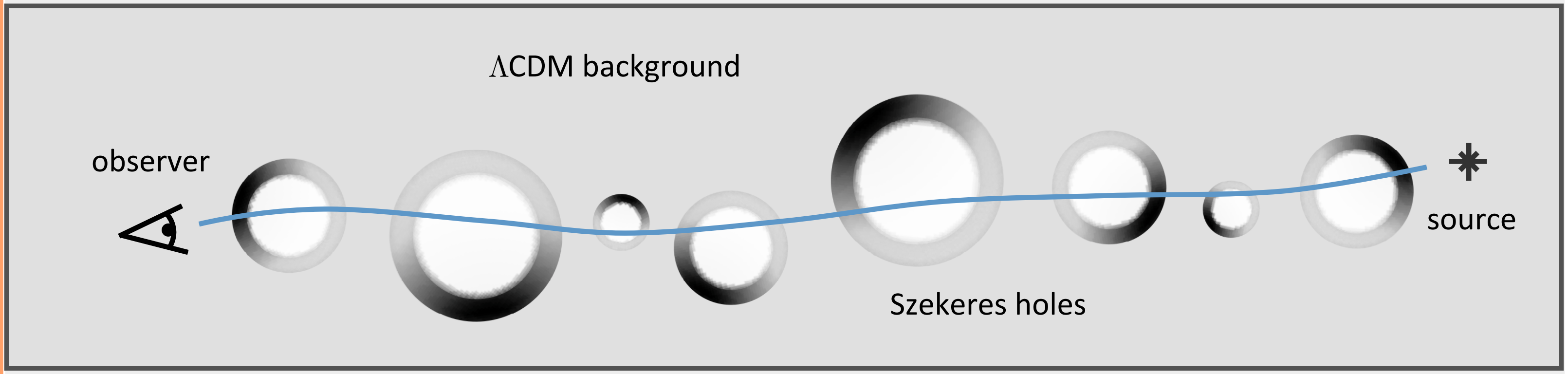
Swiss-cheese model universe



- exact inhomogeneous universe with LCDM background dynamics
- Einstein's equations continuous throughout spacetime
- lines of sight are primarily underdense
- overdense shells mimic filamentary cosmic web
- incorporates nonlinear GR effects

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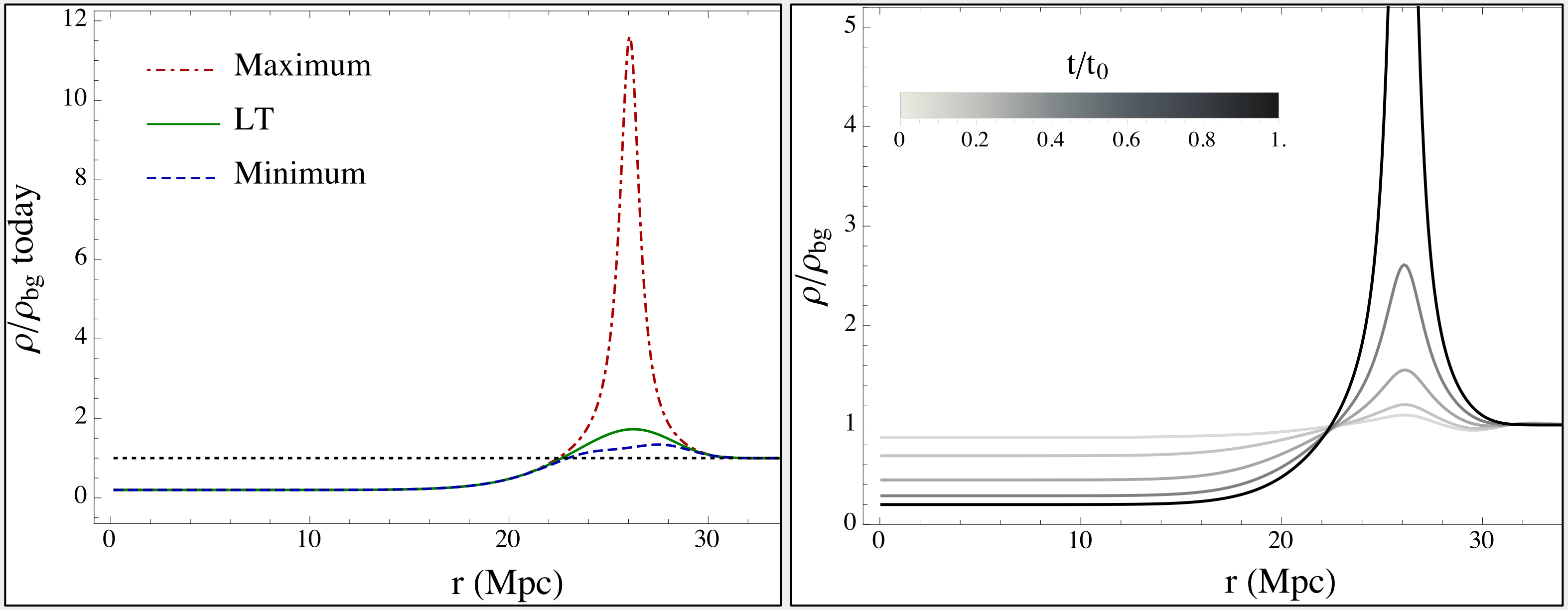
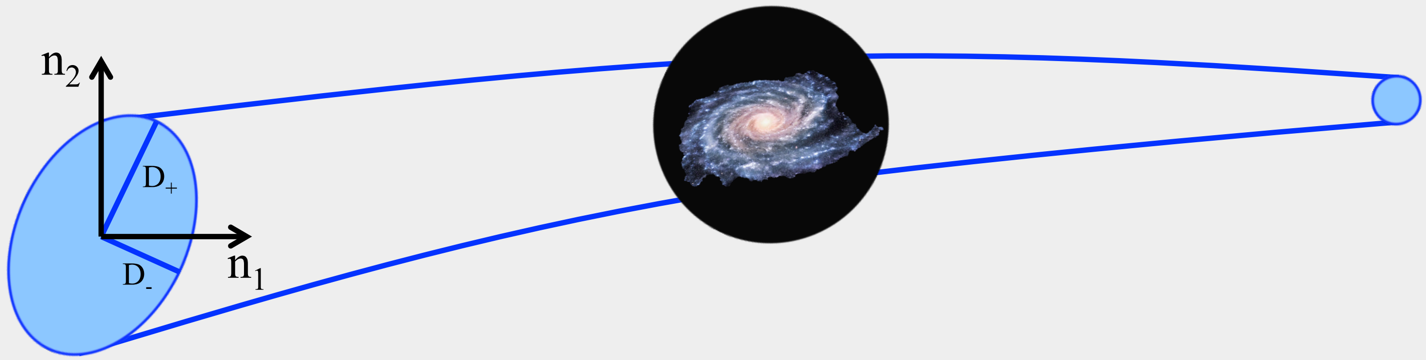
the Szekeres holes

- parameterized density profile with $\delta = -0.8$ within interior
- sizes of 30, 45, and 60 Mpc structures to mimic largest structures in the universe
- maximum: $\rho/\rho_{\rm bg} \approx 12$ minimum: $\rho/\rho_{\rm bg} \approx 1.3$



A. Peel, M. A. Troxel, and M. Ishak, Phys. Rev. D 90, 123536 (2014)

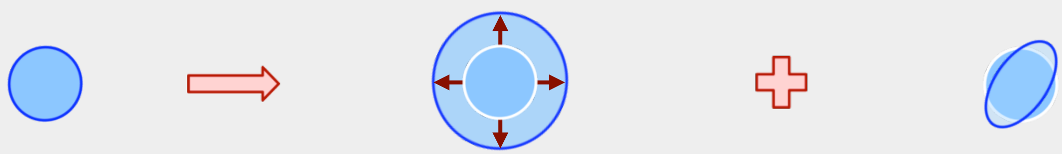
method



Sachs optical equations

$$\frac{d\theta}{ds} + \theta^2 + |\sigma|^2 = -\frac{1}{2} R_{\mu\nu} k^\mu k^\nu$$

$$\frac{d\sigma}{ds} + 2\theta\sigma = -\frac{1}{2} C_{\mu\nu\alpha\beta} m^\mu k^\nu m^\alpha k^\beta$$

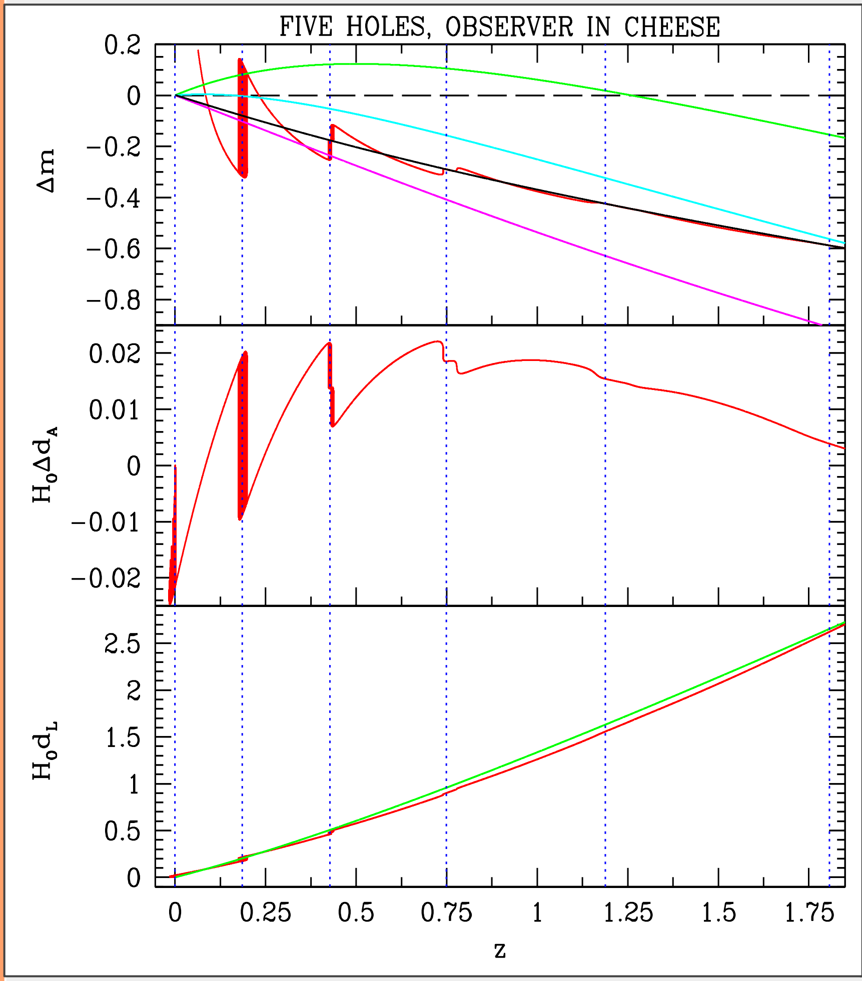


expansion θ (Ricci)

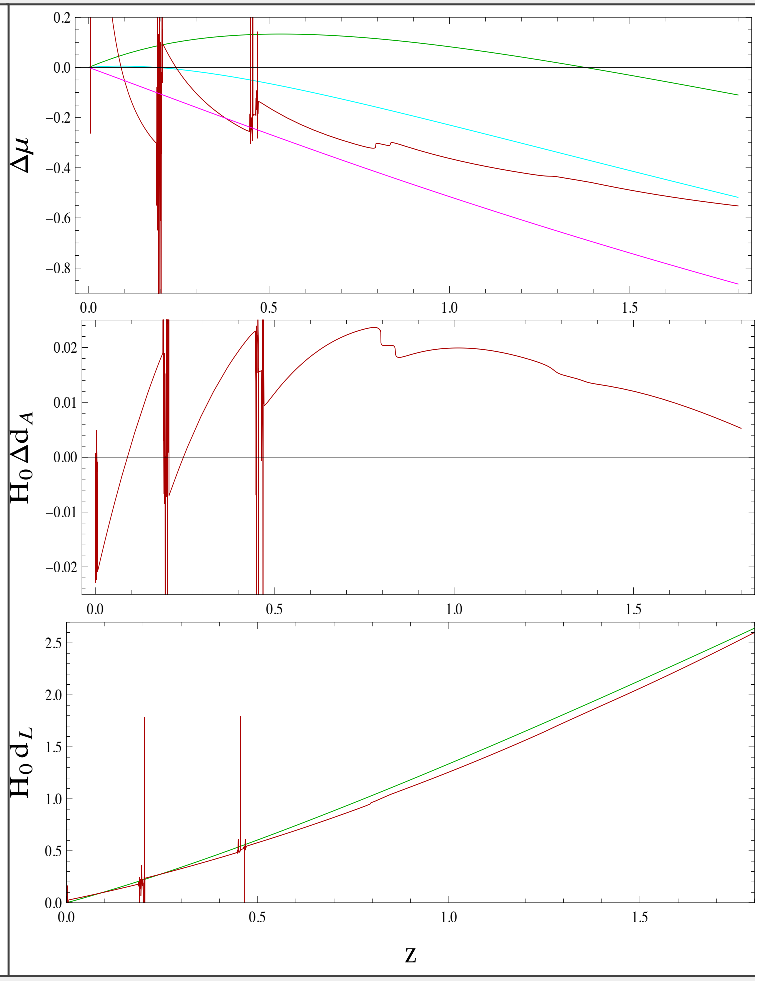
shear $\sigma = \sigma_1 + i\sigma_2$ (Weyl)

sanity check

reproduced results of spherically symmetric case



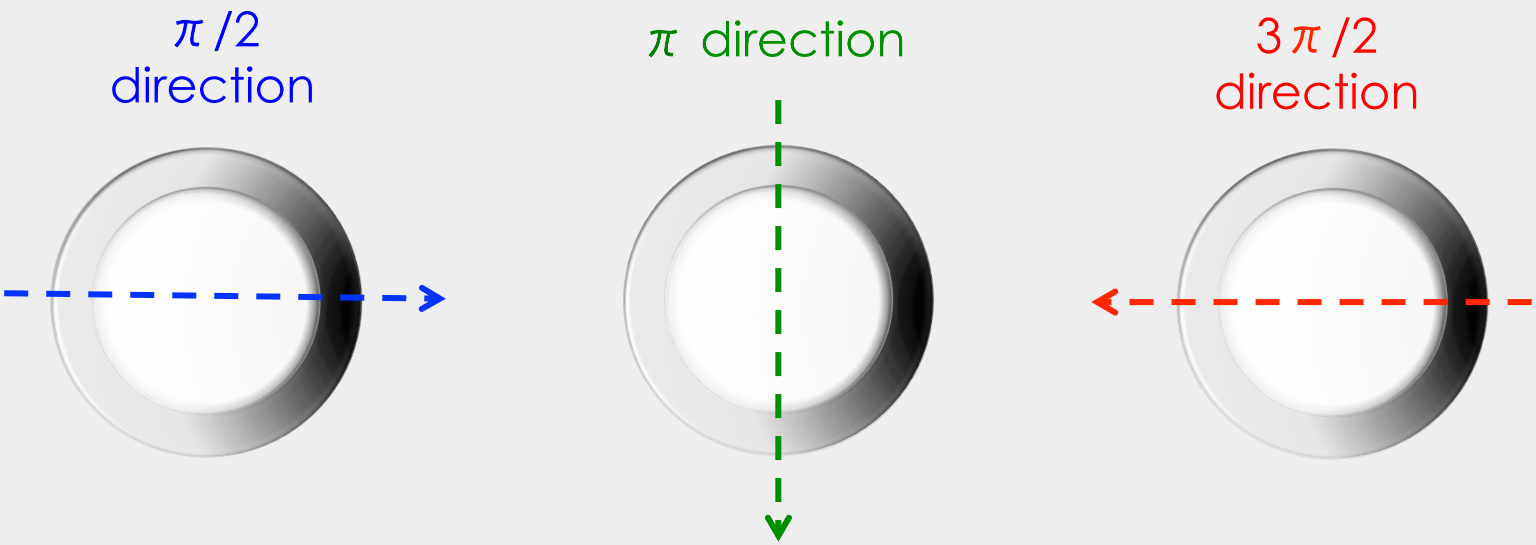
Marra, Kolb, Matarrese, Riotto, Phys. Rev. D 76, 123004 (2007)



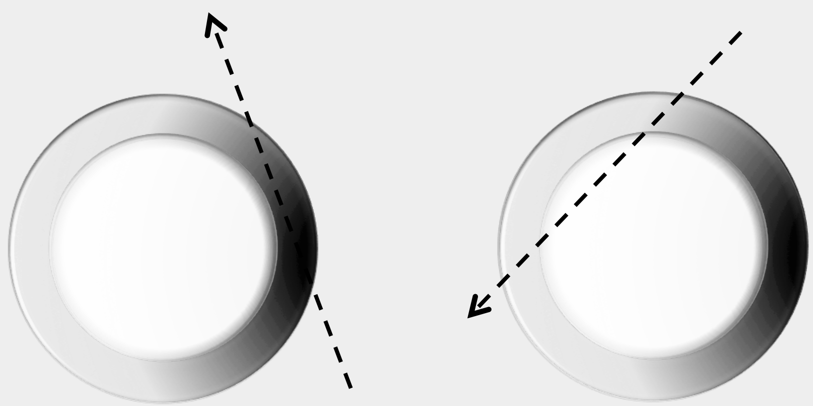
Peel, Troxel, Ishak, Phys. Rev. D 90, 123536 (2014)

various ways to traverse a hole

'radial' paths:



arbitrary:



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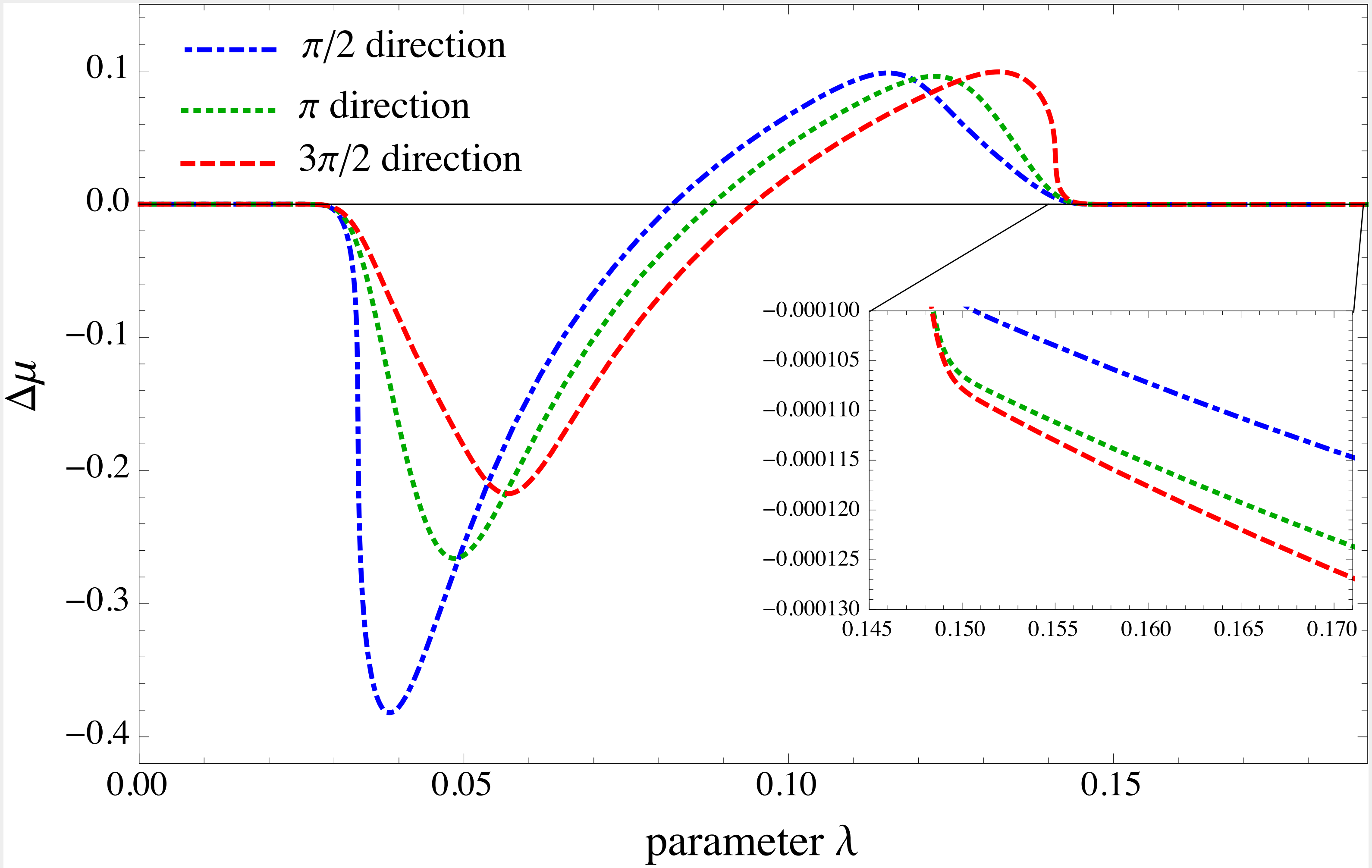
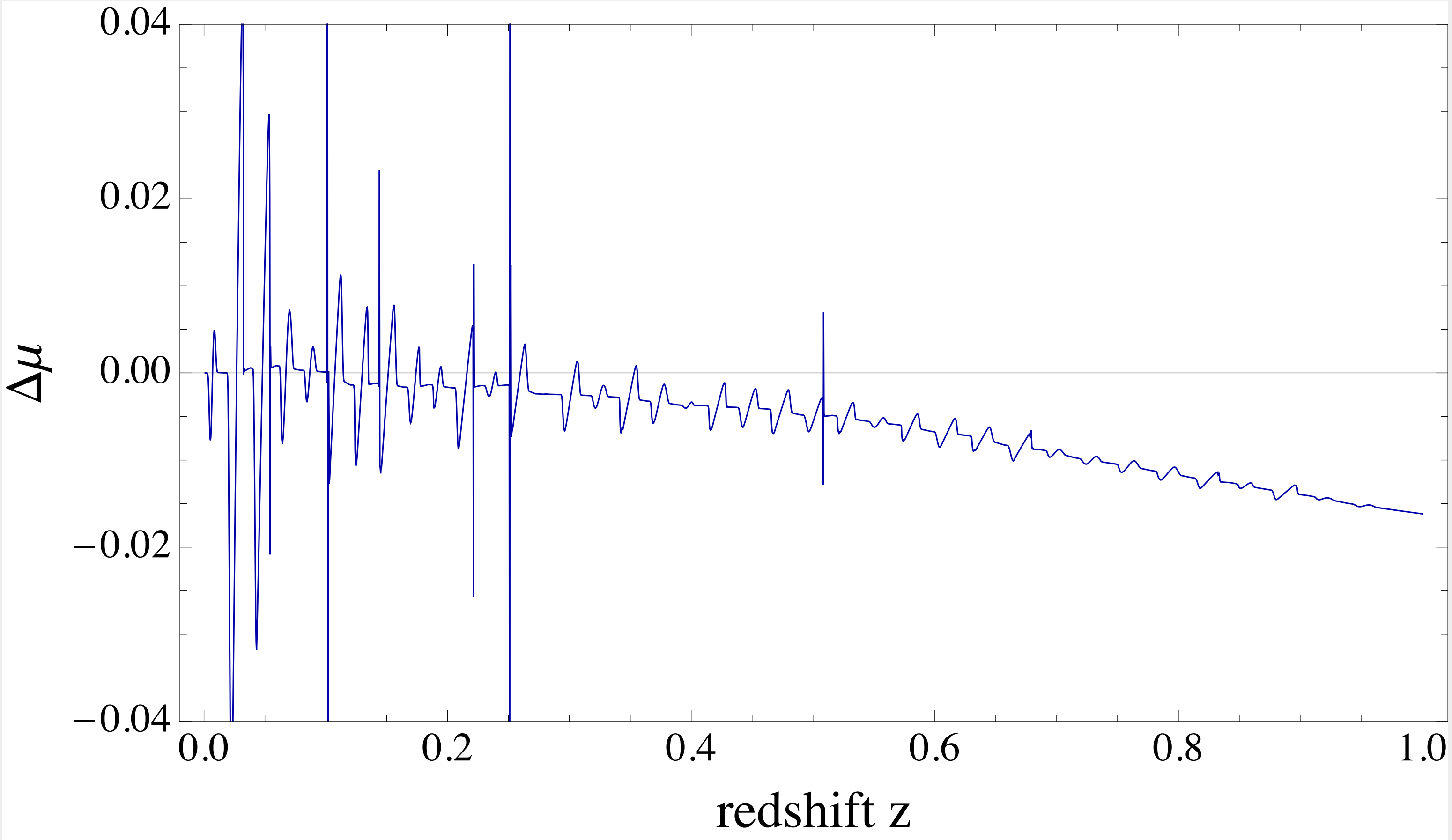
single hole 'radial' paths

$$\Delta\mu = \mu_{\Lambda\mathrm{CDM}} - \mu_{\mathrm{Swiss-cheese}}$$



A. Peel, M. A. Troxel, and M. Ishak, Phys. Rev. D 90, 123536 (2014)

multiple randomized holes



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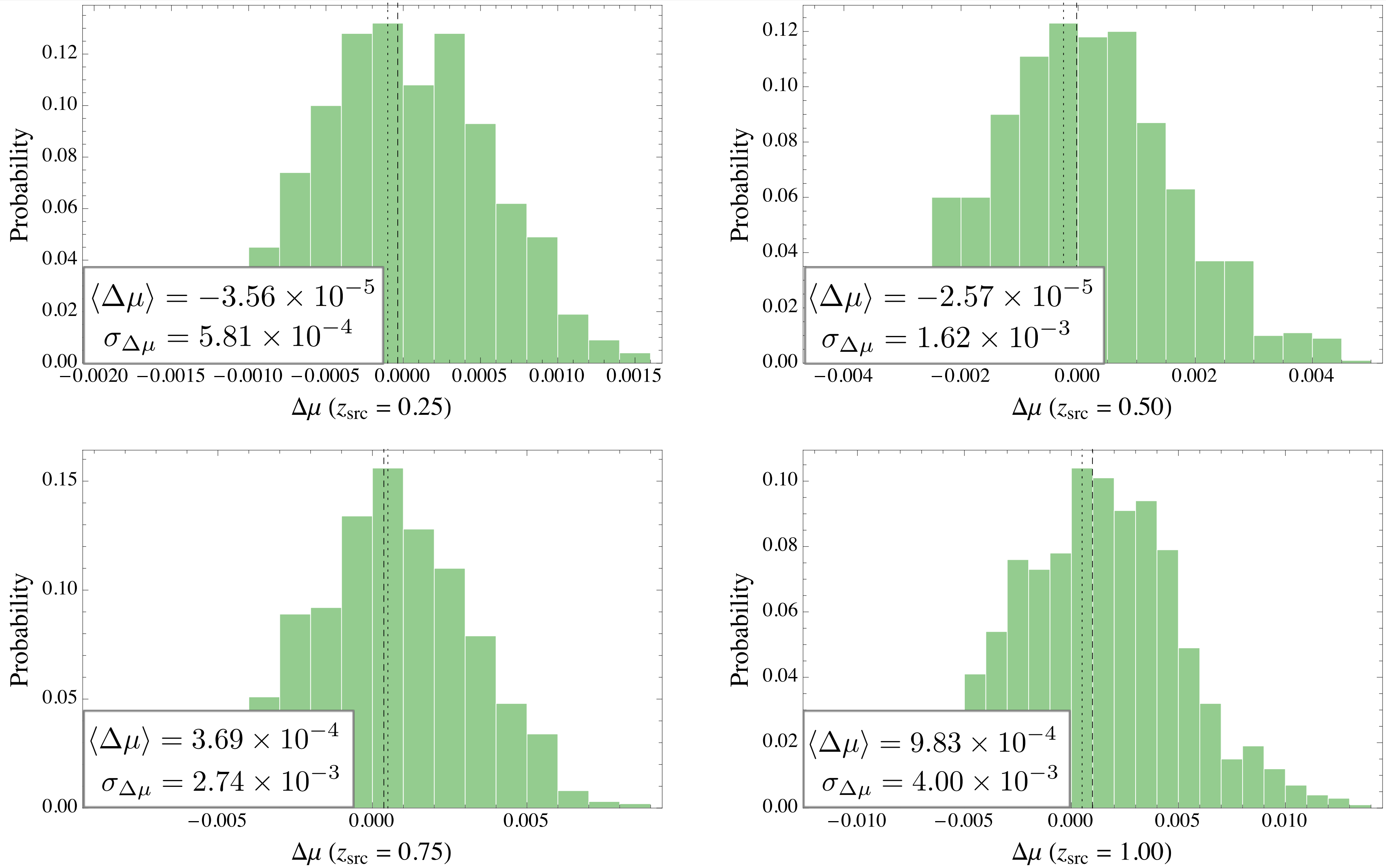
statistical results



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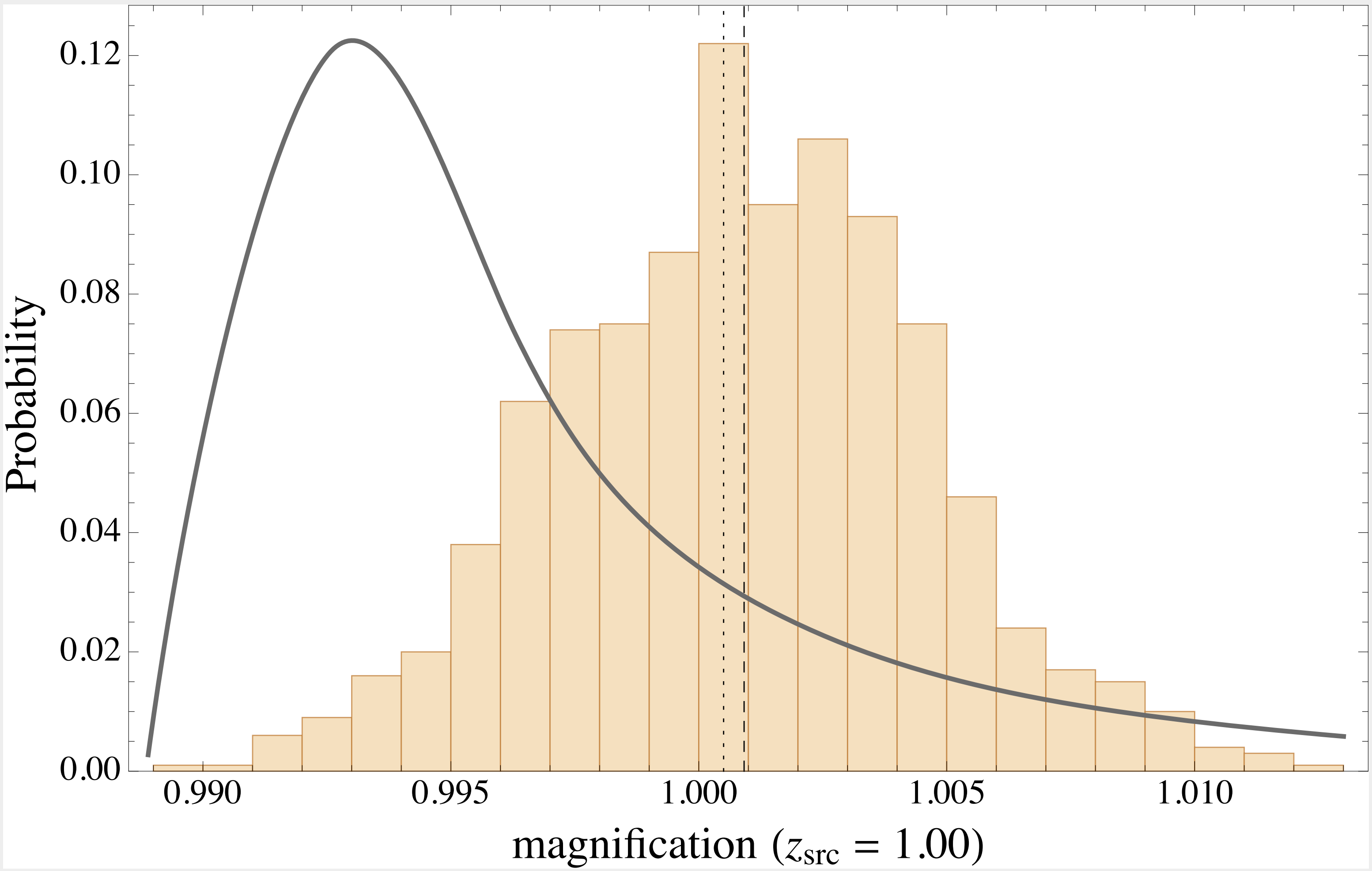
magnification expectation



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summary

- explored **two applications** of Szekeres models to precision cosmology
- found **larger growth rate** in MDE compared to FLRW and spherical collapse
- showed best-fit Szekeres model **fits growth data** as well as the standard model but with different parameters
- built a **Swiss-cheese universe** model with exact and nonlinearly evolving holes
- studied **effect of inhomogeneities** on distance measures
- saw $\langle \Delta\mu \rangle$ and $\sigma_{\Delta\mu}$ **too small** to induce systematic bias in Hubble diagram

future possibilities

- explore constant M notion in class II models
- build statistically homogeneous class II model (where $\langle\delta\rangle = 0$)
- compare results to higher-order perturbation theory

Improving the Swiss-cheese model:

- different hole density profiles
- more realistic hole sizes and distribution
- relax exact matching condition → different statistics?

Thank you!