# Sparsity based weak lensing map making

International Workshop on Cosmology and Sparsity 2 - Nice 2014

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September 9, 2014

# Layout

Mapping the 2D projected convergence

- The shear inversion problem
- The MRLens approach: inpainting and wavelet filtering
- A new combined approach

Mapping the 3D density contrast

- The 3D mapping problem
- The GLIMPSE Algorithm
- Results

Detection of clusters using 2D or 3D lensing

- Motivations for the experiment
- Choice of parameters for both algorithms
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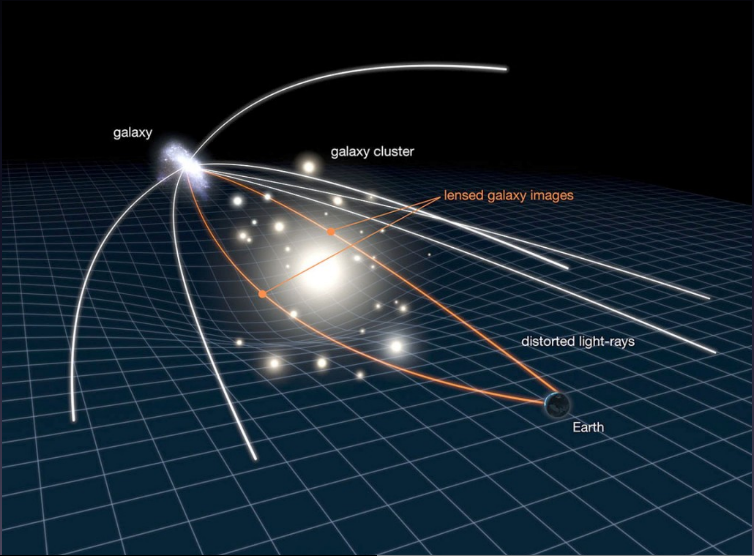
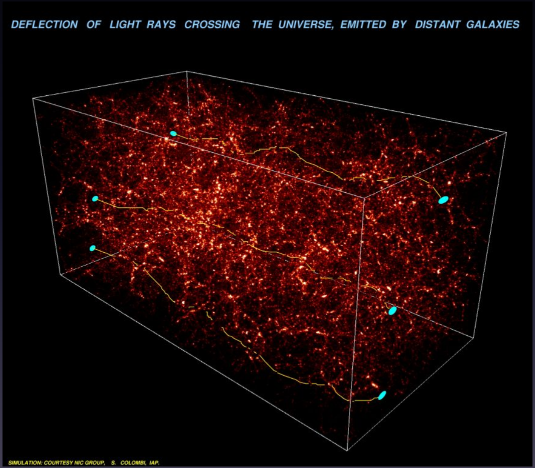

galaxy
galaxy cluster
lensed galaxy images
distorted light-rays
Earth


DEFLECTION OF LIGHT RAYS CROSSING THE UNIVERSE, EMITTED BY DISTANT GALAXIES
SIMULATION: COURTESY NIC GROUP, S. COLOMBI, IAP.

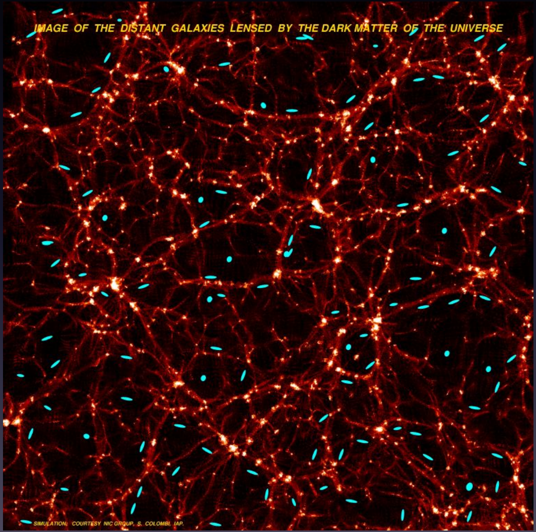
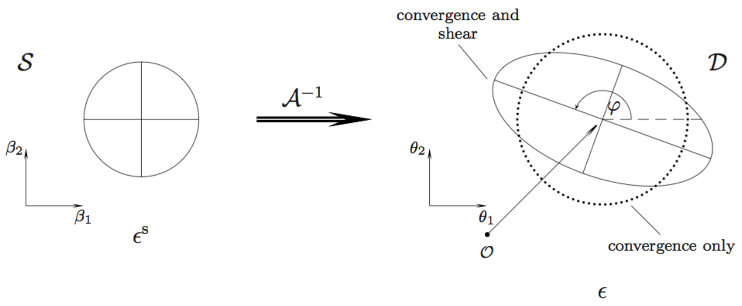


IMAGE OF THE DISTANT GALAXIES LENSED BY THE DARK MATTER OF THE UNIVERSE
SIMULATION: COURTESY NIC GROUP, S. COLOMBI, IAP.

## Impact on galaxy shapes: **Convergence** $\kappa$ and **shear** $\gamma$




$\epsilon = \epsilon_i + \gamma$ with $<\epsilon_i> = 0$

$$\Longrightarrow \quad <\epsilon> = \gamma$$

- The shear can be estimated by averaging galaxies ellipticities
  $\Longrightarrow \gamma$ is our observable
- The convergence cannot be directly estimated

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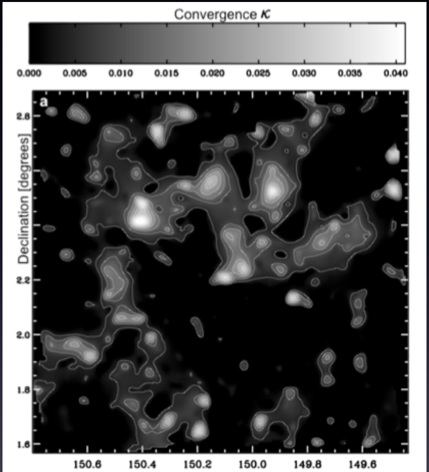


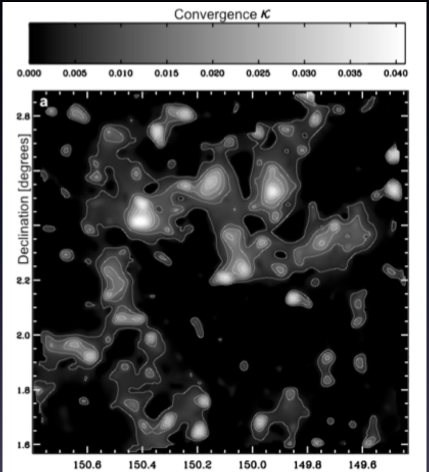

Convergence map of the COSMOS field, Massey et al. (2008)

- Why map the convergence ?

$$\kappa(\theta) = \int Q(z)\delta(\theta, z)$$

$\Rightarrow$ Projection of the 3D density contrast $\delta$

- **But** $\kappa$ is not directly observed:

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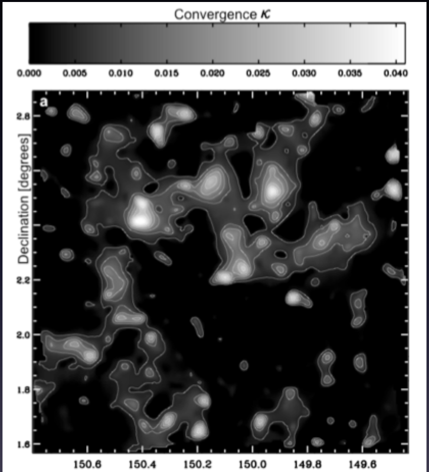


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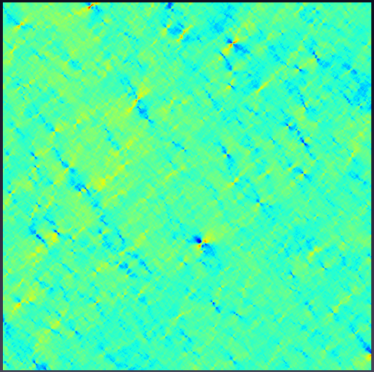
## Link between shear and convergence

$$\begin{cases} \gamma_1 = \frac{1}{2}(\partial_1^2 - \partial_2^2)\Psi \\ \gamma_2 = \partial_1\partial_2\Psi \end{cases} \qquad \kappa = \frac{1}{2}(\partial_1^2 + \partial_2^2)\Psi$$

measurable unknown

## What makes the problem difficult ?

- Noisy shear measurements
- Missing data (Bright stars, CCD defects)



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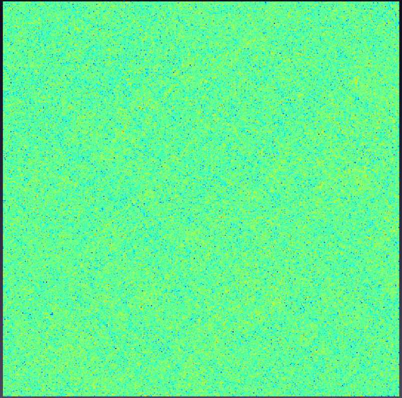

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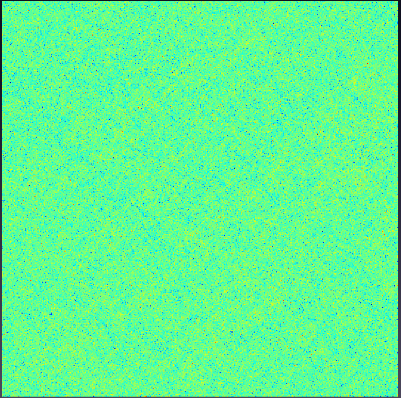

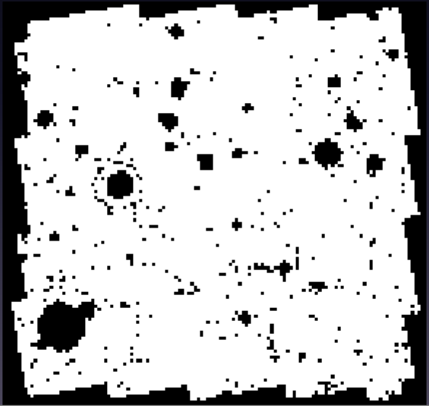

## The MRLens approach:

1. **Bin the shear** catalogue on a regular grid $\Longrightarrow$ Empty pixels define a mask $\mathrm{M}$
2. **Sparse inpainting** to reconstruct a noisy inpainted convergence map $\kappa_n$ (Pires et al. 2009):

$$\min_{\kappa} \parallel \Phi^t \kappa \parallel_0 \qquad s.t. \qquad \sum_i \parallel \gamma_i - \mathbf{M}(\mathbf{P}_i \star \kappa) \parallel_2^2 \leq \sigma$$

   solved using MCA.

3. **Multiscale entropy filtering** to clean the noise (Starck et al. 2006):

$$\min_{\kappa} \parallel \kappa_n - \kappa \parallel \qquad s.t. \qquad \sum_{j,k,l} h(w_{j,k,l})$$

   where $w_{j,k,l}$ are the wavelet coefficients of $\kappa$ and $h(w_{j,k,l}) = 0$ for significant coefficients (Multiresolution support).

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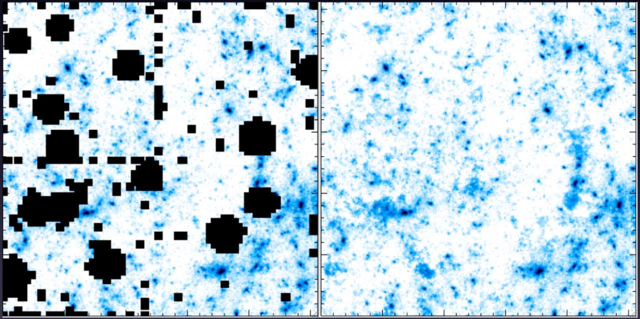

Pires et al. (2009)

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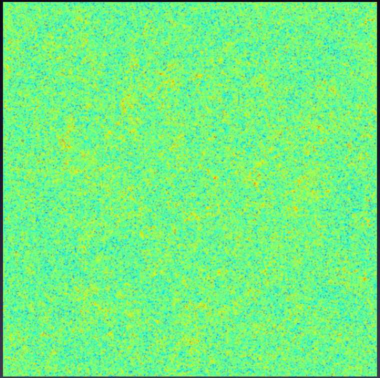

Starck et al. (2006)

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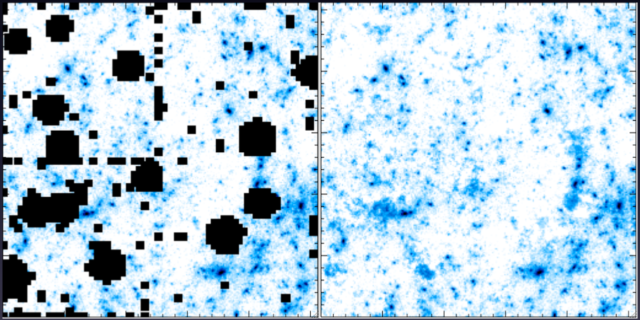

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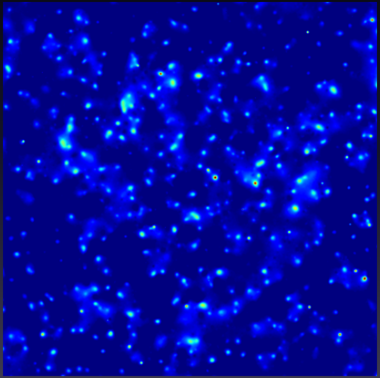
## Multiscale entropy filtering



Starck et al. (2006)

We propose a new approach, solving a single optimization problem:

$$\min_{\kappa \geq 0} \frac{1}{2} \underbrace{\| \boldsymbol{\Sigma}^{-1/2} [g - \mathbf{P}_{\kappa\gamma}\kappa] \|_2^2}_{\text{Data fidelity}} + \underbrace{\lambda \| \boldsymbol{\Phi}^t \kappa \|_1}_{\text{Sparsity constraint}}$$

where

- $g$ is the measured shear on each galaxy
- $\kappa$ is expressed on an **arbitrarily fine** grid
- $\boldsymbol{\Phi}$ is a wavelet dictionary (starlet)

## Nonequispaced FFT

The operator $\mathbf{P}_{\kappa\gamma}$ is implemented in Fourier space and evaluated at each galaxy position using **NFFT**[a]. $\mathbf{P}_{\kappa\gamma}$ is not directly invertible.

[a] http://www-user.tu-chemnitz.de/ potts/nfft

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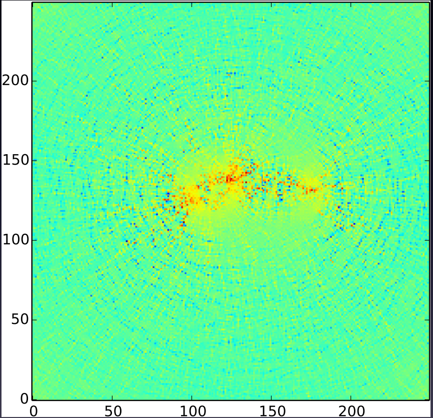
- Solved using a **Generalised Forward-Backward** (Raguet et al. 2011)
- Proximity operator of the $\ell_1$ constraint computed using a simple FISTA (Beck and Teboulle 2009)
- **Regularisation** parameter **defined with respect to the noise** level

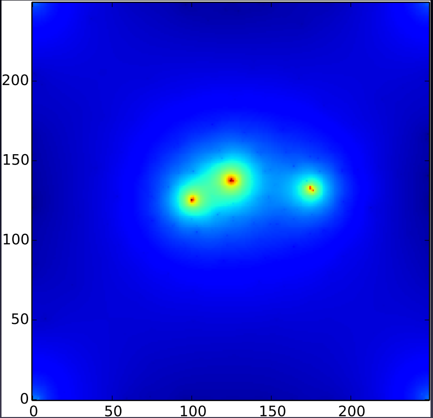
$$\lambda_j(x, y) = k\sigma_j(x, y)$$

with $\sigma_j(x, y)$ estimated for each wavelet coefficient by randomisation of the data.

Example on noise free simulation with a mean of 0.16 galaxy per pixel:

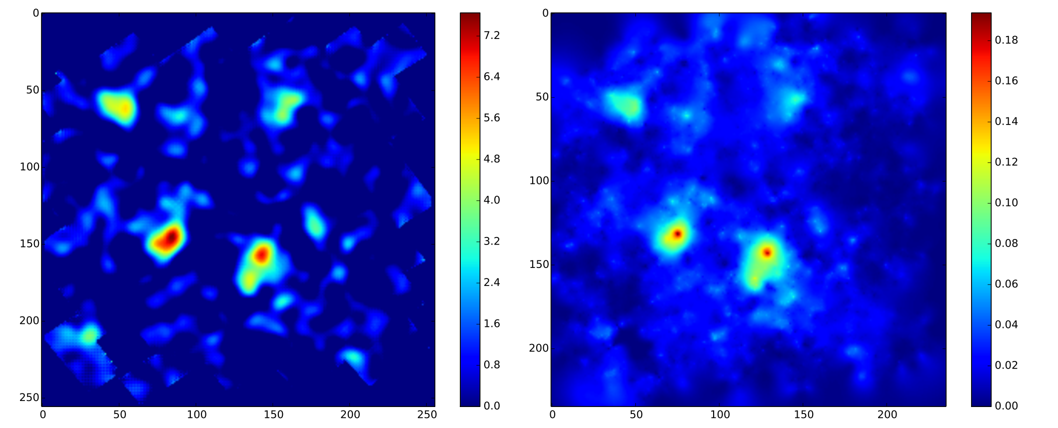



(a) $\mathbf{P}_{\kappa\gamma}{}^{*}$ applied to the data




(b) Reconstruction after convergence

# Preliminary application to STAGES Abell 901/902 Cluster




Left: Convergence Signal to Noise map from Heymans et al. (2008)
Right: Convergence reconstructed with this method

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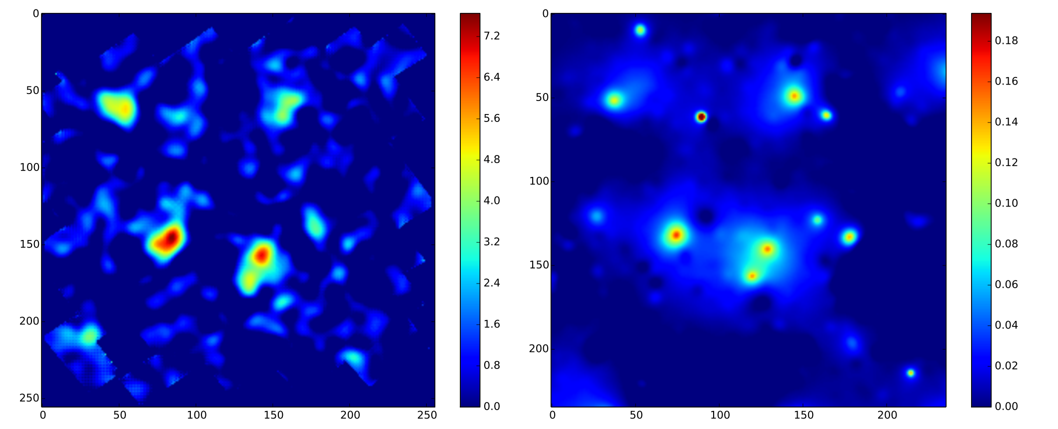


Left: Convergence Signal to Noise map from Heymans et al. (2008)
Right: Convergence reconstructed using MRLens

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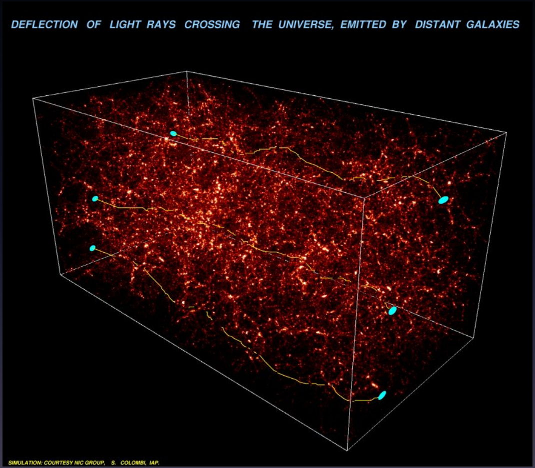
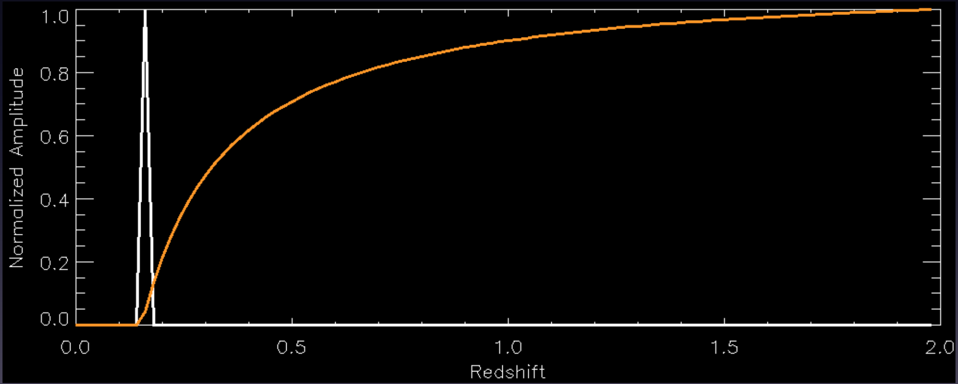



- With current and next generation lensing surveys, the redshift of the galaxies will be known from photometry.
- Combining shear and redshift we want to infer the 3D matter distribution

$$\kappa(\theta, \chi) = \frac{3H_O^2\Omega_m}{2c^2} \int_0^\chi d\chi' \frac{f_K(\chi')f_K(\chi - \chi')}{f_K(\chi)} \frac{\delta(f_K(\chi')\theta, \chi')}{a(\chi')}$$

Redshift dependence of the convergence:



- **The 3D Reconstruction Problem:**

$$\underbrace{\gamma}_{\text{shear}} = \mathbf{P} \quad \mathbf{Q} \quad \underbrace{\delta}_{\text{overdensity}} + \underbrace{n}_{\text{noise}}$$

$\mathbf{P}$ and $\mathbf{Q}$ are the **tangential** and **line of sight** lensing operators

On the bright side:

- linear problem

On the other side:

- **ill-posed** inverse problem
- extremely **noisy shears**
- **photometric redshifts** errors
- missing data

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Two linear methods have been published to approach the 3D weak lensing problem:

$$\gamma = \mathbf{R}\delta + N$$

where $N$ is assumed to be uncorrelated Gaussian noise of diagonal $\Sigma$ and $\mathbf{R} = \mathbf{P}_{\gamma\kappa}\mathbf{Q}$.

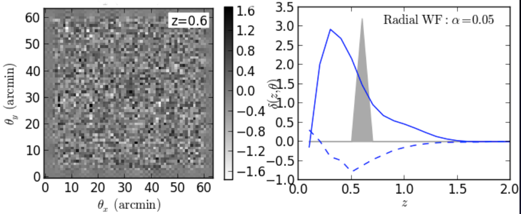
- Wiener Filtering in Simon et al. (2009):

$$\hat{s}_{MV} = [\alpha Id + SR^*\Sigma^{-1}R]^{-1}SR^*\Sigma^{-1}d$$

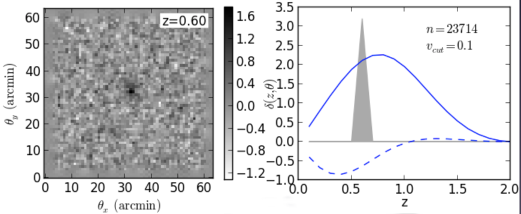
- SVD Regularization in VanderPlas et al. (2011):

$$\hat{s}_{SVD} = V\Lambda^{-1}U^*\Sigma^{1/2}d$$

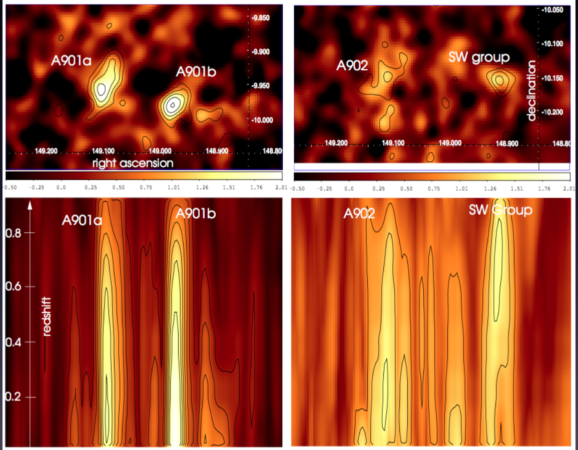
- Wiener fieltering:




- SVD:



Wiener filter reconstruction of the STAGES Abell A901/2 superclusters, from *Simon et al.* (2012)



Limitations of linear methods

- In both cases:
  - very poor redshift accuracy (structures are smeared in l.o.s.)
  - systematic bias in reconstructed redshift
  - overall noisy reconstructions
- These methods **do not aim to reconstruct the dark matter overdensity** $\delta$, only Signal to Noise Ratios.

**GLIMPSE** : Gravitational Lensing Inversion and MaPping with Sparse Estimators.

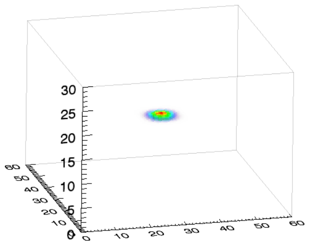
(Leonard, Lanusse, Starck 2014) **arxiv:1308.1353**

- We propose a new sparsity based approach to reconstruct the **overdensity** $\delta$
- Inversion of the lensing kernel regularised by a synthesis sparsity prior:

$$\min_{\alpha} \frac{1}{2} \underbrace{\parallel \mathbf{\Sigma}^{-1/2} [\gamma - \mathbf{PQ\Phi}\alpha] \parallel_2^2}_{\text{Data fidelity}} + \underbrace{\lambda \parallel \alpha \parallel_1}_{\text{Sparsity constraint}}$$

The 2 ingredients of the **GLIMPSE** reconstruction technique:

- a **wavelet based dictionary** adapted to dark matter halos.



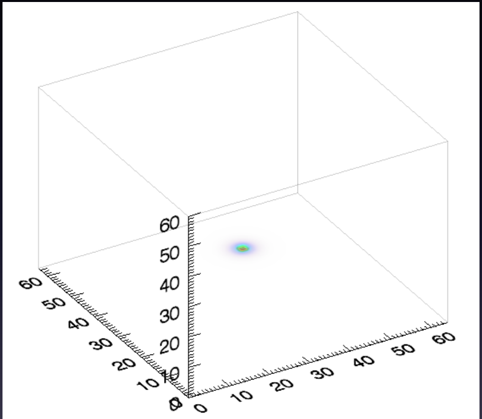

- a **Fast Iterative Soft Thresholding Algorithm** to solve the optimisation problem.

A few practical considerations:

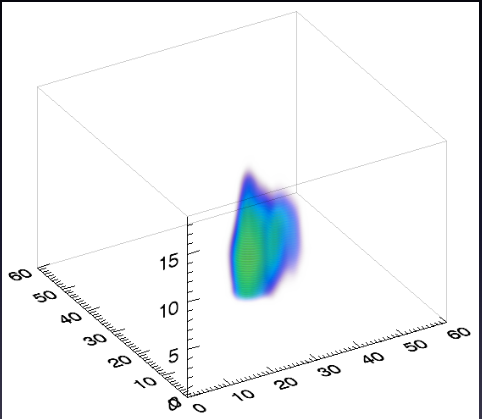
- Regularisation parameter is set according to the level of noise
- The noise on the residuals is estimated using MAD at each iteration
- We use Firm thresholding to avoid bias in the results
- The threshold level is progressively lowered to $k_{\mathrm{min}}\sigma$, typically $k_{\mathrm{min}} = 4$

The algorithm in action on an N-body simulation:

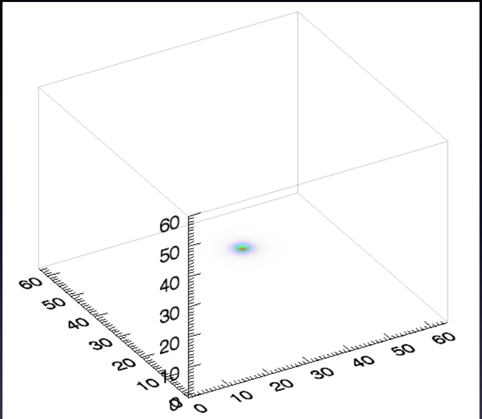
(Loading Video...)




(a) Input **simulated density contrast** for an NFW halo




(b) **SNR map** thresholded at $4.5\sigma$ using **Transverse Wiener Filtering**

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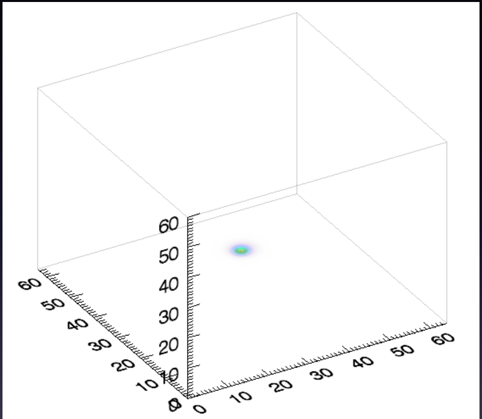



(b) **Density contrast** reconstruction using **GLIMPSE**

## Single halo simulations

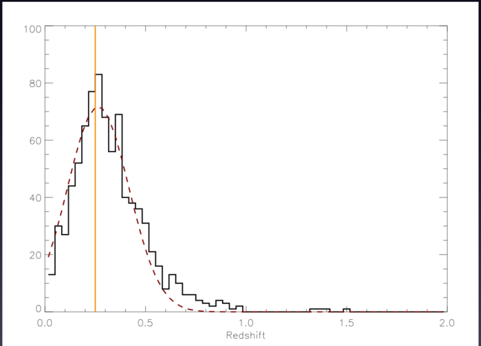
- One NFW profile at the center of a 60x60 arcmin field
- Noise and redshift errors corresponding to an Euclid-like survey
- Mass varying between $3.10^{13}$ and $1.10^{15}\ h^{-1}M_{\odot}$
- Redshifts between 0.05 and 1.55

We ran 1000 noise realisations on each of the 96 fields.

## Redshift Estimation

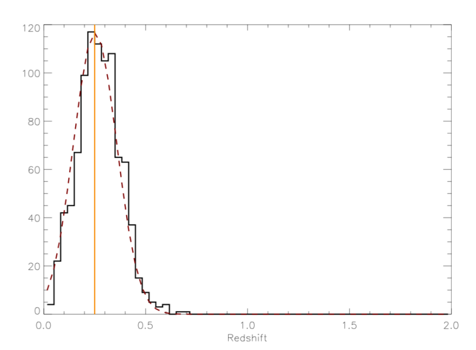
Example of 2 NFW halos at z=0.25

$$m_{vir} = 4.10^{14} h^{-1} M_{\odot}$$



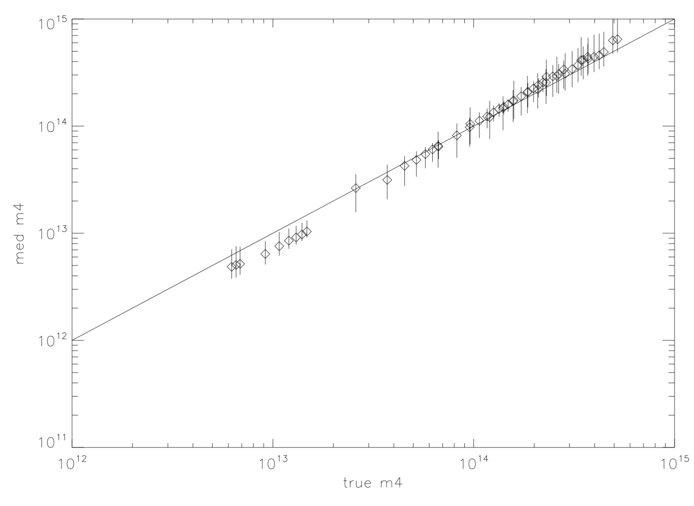

$$\sigma_z = 0.15$$

$$m_{vir} = 8.10^{14} h^{-1} M_{\odot}$$




$$\sigma_z = 0.1$$

## Mass estimation:



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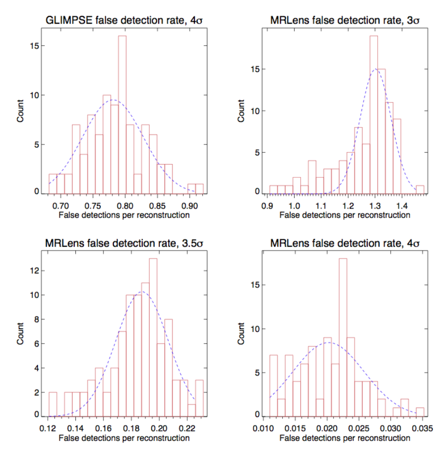
- To detect galaxy clusters from the weak lensing signal, is it better to look at the projected convergence or at the reconstructed 3D density ?

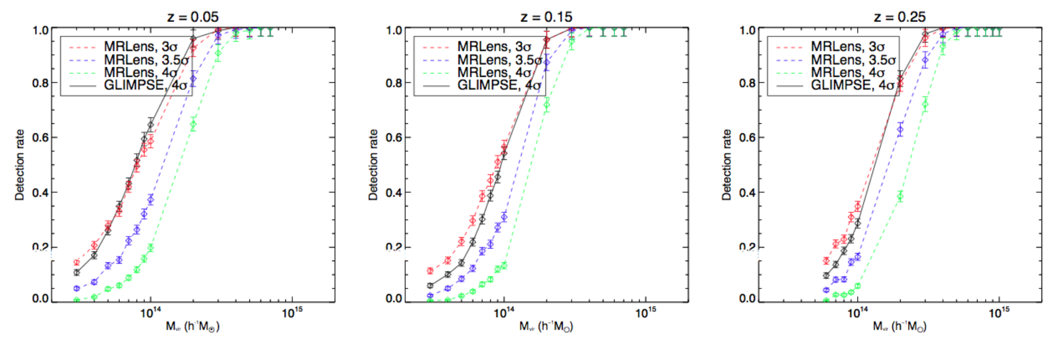
  (Leonard, Lanusse, Starck 2014), submitted, arxiv:14xx.xxxx

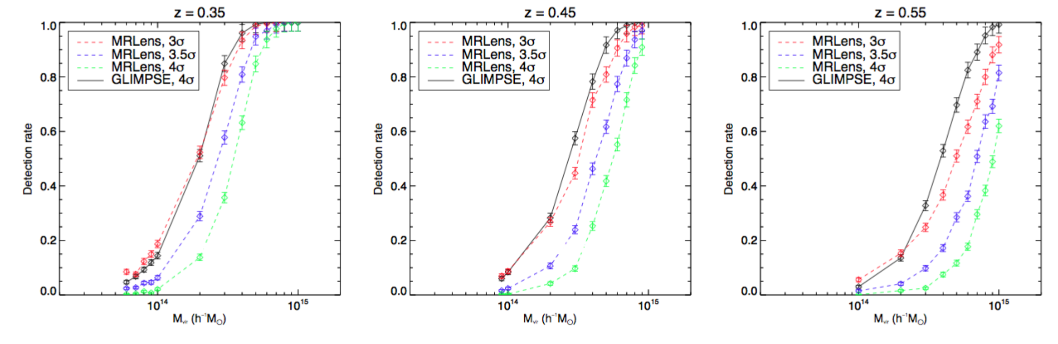
- To answer this question, we use the previous set of simulations and apply a standard MRLens denoising procedure to the projected 2D convergence maps.

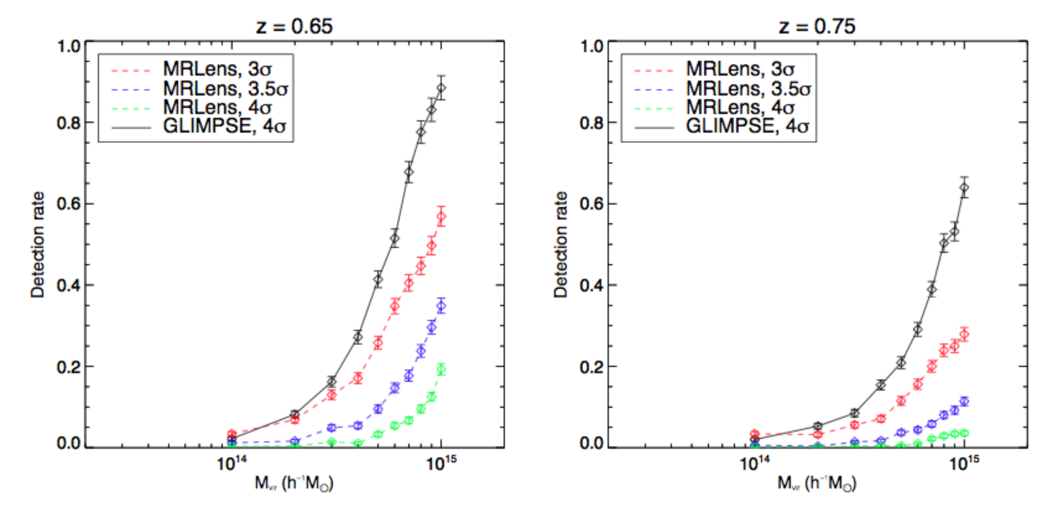
- We look at the true and false detection rate of the central cluster from the 2D and 3D reconstructions.




- We adjust the 2D detection level to yield the same false detection rate as in 3D
- 3D threshold: 4 $\sigma \Longleftrightarrow$ 2D threshold: 3 - 3.5 $\sigma$


z = 0.05
z = 0.15
z = 0.25
MRLens, 3σ
MRLens, 3.5σ
MRLens, 4σ
GLIMPSE, 4σ
Detection rate
1.0
0.8
0.6
0.4
0.2
0.0
10^14
10^15


z = 0.35
z = 0.45
z = 0.55
MRLens, 3σ
MRLens, 3.5σ
MRLens, 4σ
GLIMPSE, 4σ
Detection rate
1.0
0.8
0.6
0.4
0.2
0.0
$10^{14}$
$10^{15}$
$M_{vir}$ ($h^{-1}M_{\odot}$)


z = 0.65
z = 0.75
MRLens, 3σ
MRLens, 3.5σ
MRLens, 4σ
GLIMPSE, 4σ
Detection rate
1.0
0.8
0.6
0.4
0.2
0.0
$10^{14}$
$10^{15}$
$M_{vir}$ ($h^{-1}M_{\odot}$)

Conclusion:

- Improvement of sparsity based 2D mapping without binning of the data
- GLIMPSE represents a significant improvement over linear 3D mapping techniques (reconstruction of the density constrast)
- Strong case for the 3D reconstruction which is very competitive for detecting higher redshift clusters.

**www.cosmostat.org/glimpse.html**

Thank you for your attention.