Sparsity based tools to map the invisible universe

4th DASPAC Seminar

François Lanusse

CosmoStat Laboratory
Laboratoire AIM, UMR CEA-CNRS-Paris 7, Irfu, SAp, CEA-Saclay

May 23, 2014

Layout

Weak lensing map making

- Weak gravitational lensing
- The shear inversion problem
- The 3D weak lensing problem

Linear reconstruction of the 3D density

- Wiener filtering and SVD regularisation
- Wiener filtering reconstruction of Abell 901
- Limitations of linear methods

Sparse reconstruction of the 3D dark matter density

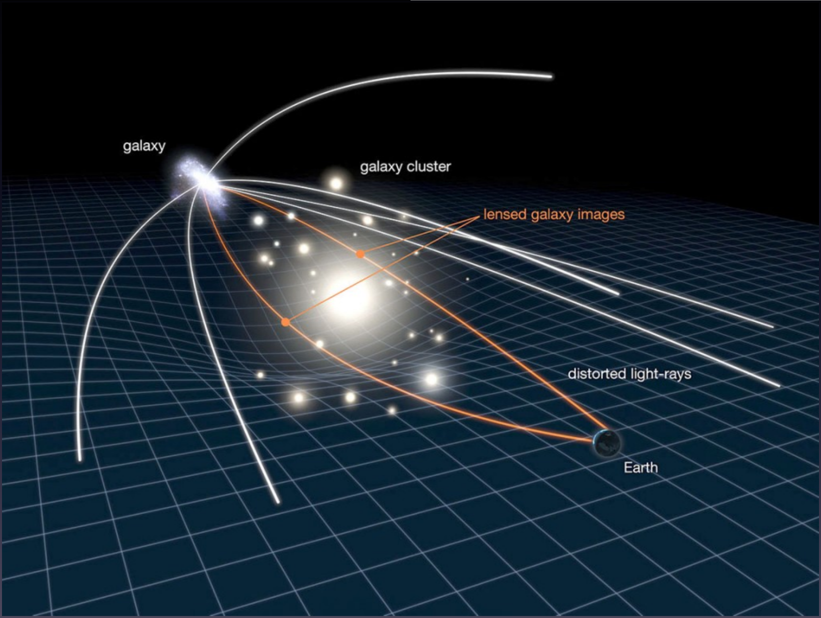
- Sparse regularisation
- The GLIMPSE Algorithm
- Results

Layout

Weak lensing map making
- Weak gravitational lensing
- The shear inversion problem
- The 3D weak lensing problem

Linear reconstruction of the 3D density
- Wiener filtering and SVD regularisation
- Wiener filtering reconstruction of Abell 901
- Limitations of linear methods

Sparse reconstruction of the 3D dark matter density
- Sparse regularisation
- The GLIMPSE Algorithm
- Results


galaxy
galaxy cluster
lensed galaxy images
distorted light-rays
Earth

The 2 regimes of gravitational lensing:

- Strong Lensing
  - near massive clusters
  - presence of arcs and rings

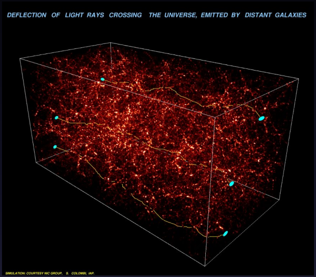
Galaxy Cluster Abell 2218 **HST** • WFPC2
NASA, A. Fruchter and the ERO Team (STScI, ST-ECF) • STScI-PRC00-08

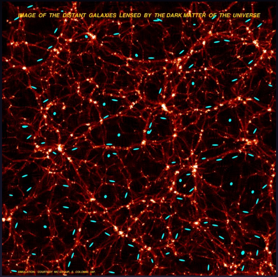
- Weak Lensing
  - generated by large scale structures (filaments, halos, ...)
  - **slight** deformation of galaxy shapes and sizes

The 2 regimes of gravitational lensing:

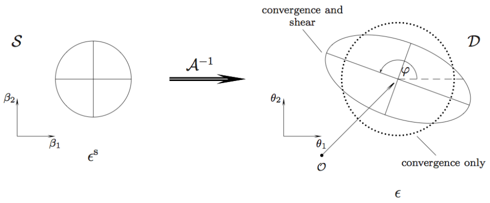
- Strong Lensing
  - near massive clusters
  - presence of arcs and rings

- Weak Lensing
  - generated by large scale structures (filaments, halos, ...)
  - **slight** deformation of galaxy shapes and sizes



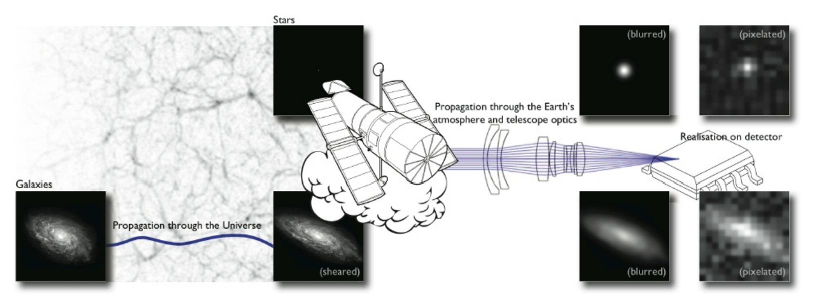


Impact on galaxy shapes: Convergence κ and shear γ



$$\epsilon = \epsilon_i + \gamma \text{ with } < \epsilon_i >= 0$$

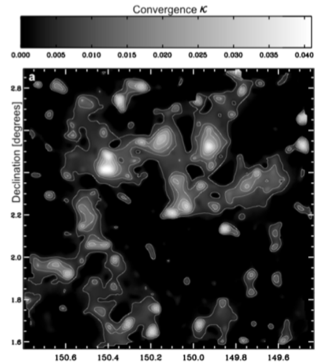
$$\Longrightarrow \quad < \epsilon >= \gamma$$



Kitching et al. (2010)

A difficult measurement in many respects

- Shear SNR on individual galaxies ≈ 0.1
- Instrumental effects: PSF, pixelisation, noise ...
- Shape measurement effects need to be tightly controlled



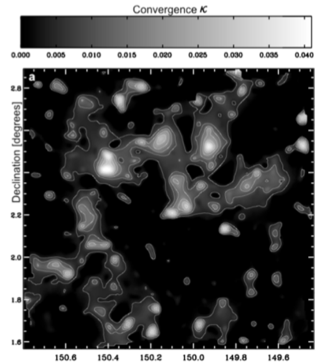
Convergence map of the COSMOS field, Massey et al. (2008)

- Why map the convergence ?

$$\kappa = \int Q(\chi)\delta(\chi)$$

$\Rightarrow$ Projection of the 3D density contrast δ

- But κ is not directly observed
$\Rightarrow$ Must be estimated from γ



Convergence map of the COSMOS field, Massey et al. (2008)

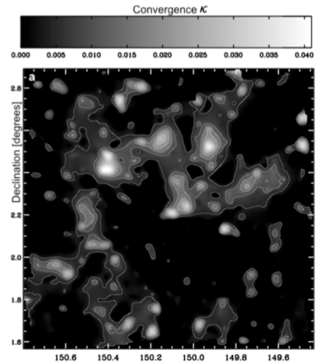
- Why map the convergence ?

$$\kappa = \int Q(\chi)\delta(\chi)$$

$\Rightarrow$ Projection of the 3D density contrast δ

- But κ is not directly observed

$\Rightarrow$ Must be estimated from γ



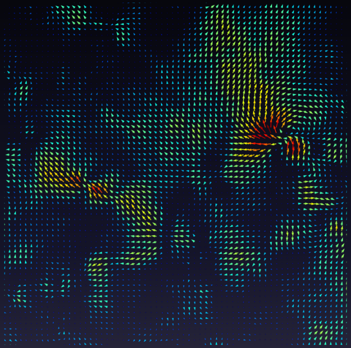
Convergence map of the COSMOS field, Massey et al. (2008)

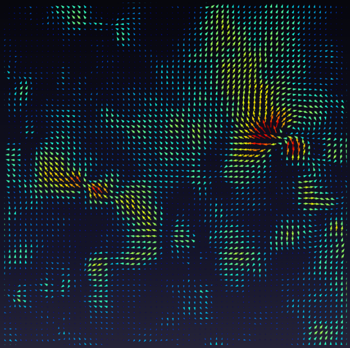
- Why map the convergence ?

$$\kappa = \int Q(\chi)\delta(\chi)$$

$\Rightarrow$ Projection of the 3D density contrast δ

- **But** κ is not directly observed
$\Rightarrow$ Must be estimated from γ

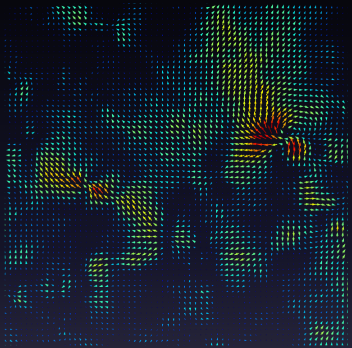


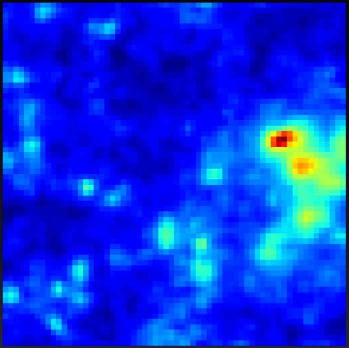


Relationship between shear and convergence

$$\begin{cases} \gamma_1 = \frac{1}{2}(\partial_1^2 - \partial_2^2) \Psi \\ \gamma_2 = \partial_1 \partial_2 \Psi \end{cases} \qquad \kappa = \frac{1}{2}(\partial_1^2 + \partial_2^2)\Psi$$

measurable unknown





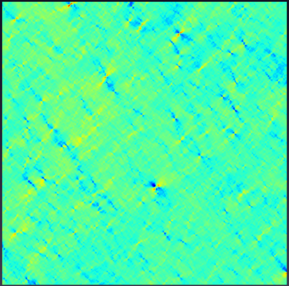
Relationship between shear and convergence

$$\begin{cases} \gamma_1 = \frac{1}{2}(\partial_1^2 - \partial_2^2) \Psi \\ \gamma_2 = \partial_1 \partial_2 \Psi \end{cases} \qquad \kappa = \frac{1}{2}(\partial_1^2 + \partial_2^2)\Psi$$

measurable unknown

What makes the problem difficult ?

- Noisy shear measurements
- Missing data (Bright stars, CCD defects)

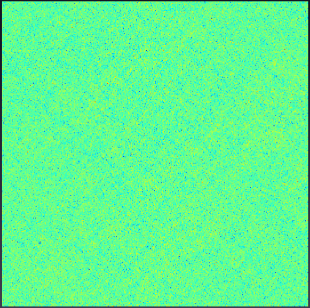


The inversion method needs to satisfy:

- No noise amplification
- Robustness to masks

What makes the problem difficult ?

- Noisy shear measurements
- Missing data (Bright stars, CCD defects)

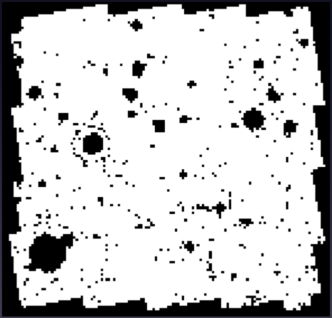


The inversion method needs to satisfy:

- No noise amplification
- Robustness to masks

What makes the problem difficult ?

- Noisy shear measurements
- Missing data (Bright stars, CCD defects)



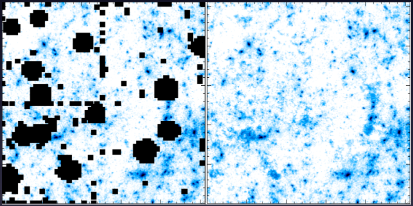
The inversion method needs to satisfy:

- No noise amplification

- Robustness to masks

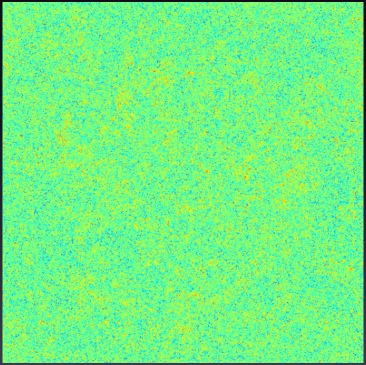
Sparsity based solutions developed at CosmoStat:

Sparse inpainting



Pires et al. (2009)

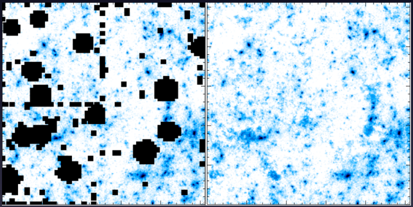
Multiscale entropy filtering



Starck et al. (2006)

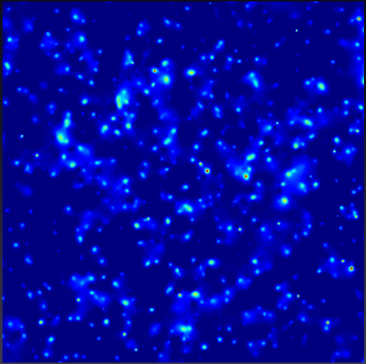
Sparsity based solutions developed at CosmoStat:

Sparse inpainting



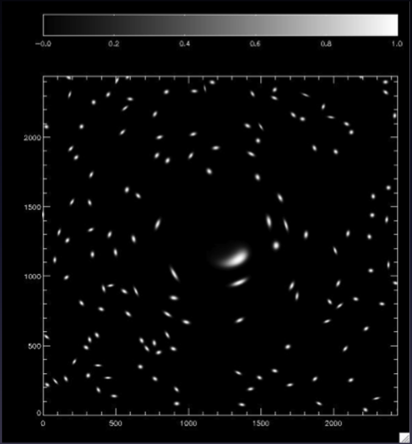
Pires et al. (2009)

Multiscale entropy filtering



Starck et al. (2006)

The 3D mapping problem

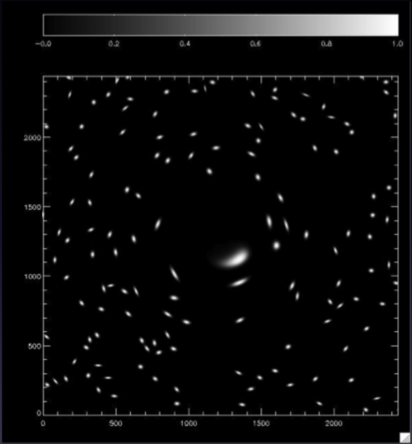


$\Longrightarrow$

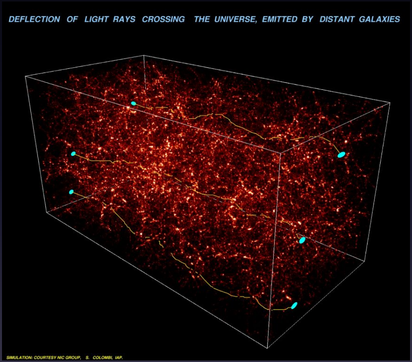
From measurements:

- shear
- redshift

The 3D mapping problem



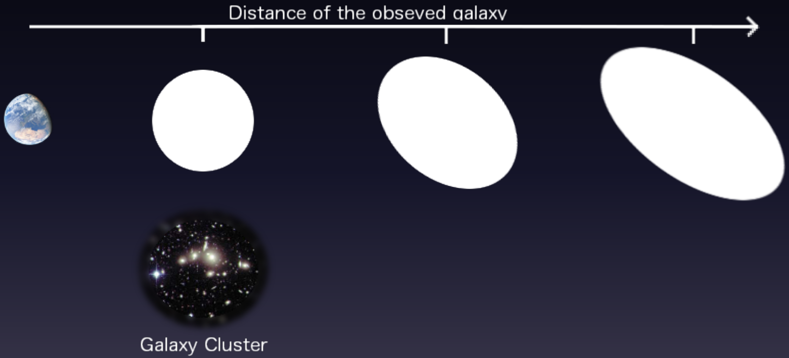
$\Longrightarrow$



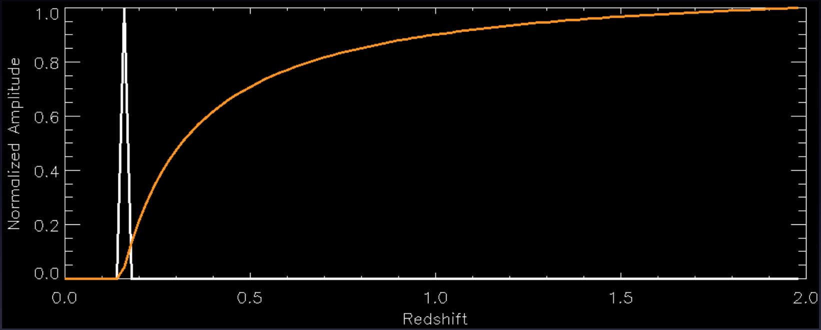
From measurements:

- shear
- redshift

Infer the **3D distribution** of dark matter


Distance of the obseved galaxy
Galaxy Cluster

Redshift dependence of the convergence:



$$\kappa(\chi) = \int d\chi' Q(\chi, \chi') \delta(\chi')$$

- **The 3D Reconstruction Problem:**

$$\underbrace{\gamma}_{\text{shear}} = \mathbf{P} \quad \mathbf{Q} \quad \underbrace{\delta}_{\text{overdensity}} + \underbrace{n}_{\text{noise}}$$

P and Q are the **tangential** and **line of sight** lensing operators

On the bright side:

- linear problem

On the other side:

- **ill-posed** inverse problem
- extremely **noisy shears**
- **photometric redshifts** errors
- missing data

- **The 3D Reconstruction Problem:**

$$\underbrace{\gamma}_{\text{shear}} = \mathbf{P} \quad \mathbf{Q} \quad \underbrace{\delta}_{\text{overdensity}} + \underbrace{n}_{\text{noise}}$$

$\mathbf{P}$ and $\mathbf{Q}$ are the **tangential** and **line of sight** lensing operators

On the bright side:

- linear problem

On the other side:

- **ill-posed** inverse problem
- extremely **noisy shears**
- **photometric redshifts** errors
- missing data

Layout

- Weak lensing map making
  - Weak gravitational lensing
  - The shear inversion problem
  - The 3D weak lensing problem
- Linear reconstruction of the 3D density
  - Wiener filtering and SVD regularisation
  - Wiener filtering reconstruction of Abell 901
  - Limitations of linear methods
- Sparse reconstruction of the 3D dark matter density
  - Sparse regularisation
  - The GLIMPSE Algorithm
  - Results

Two linear methods have been published to approach the 3D weak lensing problem:

$$\gamma = \mathbf{R}\delta + N$$

where N is assumed to be uncorrelated Gaussian noise of diagonal Σ and $\mathbf{R} = \mathbf{P}_{\gamma\kappa}\mathbf{Q}$.

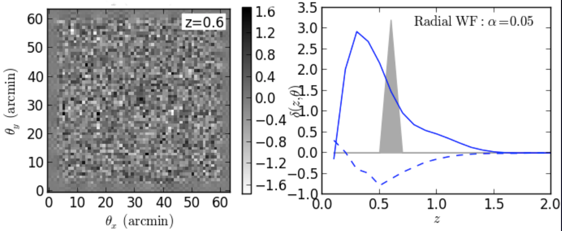
- Wiener Filtering in Simon et al. (2009):

$$\hat{s}_{MV} = [\alpha Id + SR^{*}\Sigma^{-1}R]^{-1}SR^{*}\Sigma^{-1}d$$

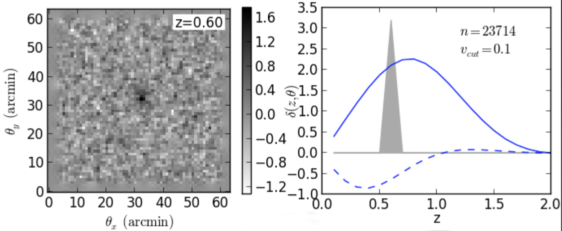
- SVD Regularization in VanderPlas et al. (2011):

$$\hat{s}_{SVD} = V\Lambda^{-1}U^{*}\Sigma^{1/2}d$$

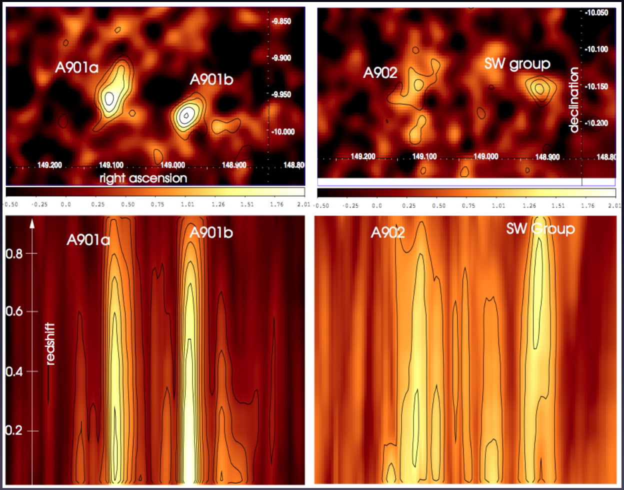
- Wiener fieltering:



- SVD:



Wiener filter reconstruction of the STAGES Abell A901/2 superclusters, from *Simon et al.* (2012)



Limitations of linear methods

- In both cases:
  - very poor redshift accuracy (structures are smeared in l.o.s.)
  - systematic bias in reconstructed redshift
  - overall noisy reconstructions

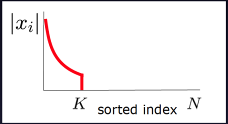
- These methods **do not aim to reconstruct the dark matter overdensity** δ, only Signal to Noise Ratios.

Layout

- Weak lensing map making
  - Weak gravitational lensing
  - The shear inversion problem
  - The 3D weak lensing problem
- Linear reconstruction of the 3D density
  - Wiener filtering and SVD regularisation
  - Wiener filtering reconstruction of Abell 901
  - Limitations of linear methods
- Sparse reconstruction of the 3D dark matter density
  - Sparse regularisation
  - The GLIMPSE Algorithm
  - Results

Sparsity in a few words:

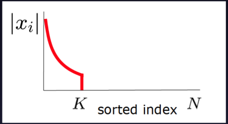
- **Sparse signals**
  Signals that can be represented by a small number of coefficients in an appropriate dictionary.



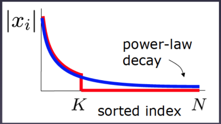
- **Compressible signals**
  Signals that exhibits a power law decay of the amplitude of

Sparsity in a few words:

- **Sparse signals**
  Signals that can be represented by a small number of coefficients in an appropriate dictionary.



- **Compressible signals**
  Signals that exhibits a power law decay of the amplitude of their coefficients in an appropriate dictionary.



Example of a **compressible signal** in the wavelet domain:

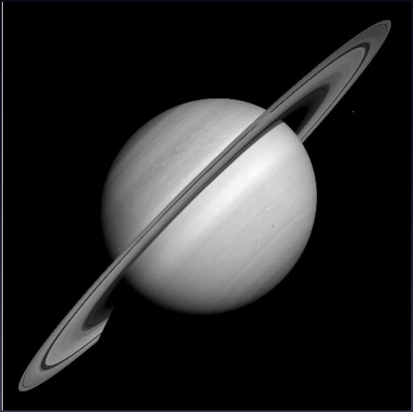


Figure : Original image

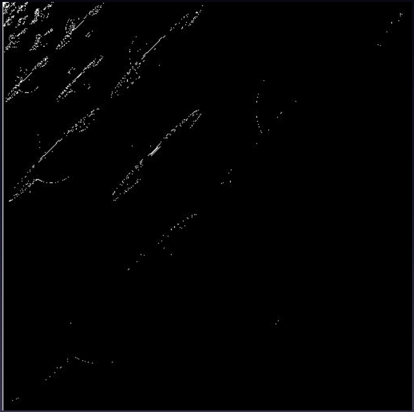


Figure : First 1% of the highest wavelet coefficients

Example of a **compressible signal** in the wavelet domain:

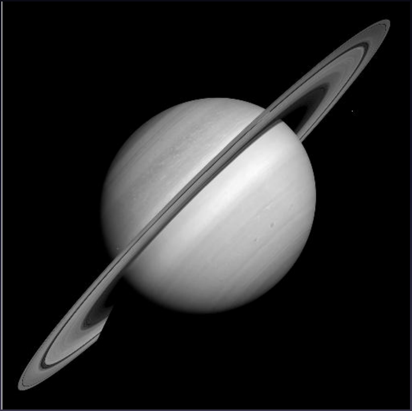


Figure : Original image

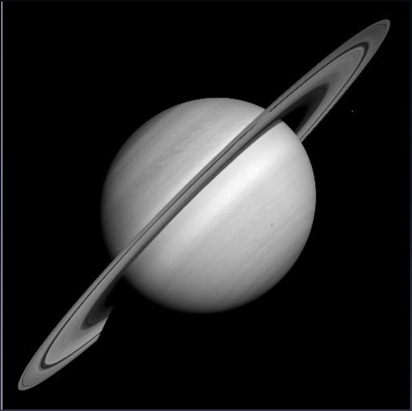


Figure : Reconstructed image from only 1% of the coefficients

Considering a general linear problem of the form:

$$Y = \mathbf{A}X_0 + N$$

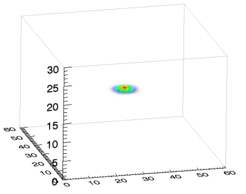
An approximation of X_0 can be recovered by imposing a sparsity promoting penalty on the solution in a dictionary Φ.

$$\boxed{\min_{\alpha} \frac{1}{2} \parallel Y - \mathbf{A\Phi}\alpha \parallel_2^2 + \lambda \parallel \alpha \parallel_1} \qquad \text{with } \tilde{X} = \Phi\alpha$$

Simple example: **Deblurring**

The 2 ingredients of the **GLIMPSE** reconstruction technique:

- a **wavelet based dictionary** adapted to dark matter halos.



- a **Fast Iterative Soft Thresholding Algorithm** to solve the optimisation problem:

$$\min_{\alpha} \frac{1}{2} \underbrace{\| \, \mathbf{\Sigma}^{-1/2} \left[\gamma - \mathbf{PQ\Phi}\alpha \right] \|_2^2}_{\text{Data fidelity}} + \underbrace{\lambda \| \, \alpha \, \|_1}_{\text{Sparsity constraint}}$$

Leonard, Lanusse, Starck (2014)

The iterative algorithm:

$$\alpha_{n+1} = ST_{k\sigma}\left(\alpha_n + \mu \mathbf{\Phi}^t \mathbf{Q}^t \mathbf{P}_{\gamma\kappa}^t \left[\gamma - \underbrace{\mathbf{P}_{\gamma\kappa}\mathbf{Q}\mathbf{\Phi}\alpha_n}_{\text{shear estimate}}\right]\right)$$

- Compute **shear estimate** from current estimation of δ
- Compute **residuals** on the shear
- Compute update to the wavelet coefficients of δ to **minimize the data fidelity**
- Update wavelet coefficients of δ after applying a wavelet shrinkage at $k\sigma$ to **promote the sparsity of the solution**

The iterative algorithm:

$$\alpha_{n+1} = ST_{k\sigma}\left(\alpha_n + \mu \boldsymbol{\Phi}^t \mathbf{Q}^t \mathbf{P}^t_{\gamma\kappa} \underbrace{[\gamma - \mathbf{P}_{\gamma\kappa}\mathbf{Q}\boldsymbol{\Phi}\alpha_n]}_{\text{Residuals on the shear}}\right)$$

- Compute **shear estimate** from current estimation of δ
- Compute **residuals** on the shear
- Compute update to the wavelet coefficients of δ to **minimize the data fidelity**
- Update wavelet coefficients of δ after applying a wavelet shrinkage at $k\sigma$ to **promote the sparsity of the solution**

The iterative algorithm:

$$\alpha_{n+1} = ST_{k\sigma}\left(\alpha_n + \underbrace{\mu \mathbf{\Phi}^t \mathbf{Q}^t \mathbf{P}^t_{\gamma\kappa}\left[\gamma - \mathbf{P}_{\gamma\kappa}\mathbf{Q}\mathbf{\Phi}\alpha_n\right]}_{\text{Gradient step update}}\right)$$

- Compute **shear estimate** from current estimation of δ
- Compute **residuals** on the shear
- Compute update to the wavelet coefficients of δ to **minimize the data fidelity**
- Update wavelet coefficients of δ after applying a wavelet shrinkage at $k\sigma$ to **promote the sparsity of the solution**

The iterative algorithm:

$$\alpha_{n+1} = ST_{k\sigma}\left(\alpha_n + \mu \mathbf{\Phi}^t \mathbf{Q}^t \mathbf{P}_{\gamma\kappa}^t \left[\gamma - \mathbf{P}_{\gamma\kappa} \mathbf{Q} \mathbf{\Phi} \alpha_n\right]\right)$$

- Compute **shear estimate** from current estimation of δ
- Compute **residuals** on the shear
- Compute update to the wavelet coefficients of δ to **minimize the data fidelity**
- Update wavelet coefficients of δ after applying a wavelet shrinkage at $k\sigma$ to **promote the sparsity of the solution**

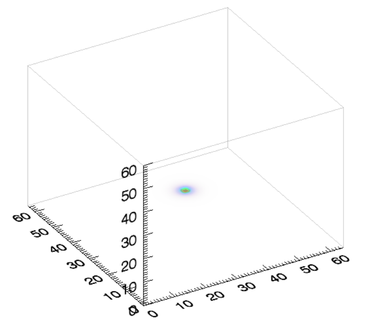
A few practical considerations:

- Regularisation parameter is set according to the level of noise
- The noise on the residuals is estimated using MAD at each iteration
- We use Firm thresholding to avoid bias in the results
- The threshold level is progressively lowered to $k_{\min}\sigma$, typically $k_{\min} = 4$

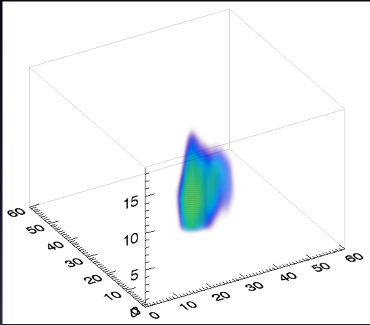
The algorithm in action on an N-body simulation:

(Loading Video...)

Comparison to previous methods on a single halo field:

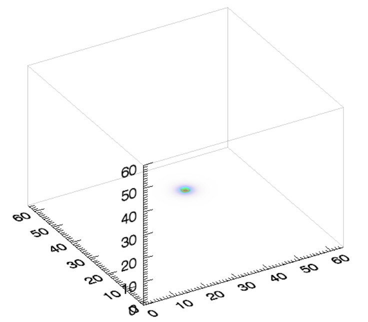


(a) Input **simulated density contrast** for an NFW halo

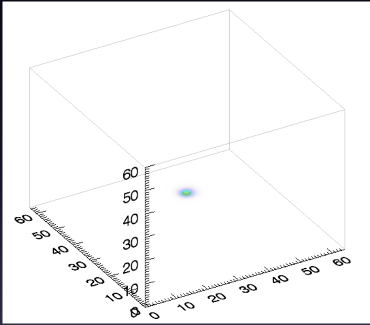


(b) **SNR map** thresholded at 4.5σ using **Transverse Wiener Filtering**

Comparison to previous methods on a single halo field:



(a) Input **simulated density contrast** for an NFW halo



(b) **Density contrast** reconstruction using **GLIMPSE**

Single halo simulations

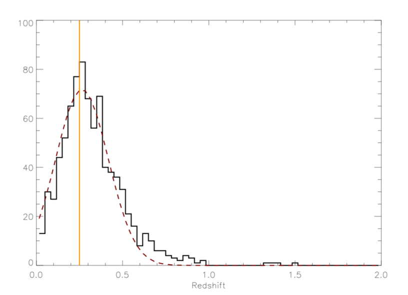
- One NFW profile at the center of a 60x60 arcmin field
- Noise and redshift errors corresponding to an Euclid-like survey
- Mass varying between 3.10^{13} and $1.10^{15}\ h^{-1}M_{\odot}$
- Redshifts between 0.05 and 1.55

We ran 1000 noise realisations on each of the 96 fields.

Redshift Estimation

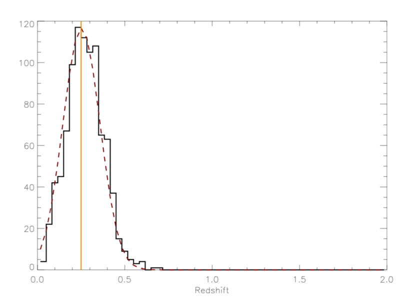
Example of 2 NFW halos at z=0.25

$$m_{vir} = 4.10^{14} h^{-1} M_{\odot}$$



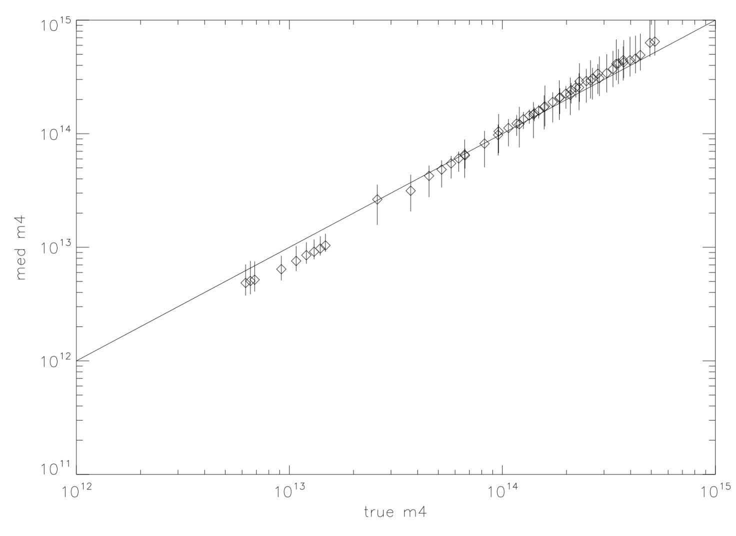
$$\sigma_z = 0.15$$

$$m_{vir} = 8.10^{14} h^{-1} M_{\odot}$$



$$\sigma_z = 0.1$$

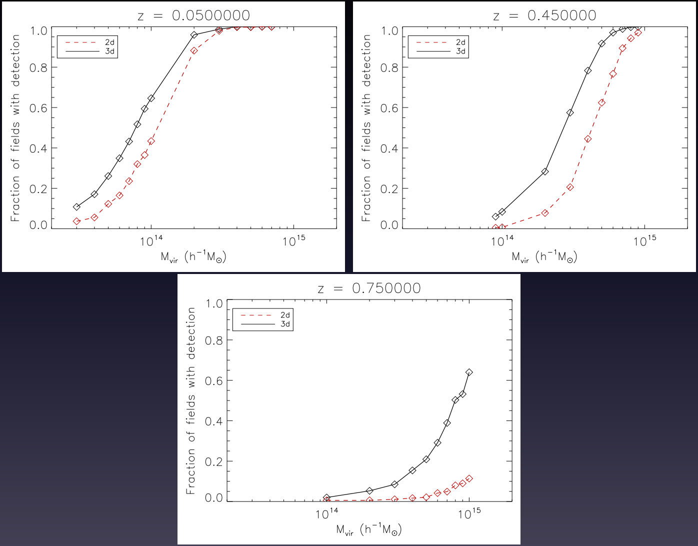
Mass estimation:



We compared our ability to detect a halo in 3D using GLIMPSE and in 2D using MRLens.

- MRLens and GLIMPSE are applied using equivalent threshold levels
- Halos are detected in the outputs within the central 4 pixels

Preliminary results



$\Longrightarrow$ More halos are detected in 3D, especially at high redshift

Conclusion:

- Linear methods are outperformed on all fronts
- Sparse regularisation provides a very robust framework to address 3D weak lensing
- GLIMPSE allows to reconstruct density contrasts and estimate masses
- Using 3D information may improve over standard 2D mapping techniques

Thank you for your attention.