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Towards a fast, model-independent Cosmic Microwave Background bispectrum estimator

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Abstract

The measurement of the statistical properties of the Cosmic Microwave Background (CMB) fluctuations enables to probe the physics of the very early Universe especially at the inflation epoch. A particular interest lays on the detection of non-Gaussianity in the CMB because it can be used to constrain proposed models of inflation and structure formation, or possibly point out new models. The bispectrum is a natural and widely studied tool for measuring non-Gaussianity in a model-independent way. Most studies to measure non-Gaussian signatures in the CMB are highly model-dependent, focusing on the measurement of a single parameter ($f_{\rm NL}^{\rm local}$) characterizing the amplitude of the bispectrum. This paper sets the grounds for a full bispectrum estimator based on the decomposition of the sphere into projected patches. The mean bispectrum estimated this way can be calculated quickly and is model-independent. This approach is very flexible, allowing exclusion of some patches in the processing or consideration of just a specific region of the sphere.

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1. Introduction

The paradigm of Inflation that consists in an early short period of rapid expansion is now becoming the leading theory to describe the physics of the very early Universe and to explain the origin of primordial perturbations. Inflation can make the primordial curvature perturbation $\Phi(x)$ weakly non-Gaussian. Many of the non-Gaussian models of inflation are writing the primordial curvature perturbation $\Phi(x)$ using the local model [9] as follows:

$$\Phi(x) = \Phi_{\rm L}(x) + f_{\rm NL}^{local}(\Phi_{\rm L}^2(x) - \langle\Phi_{\rm L}^2(x)\rangle), \tag{1}$$

where $f_{\rm NL}^{\rm local}$ is the non-linear coupling **constant** in the local model. **The local model is the more currently used because of its simplicity and because**

all the higher-order moments are determined in terms of this $f_{\rm NL}^{\rm local}$ parameter. Most of models of inflation are only doing predictions for this form of non-Gaussianity. However, other models have been investigated given rise to different types of deviation from Gaussianity and detailed calculations of other form of non-Gaussianity have been carried out (see for example [1, 5]) .

The observed CMB temperature fluctuations $\Delta T/T$ are related to the primordial curvature perturbation $\Phi(x)$ through the following non-linear relation [10]:

$$\frac{\Delta T}{T} \approx g_{\rm T}(\Phi(x)), \tag{2}$$

where $g_{\rm T}$ is the radiation transfer function. At small scales, the radiation transfert function $g_{\rm T}$ is somewhat complex but it can be evaluated numerically by solving the Boltzmann transport equation. On very large scale, in the Sachs-Wolfe regime ($l < 10$), the relation becomes very simple $\frac{\Delta T}{T} = \frac{\Phi(x)}{3}$. A number of theories of inflation have been proposed that make different predictions about the CMB fluctuations. Thus the measurement of the statistical properties of the CMB are a direct test of inflation and will help to rule out most of these models. The observed CMB anisotropies $\Delta T/T$ can be expanded into spherical harmonics:

$$\Delta T/T = \sum_{lm} a_{lm} Y_{lm}. \tag{3}$$

The power spectrum is the more currently used statistics to characterize the CMB. If the CMB is Gaussian, it is fully described by its angular power spectrum:

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{+l} |a_{lm}|^2. \tag{4}$$

But many realistic models predict deviations from Gaussianity. Even if, the level of non-Gaussianity is predicted to be very small in single field slow-roll inflation model, there is a large class of more general models that predict a substantially high level of primordial non-Gaussianity. If these models are true, the power spectrum provides a limited insight on the very early Universe **and we need higher-order estimators to probe the CMB non-Gaussianity. The bispectrum is a natural model-independent statistics to probe the small departure from Gaussianity that could originate during inflation. The CMB angular bispectrum may be calculated from product of three** spherical harmonic coefficients of the CMB temperature field. For Gaussian fields, the expectation value is exactly zero. Given statistical isotropy of the universe, the angular bispectrum $B_{l_1 l_2 l_3}$ is given by (Komatsu et al, 2001):

$$B_{l_1 l_2 l_3} = \sum_{m_1, m_2, m_3} \left(\begin{array}{ccc} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{array} \right) a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}, \tag{5}$$

where the matrix denotes the Wigner-3j symbol, $B_{l_1 l_2 l_3}$ satisfy the triangle condition: $|l_1 - l_2| \leq l_3 \leq l_1 + l_2$ and the parity invariance: $l_1 + l_2 + l_3 =$ even. Then, a bispectrum non equal to zero is a signature that the inflation period is more complex than a simple inflation slow-roll model.

However, the full bispectrum estimation is too long to compute and it is usually assumed that the signal will be too weak to measure individual multipoles with any significance. Instead a least squares fit to compare the observed bispectrum with a particular (separable) theoretical bispectrum is used. Thus, most non-Gaussianity studies focus on estimating the $f_{\rm NL}^{\rm local}$ parameter because the bispectrum is fully specified by this parameter assuming a local model (1). A fast estimator for $f_{\rm NL}^{\rm local}$ has been developed by [11] and improved by [1]:

$$f_{\rm NL}^{\rm local} \simeq \left[\sum_{l_1 \leq l_2 \leq l_3}^{l_{\max}} \frac{(\mathcal{B}^{\rm prim}_{l_1 l_2 l_3})^2}{C_{l_1} C_{l_2} C_{l_3}} \right]^{-1} S_{\rm prim}, \tag{6}$$

where C_l is the theoretical power spectrum and $\mathcal{B}^{\rm prim}_{l_1 l_2 l_3}$ is the theoretical bispectrum [9] for $f_{\rm NL}^{\rm local} = 1$. The statistics $S_{\rm prim}$ defined in [8] has only a complexity of $\mathcal{O}(N^{3/2})$ whereas the full bispectrum analysis is $\mathcal{O}(N^{5/2})$. Non-Gaussianities of the local type are generated in standard inflation and for multi-field models. A detection of $f_{\rm NL}^{\rm local} > 10$ will rule out most of the existing inflation models.

However, an expression like equation (1) is not general and there are many other inflationary models that predict different types of deviations from Gaussianity. Other models have been investigated and detailed calculations of other form of non-Gaussianity have been carried out (see for example [1, 5]) But all these estimators are highly model-dependent and some non-Gaussian signatures may be missed. In this paper, we will be interested by a more general method to probe the CMB non-Gaussianities: the bispectrum that is model-independent. If the non-Gaussianities are of the local type, the bispectrum will reach a maximum for squeezed configurations (i.e. one wave vector is much smaller than the other two).

The problem with estimating the full bispectrum on spherical data comes from its time to compute using the spherical harmonics definition (5). This paper introduces a promising approach for accelerating the calculation of the fully general bispectrum. This method is based on the projection of the sphere into small-field projected maps. A Fourier decomposition is then used to estimate the bispectrum for each projected map. And a mean bispectrum estimator can be obtained by combining results from all the patches. **This paper sets the grounds of the method but to go further and fully optimize the estimator, we need more accurate non-Gaussian simulations that are not yet available** and we need also a code to compute the analytical predictions of the full bispectrum for a given level of non-Gaussianity. The outline of the paper is as follows. In §2, the main issues of the method are pointed out and the method is optimized to do power spectrum estimation. In §2.4 the same method is used to accelerate the full bispectrum estimation. Some adaptations of the method are required to optimize the bispectrum estimation.

2. Decomposition of the sphere into rectangular Cartesian maps

In this section, we will describe a method to speed up the spectral analysis that consists in decomposing the sphere into patches. Such method is similar to the Welch's method [12, 15] that is commonly used to reduce the variance in spectral analysis of 1D data. For a sphere, the size and the repartition of the patches have to be decided. Then, the pixels of the patches have to be projected into rectangular Cartesian maps. A spectral estimator is then obtained by averaging the result of the spectral analysis on each rectangular Cartesian map. Such methods have a bias due to the limited size of the projected patches. In a future paper, we will consider multi-taper techniques [2] that consist in averaging over different tapers using the full data. In these methods, the bias is smaller since the data length is not shortened.

In this section, we will follow the basic approach that consists in decomposing the sphere into patches. A number of issues has to be solved before having an optimal estimator.

2.1. Some issues of the method

The first issue consists in finding an optimal **tessellation of the sphere**. **To facilitate the post-processing especially the FFT required to do spectral analysis, the patches ought to be rectangular and have preferably the same orientation**. However, there do not exist any regular tessellation of the sphere by rectangles. Anyway, a pseudo regular tessellation with rectangles can be obtained by allowing the patches to overlap. Besides, an overlapping is recommended to do power spectrum estimation. Indeed, according to the Welch's overlapped segment method [15, 12], estimating the power spectrum of a signal by splitting this one into overlapping windowed patches reduces the noise in the estimated power spectra by reducing the frequency resolution. We still have to decide the size of the patches and the position of the centers. To reduce the distortions, the size of the patches should depend on the map projection and is a compromise between the distortions introduced by the projection and the window function effects.

The **window function effect** is another issue that has to be considered. It comes from the limited size of the projected maps. Thus, instead of analyzing the signal $s(x,y)$, we will be analyzing the truncated signal: $s_h(x,y) = s(x,y)h(x,y)$. In the frequency domain, we obtain the following convolution product $S_h(u,v) = S(u,v) * H(u,v)$ where $H(u,v)$ is the Fourier transform of the window function. By default the window function is a rectangular window, constant inside the patch and zero elsewhere (see the left panel of the Fig.5).

But, it appears that a window function with better frequency response has to be used. The ideal window function is one whose frequency response is a Dirac delta function. It corresponds to an infinite rectangular window function which is impossible in practice. Instead of a delta function, the frequency response has normally a main lobe and side lobes (see the right panel of the Fig.5). To be close to a Dirac function, the main lobe ought to be the highest and the narrowest to increase the frequency resolution and the side lobes have to be the lowest to limit the mode-to-mode interaction. The rectangular window function is usually not recommended because of its important sides lobes.

An additional issue comes from the **projection effects**. The method that we have developed is based on the decomposition of the sphere into a few rectangular Cartesian maps. It assumes to do pixel projections from the HEALPix map to rectangular Cartesian maps. No matter how sophisticated the projection process will be, distortions are inherent in flattening the sphere. Some classes of map projections maintain areas, and others preserve local shapes, distances, and/or directions... No projection, however, can preserve all these characteristics. Choosing a projection thus always requires compromising accuracy in some way, and that is one reason why so many different map projections have been developed.

Whatever the projection, if we want to keep this projection exact, we will have to deal with rectangular Cartesian maps having a **non-regular grid**. This will be a problem especially for the power spectrum and bispectrum estimation because we need to estimate the Fourier coefficients from a signal $f(x, y)$ at arbitrary nodes (x, y).

Assuming the signal f is a periodic function, it can be decomposed into Fourier series as follows:

$$f(x, y) = \sum_{n_1=-\infty}^{+\infty} \sum_{n_2=-\infty}^{+\infty} C_{n_1,n_2}(f) e^{2\pi i \frac{n_1}{T} x} e^{2\pi i \frac{n_2}{T} y} \tag{7}$$

The approximation that has been used in this paper consists in finding the coefficients $C_{n_1,n_2}(f)$ of the Fourier series (7). The problem can be solved efficiently by noticing the system matrix is a Toeplitz matrix i.e. a diagonal-constant matrix [see 4, for more details about the use of Toeplitz matrices in irregular sampling problems]

2.2. Optimization of the method for power spectrum estimation

As discussed in the previous section, a number of issues has to be solved in order to use the previous method for spectral estimation. In this section, the method is optimized to perform power spectrum estimation. For this purpose, we have generated simulations of the full sky CMB temperature as realizations of a random Gaussian field with a prescribed power spectrum. The cosmology adopted in these simulations is consistent with the WMAP parameter measurements. We use the HEALPix pixelisation as WMAP and Planck missions with a resolution parameter of $n_{\mathrm{side}} = 1024$.

2.2.1. The tiling of the sphere

The first issue consists in finding an optimal tiling of the sphere. Following the ideas discussed in §2, several decompositions have been tested.

An interesting tiling consists in decomposing the sphere into rectangular patches distributed into lines of same latitude (see Fig.1). **This decompo-**

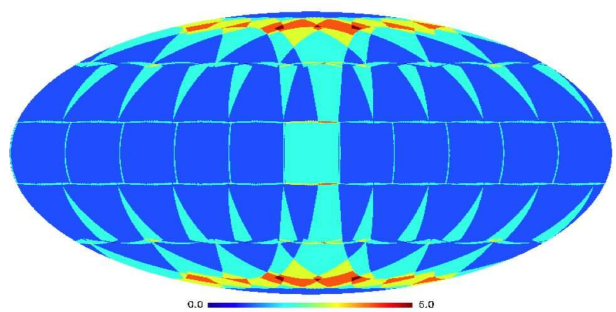

Figure 1: Sphere tiling using an equi-latitude decomposition

sition introduces a substantial overlapping at the pole. For power spectrum estimation, this effect can be neglected by assuming the CMB field is isotropic. But this decomposition should not be used to detect non-Gaussianity because some types of non-Gaussianity can produce localized hot spots or other structures.

Another tiling consists in decomposing the sphere into rectangular patches located at the HEALPix centers of a lower resolution (see Fig.2). This decomposition insures a good repartition of the patches in the sky and should be preferred for bispectral estimation.

Once, we have selected the optimal tiling of the sphere, the pixels of the patches are projected into rectangular Cartesian maps using a projection that will be described in §2.2.2. To perform the decomposition, the size of the patches has to be fixed in such way the distortions are reduced. To reduce the spectral leakage, the size of the patch has to be increased to narrow the main lobe of the frequency response. But increasing the size of the patch will increase the distortions effects due to projection effects.

In Fig.3, the mean power spectrum has been estimated using three different sizes of field: 10°x10°, 20°x20° and 30°x30°. A Hann window has also been used to reduce the spectral leakage. Then, these power spectra (red) have been compared to the theoretical power spectrum used to simulate the full-sky CMB (black).

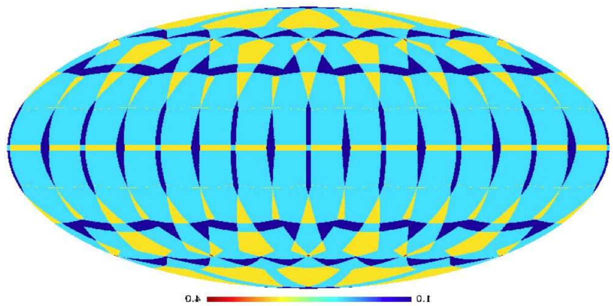

Figure 2: Sphere tiling using the HEALPix centers of a lower resolution.

As expected, an important spectral leakage is observed on the mean power spectrum estimated with patches of 10°x10° (see top panel of Fig. 3). With patches of 30°x30° (see bottom panel of Fig. 3), the spectral leakage is severely dampened but some distortions are visible at large scales, most certainly due to projection effects. The best result is obtained with a field of 20°x20° (see middle panel of Fig. 3) which is a good compromise to both minimize the spectral leakage and reduce the distortions introduced by the map projection.

2.2.2. Map projection

The map projection introduces distortions of various classes, as it has been discussed in §2.1. The choice of the map projection has to be done in such way that it reduces the error in the estimated power spectrum. Two projections have been compared:

1. The gnomonic projection is constructed by projecting every point of the sphere onto patches from the center of the sphere. Assuming the patch is tangent to the point S ($\theta_\circ$, $\psi_\circ$), the coordinate transformation is the following:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{\cos\theta\sin\theta_\circ - \sin\theta\cos\theta_\circ\cos(\psi-\psi_\circ)}{\cos\theta\cos\theta_\circ + \sin\theta\sin\theta_\circ\cos(\psi-\psi_\circ)} \\ \frac{\sin\theta\sin(\psi-\psi_\circ)}{\cos\theta\cos\theta_\circ + \sin\theta\sin\theta_\circ\cos(\psi-\psi_\circ)} \end{pmatrix}, \quad (8)$$

where θ is the longitude and ψ is the latitude.

2. The stereographic projection is constructed by projecting every point of the sphere onto patches from the sphere north pole in a plane tangent to the south pole. Assuming the patch is tangent to the point S ($\theta_\circ$, $\psi_\circ$), the coordinate transformation is the following:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2(\cos\theta\sin\theta_\circ - \sin\theta\cos\theta_\circ\cos(\psi-\psi_\circ))}{1+\cos\theta\cos\theta_\circ + \sin\theta\sin\theta_\circ\cos(\psi-\psi_\circ)} \\ \frac{2\sin\theta\sin(\psi-\psi_\circ)}{1+\cos\theta\cos\theta_\circ + \sin\theta\sin\theta_\circ\cos(\psi-\psi_\circ)} \end{pmatrix}. \quad (9)$$

In Fig.4, we show the mean power spectra estimated from patches of 20°x20° with a Hann window using two different projections: the gnomonic projection

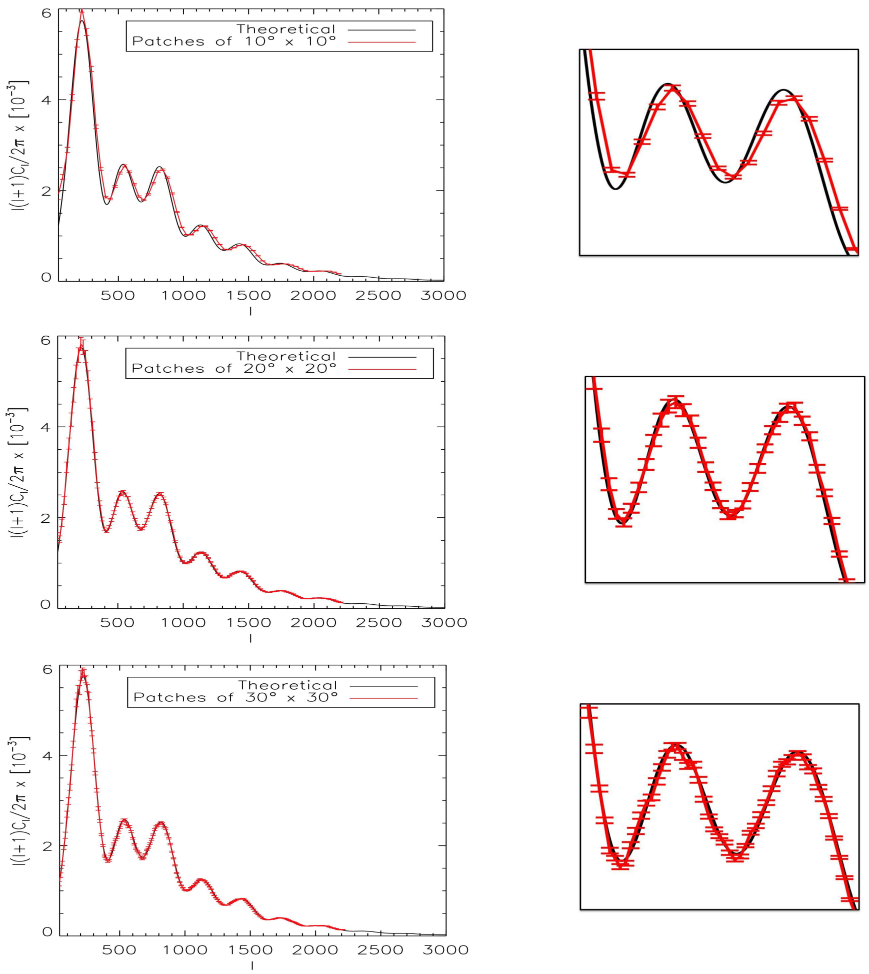

Figure 3: **The effect of the size of the patches in the power spectrum estimation:** a power spectrum (with error bars) has been estimated using three different size of patches (Note: a Hann window function is used to reduce the spectral leakage): 10°x10° (top left), 20°x20° (middle left) and 30°x30° (bottom left). Then, these three power spectra (red) have been compared to the theoretical power spectrum (black). The right column corresponds to a zoom in on the left power spectrum.

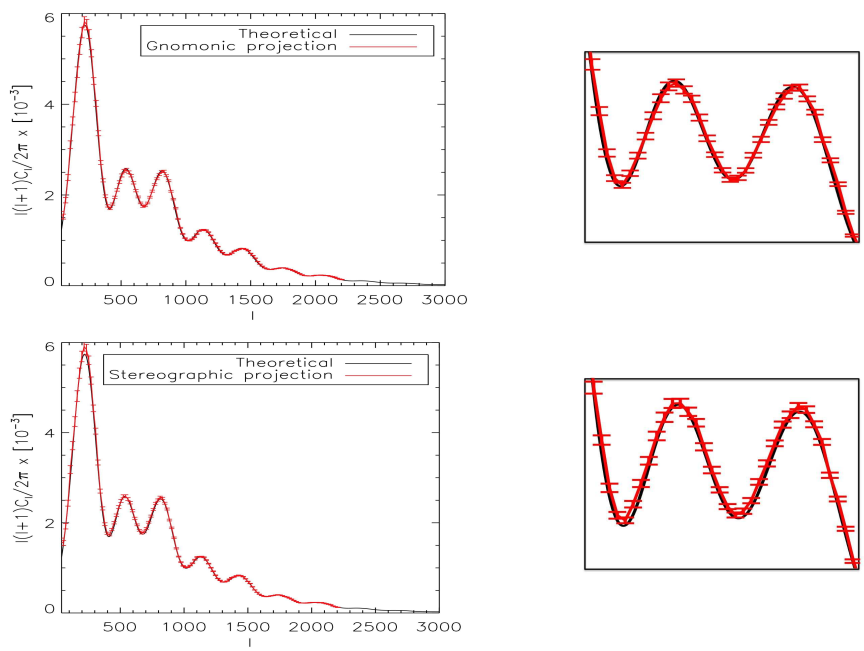

Figure 4: **The map projection effect in the power spectrum estimation:** Power spectra estimated with two different projections: the gnomonic projection (left) and the stereographic projection (right). These power spectra (red) are compared to the theoretical power spectrum (black). The right column corresponds to a zoom of the left power spectrum.

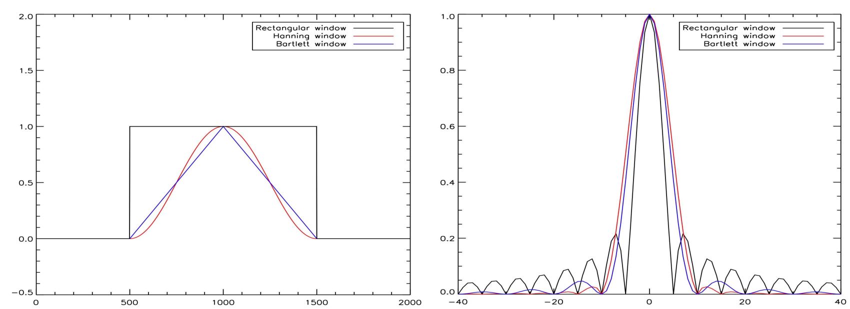

Figure 5: Window functions commonly used in FFT power spectrum estimation (left) and its frequency response (right). A signal whose frequency is actually located at zero offset leaks into neighboring frequencies with the amplitude shown. The rectangular window which is equivalent to no windowing is the least recommended because of its large sidelobes.

(left) and the stereographic projection (right). The two mean power spectra (red) are compared to the theoretical power spectrum (black). The better result is obtained with the gnomonic projection that will be used as the default projection for power spectrum estimation. However, we have to note that the map projection errors have already been reduced by fixing the size of the field to 20°x20° (see §2.2.1).

2.2.3. Windowing

After projection, the full-sky CMB is then decomposed into rectangular Cartesian maps of 20°x20°, but, as it has been discussed §2.1, the analysis of a finite signal affects the frequency analysis. The simplest way to model a patch of a finite size is through the usage of a rectangular window. But, this default window introduces an important spectral leakage. There is a lot of possible other window functions that can be used to reduce the spectral leakage in the power spectrum estimation. A few of the more common window functions have been compared in this study (see Fig.5):

1. The Rectangular window that is the default window:

$$h(x,y) = 1. \tag{10}$$

2. The Hann window that has been used in §2.2.1:

$$h(x,y) = \cos(\pi x)^2 \cos(\pi y)^2. \tag{11}$$

3. The Bartlett window:

$$h(x,y) = 1 - |\frac{x}{2}| - |\frac{y}{2}|. \tag{12}$$

In Fig.6, the mean power spectrum has been estimated by decomposing the CMB full-sky into patches of 20°x20° using the three windows described above.

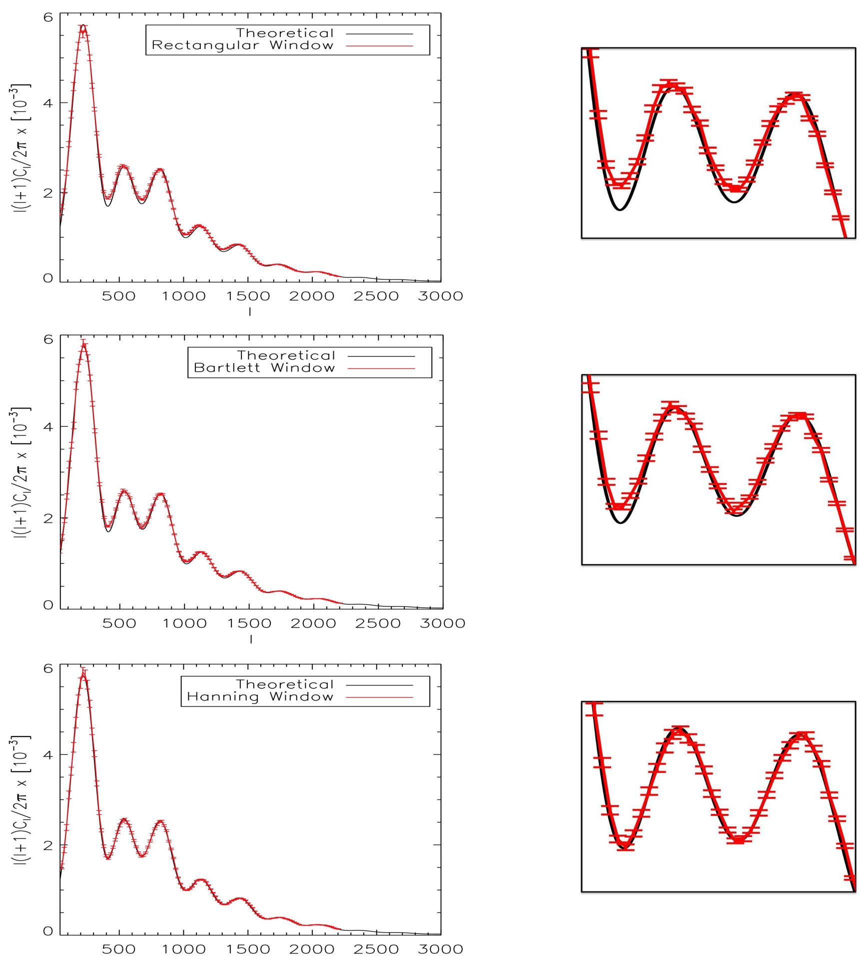

Figure 6: **The window function effect in the power spectrum estimation:** a mean power spectrum (with error bars) is obtained by decomposing the sphere into patches of 20°x20° using a rectangular window (top left), a Bartlett window (middle left) and a Hann Window (bottom left). These power spectra (red) are compared to the theoretical power spectrum (black). The right column corresponds to a zoom in on the left power spectrum.

These mean power spectra (in red) have been compared to the theoretical power spectrum that has been used to simulate the CMB full-sky (in black). As expected, a spectral leakage is observed with the default rectangular window due to the finite size of the signal (see the top panel of Fig.6). The spectral leakage has been reduced by applying the previously described non-rectangular window functions (see the middle and bottom panel of Fig.6). However, the better result is obtained when a Hann window is applied to the signal (see the bottom panel of Fig.6). Indeed, the Hann window is known to produces moderate side lobes (see the bottom panel of Fig.5) and to have high frequency resolution which is close to an ideal window function (see §2.1). Therefore, the Hann window will be used as the default window function for power spectrum estimation. As said previously, in this paper, we haven't tested the multitaper approach [2] that seems to be optimal to reduce the spectral leakage in the power spectrum estimated from small patches of the sky. This will be done in a future work.

2.3. Validation of the method with the power spectrum

In the previous section, we have done an optimization of the method for power spectrum estimation. More can be done to still improve the method but we have reached a satisfactory level for the purpose of this paper. The best power spectrum estimation has been obtained by decomposing the sphere into patches of 20°x20°, by projecting the pixels of the patch into rectangular Cartesian maps using a gnomonic projection and by multiplying each projected map by a Hann window. The power spectrum estimated by this method is now compared with the power spectrum estimated from the spherical harmonics formula (equation 4).

In Fig.7, we have estimated a mean power spectrum from 100 full-sky CMB simulated maps using the two different methods. On top left, we have the mean power spectrum estimated from spherical harmonics coefficients (in red) and in the top right, the optimized mean power spectrum obtained by the method based on the decomposition of the sphere into patches (in pink). These two mean power spectra have been compared to the theoretical power spectrum used to do the simulations (in black). The two curves on the left panel are almost superimposed while the two curves in the right panel show a small shift certainly due to a small leakage that remains after windowing. This kind of bias can be corrected [7] or reduced using a multitaper approach [2]. The standard deviation is significantly smaller in the per patch method than in the full-sky method (see the bottom panel of Fig. 7), this is because the frequency resolution has been reduced.

It could be thought, the per patch method is not recommended for power spectrum estimation because the estimation of the power spectrum of a full-sky CMB map from the spherical harmonics is quite fast and on the other hand, the decomposition of the sphere into patches takes some time. But in practice, we never have access to a full-sky CMB map because of the contamination by residual

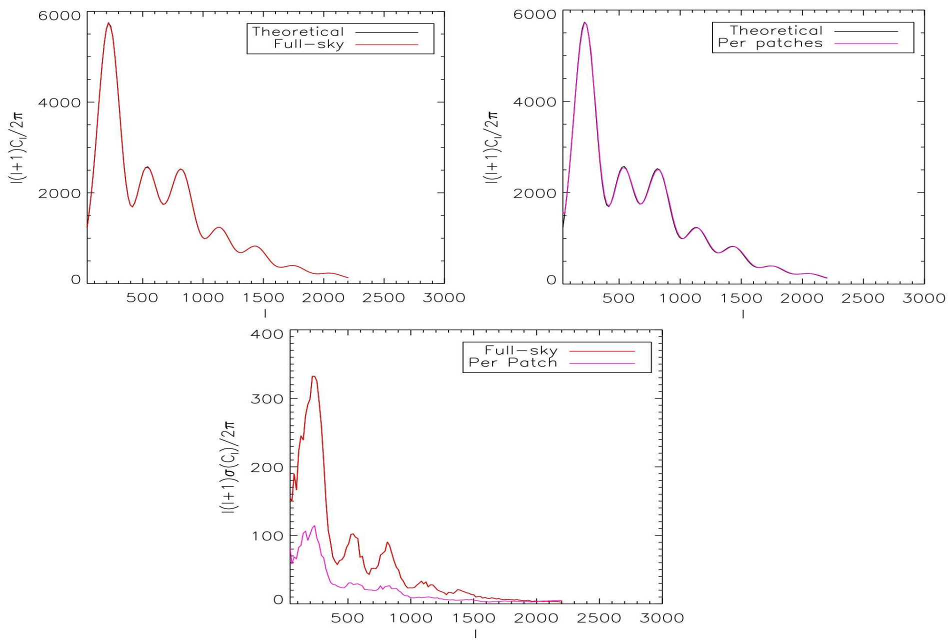

Figure 7: **Mean power spectra estimated from 100 full-sky CMB simulated maps using two different methods:** using the full-sky method based on the spherical harmonic coefficients (top left panel in red) and using the method based on the decomposition of the sphere into patches (top right panel in pink). These two mean power spectra are compared to the theoretical power spectrum used to produce the simulations (in black). The bias in the mean power spectrum estimated with the full-sky method is about 0.5% while it is about 3% with the per patch method. The empirical standard deviation is given for the two methods in the bottom panel.

foregrounds. For this reason, the per patch method can be preferred for some applications.

2.4. Application of the method

As said previously, in practice, we never have access to a full-sky CMB map. Indeed, the CMB map obtained by a method of component separation is always partially masked to discard pixels where the level of contamination by residual foregrounds is expected to be important. About 15-20% of the most contaminated data is removed mostly in the galactic plane. The power spectrum estimation using the spherical harmonics coefficients of these incomplete CMB maps suffers from the existence of gaps. It exists some methods to handle this missing data problem. The more commonly used is the MASTER method that has been proposed by [7] that uses apodization windows. The method that has been proposed in this paper is an alternative to methods like MASTER that try to correct the power spectrum from the effect of missing data. The approach is

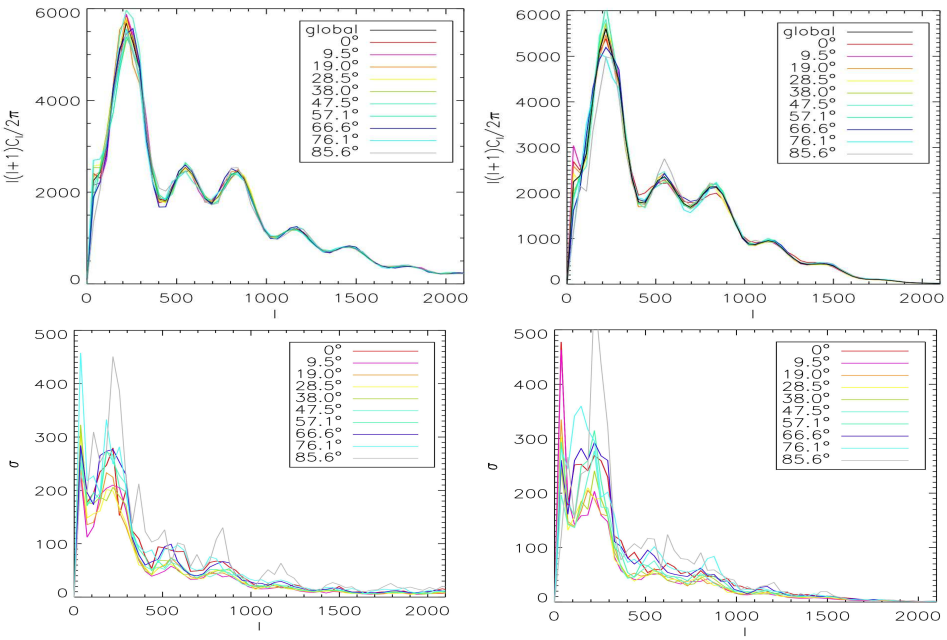

Figure 8: **Power spectrum per latitude (top) and corresponding errors (bottom)** (patches of 10°x10°) estimated from a simulated Gaussian CMB map ($n_{side} = 2048$) without (left) and with (right) contamination of foregrounds. The north and south contributions have been merged.

totally different because the problem of missing data is solved by avoiding the patches with important contaminations on the power spectrum estimation.

Following the same idea, we can imagine to use this method as a diagnosis of the component separation quality. An estimator of the power spectrum per latitude can be obtained by averaging the result of the spectral analysis on each patch located at the same latitude (see Fig. 1). The contamination by residual foregrounds introduces distortions on the power spectrum that should increase close to the galactic plane. The level of contamination per latitude is an indicator of the efficiency of the component separation method. Different component separation methods can be evaluated and compared by this way using simulated data.

Fig. 8 shows the effect of residual foregrounds in the power spectrum estimation. The left panel shows the power spectra estimated from a simulated CMB map purely Gaussian for different latitudes. The variance is only due to the number of patches per latitude that decreases moving to the poles (see Fig. 2). This explains that the statistical variance becomes important for power spectra estimated for latitudes close to the poles (larger than |70°|).

The right panel of Fig. 8 shows the power spectra per latitude estimated from a simulated CMB map with a significant level of residual foregrounds in the galactic region. This CMB map has been obtained by applying an optimized component separation method to simulations of Planck observations. The quality of the component separation method can be evaluated by comparing this result to the previous result. As previously, we observe distortions in the power spectrum close to the pole due to the statistical variance (latitude larger than $|70°|$). But some important distortions are also present close to the galactic plane (latitude equal to $0°$ - red line and $9.5°$ - pink line) clearly due to residual foregrounds. This method can be used to select the better component separation method and help to define the region to be masked. **Obviously, this only could be accomplished on simulated data for which the true power spectrum is known.**

3. Full bispectrum estimation

The primary goal of this paper is to provide a method for accelerating the calculation of the full bispectrum that can not be estimated with spherical harmonics in a reasonable time. In the previous section, we have introduced a promising approach that have been tested on the power spectrum estimation. The previous methodology will now be applied to estimate a full bispectrum. Obviously, the method needs now to be optimized for the bispectral estimation.

3.1. Limitations of the study

Unfortunately, the optimization will be limited by two points. The first limitation is the resolution of the non-Gaussian simulations of CMB currently available. The bispectrum of a Gaussian field being null, non-Gaussian CMB simulations have to be used to test the validity of the method. For that purpose, we have used the CMB non-Gaussian simulations of local type provided at the following address: http://planck.mpa-garching.mpg.de/cmb/fnl-simulations. A non-Gaussian CMB temperature map can easily be computed with any desired level of non-Gaussianity ($f_{\mathrm{NL}}^{\mathrm{local}}$) by linear combination of the a_{lm} provided. More details about the simulations can be found in [3]. A major problem in our study is that the HEALPix resolution parameter of these simulations is $n_{\mathrm{side}} = 512$ which is quite low compared to Planck data ($n_{side} = 2048$). This will seriously limit our study. But, there exists no other non-Gaussian simulations of CMB publicly available with a better resolution.

The second limitation is that there exists no code publicly available to compute the theoretical full bispectrum for a given level of non-Gaussianity, certainly because it is extremely complex to write a user friendly algorithm. The comparison with theory is required to be sure that the processing is not introducing important errors in the mean bispectrum. An analytic prediction of the bispectrum has been given in [10] for non-Gaussianities of the local-type ($F_{\mathrm{NL}}^{\mathrm{local}}$) but only for the equilateral configuration.

In this paper, the comparison with the theoretical bispectrum has been barely done on the equilateral configuration. However, even focusing on a particular configuration like the equilateral configuration, our method still remains more powerful than $f_{\mathrm{NL}}^{\mathrm{local}}$ estimation methods because all modes of the equilateral bispectrum are reconstructed compared to a single parameter.

3.2. Optimization of the method for bispectrum estimation

In this section, the optimization of the method for bispectral estimation will be done by comparing the equilateral bispectrum estimated by our method on the CMB non-Gaussian simulations described previously with the analytic prediction given in [10]. However, the bad resolution of the non-Gaussian simulations will considerably limit this study. Consequently, more work will have to be done as soon as non-Gaussian simulations with a better resolution will be available.

As for power spectrum estimation, a number of issues has to be solved in order to adapt the previous method to do bispectral estimation. The first issue is the tiling of the sphere. As said previously, for bispectral estimation, the decomposition of the sphere into rectangular patches located at the HEALPix centers of a lower resolution (see Fig. 2) is preferred because this decomposition insures a good repartition of the patches in the sky. But the size of the patches has to be reduced significantly compared to previous decomposition because the bispectrum is very sensitive to the non-Gaussianities introduced by the projection effects. As a result, the gnomonic projection will be preferred to do the projection from the pixels of the patches into the rectangular Cartesian maps because this projection introduces very little distortions for small patches [6]. However, the size of the patches has to be fixed in such way the distortions are reduced. In Fig.9, the mean bispectrum has been estimated using three different size of field: 7°x7°, 10°x10°, 17°x17°. The pixel projection introduces non-Gaussianities in the projected maps that appears in the bispectrum. For patches of 7°x7°, the amplitude and the location of the acoustic oscillations are quite well detected. But, the larger the field is, the more important the projection effects are. For patches of 17°x17°, the CMB non-Gaussianities of the local type almost disappear. There is still more power at the location of the acoustic oscillations but the distortion effects introduce an important noise in the mean bispectrum. This experiment should be redone with simulations with a better resolution.

Another issue is the window function. As for spectral estimation, it doesn't exist a perfect window function for bispectral estimation. It should be a trade-off between bispectral resolution and leakage effect. In Fig. 10, we have compared the Hann window to the rectangular window to do bispectral estimation and we obtain a better result with the default rectangular window function (left). Indeed, the amplitude of the acoustic oscillations are better recovered with the default rectangular window. In a future work, a window function dedicated to

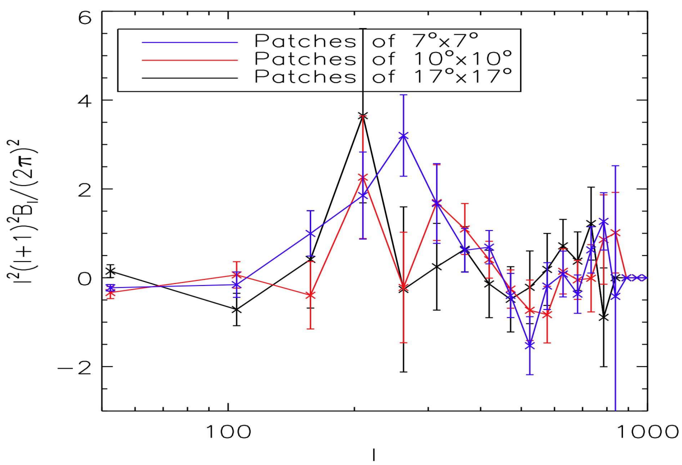

Figure 9: **The effect of the size of the patches in the bispectrum estimation:** a mean bispectrum has been estimated using three different size of patches (7°x7°, 10°x10°, 17°x17°) from 100 full-sky non-Gaussian CMB maps ($F_{\rm NL}^{\rm local} = 100$).

the CMB data has to be designed to improve the bispectral estimation. Some authors have already tried to address the problem of finding an optimal bispectral window [see 13, 16].

3.3. Preliminary Results

The best equilateral bispectral estimation has been obtained by decomposing the sphere into patches of 7° x 7° and by projecting the pixels of the patch into rectangular Cartesian maps using a gnomonic projection. The code FASTLens [14] publicly available has been used to compute the bispectrum in the Cartesian maps. **This study has been limited to the equilateral configuration of the bispectrum. As soon as a code to compute analytical prediction for the full CMB bispectrum will be released, the same study could be extended to all the configurations of the bispectrum.** Furthermore, the study will be improved as soon as non-Gaussian simulations of CMB with a better resolution will be available.

Anyway, the equilateral bispectrum computed (see the left panel of Fig. 10) shows the expected acoustic oscillations and the Silk damping predicted by theory for a CMB with non-Gaussianity of the local-type [10]. Therefore, this preliminary result shows that we can compute an equilateral bispectrum which is in good agreement with the analytical predictions.

4. Conclusion

The goal of this paper is to compute quickly the bispectrum of the full-sky CMB map. It should be pointed out that it is currently

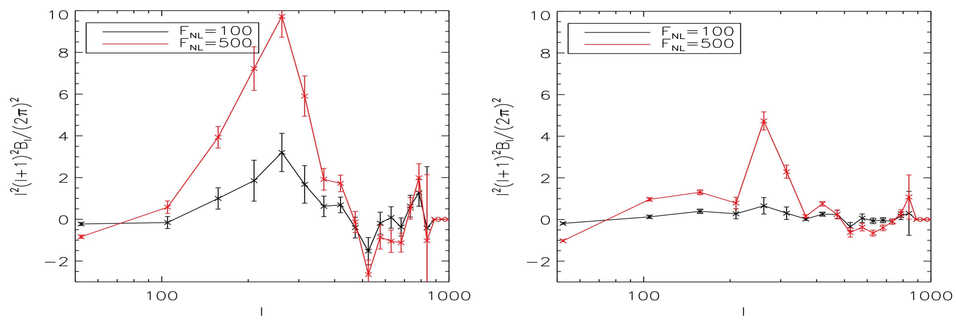

Figure 10: **The effect of the window function in the bispectrum estimation:** a mean bispectrum has been estimated by decomposing the sphere into patches of 7°x7° using a rectangular window (left) and a Hann window (right) from 100 full-sky non-Gaussian CMB maps with $F_{\rm NL}^{\rm local} = 100$ and $F_{\rm NL}^{\rm local} = 500$.

too hard to directly measure this full CMB bispectrum. This paper introduces a promising approach for accelerating the estimation of the full bispectrum on the sphere. This method involves the decomposition of the HEALPix map into small projected Cartesian maps. A mean full bispectrum can then be estimated by combining results from all the projected maps.

First, this approach has been used to estimate the power spectrum of a full-sky CMB map. A number of optimizations have been done to obtain the best power spectrum estimator. An interesting application of the method in order to test the quality of the CMB component separation on the galactic region has then been presented.

The approach has then be applied to the full bispectrum estimation to accelerate its computation. The approach presented in this paper enables a fast reconstruction of the whole bispectrum directly from the observational data. A number of optimizations have been performed to improve the quality of the bispectral estimation. However, these optimizations has been only tested on the equilateral configuration of the bispectrum because of the lack of analytical predictions for the full bispectrum. Anyway, this study could be easily extended to other configurations as soon as a code to compute analytical predictions for the full CMB bispectrum will be released. Another limitation of the study comes from the resolution of the non-Gaussian CMB simulations used for the analysis. This is a preliminary result that should be improved as earlier as is feasible.

However, the equilateral bispectrum that is computed from the non-Gaussian CMB simulations using this approach is in very good agreement with the analytical predictions. Indeed, the features expected by the theory are present despite the poor resolution of the estimated bispectrum. Thus, this approach appears very promising to constrain the CMB non-Gaussianity in a model-independent way.

Acknowledgment

This work has been supported by the European Research Council grant SparseAstro (ERC-228261). Some of the results in this paper have been derived using the HEALPix package (Gorski, Hivon, and Wandelt 1999). We wish also to thank F. Sureau for useful discussions.

References

[1] Creminelli, P., Nicolis, A., Senatore, L., Tegmark, M., and Zaldarriaga, M.: 2006, *Journal of Cosmology and Astro-Particle Physics* **5**, 4

[2] Das, S., Hajian, A., and Spergel, D. N.: 2009, *Phys. Rev. D* **79(8)**, 083008

[3] Elsner, F. and Wandelt, B. D.: 2009, *ApJS* **184**, 264

[4] Feichtinger, H. G., Gröchenig, K., and Strohmer, T.: 1995, *Numer. Math.* **69(4)**, 423

[5] Fergusson, J. R. and Shellard, E. P. S.:2009, *Phys. Rev. D* **80**, 043510

[6] Goldberg, D. M. and Gott, III, J. R.: 2006, *ArXiv Astrophysics e-prints*

[7] Hivon, E., Górski, K. M., Netterfield, C. B., Crill, B. P., Prunet, S., and Hansen, F.: 2002, *ApJ* **567**, 2

[8] Komatsu, E., Kogut, A., Nolta, M. R., Bennett, C. L., Halpern, M., Hinshaw, G., Jarosik, N., Limon, M., Meyer, S. S., Page, L., Spergel, D. N., Tucker, G. S., Verde, L., Wollack, E., and Wright, E. L.: 2003, *ApJS* **148**, 119

[9] Komatsu, E. and Spergel, D. N.: 2001, *Phys. Rev. D* **63(6)**, 063002

[10] Komatsu, E. and Spergel, D. N.: 2002, in V. G. Gurzadyan, R. T. Jantzen, & R. Ruffini (ed.), *The Ninth Marcel Grossmann Meeting*, pp 2009–2010

[11] Komatsu, E., Spergel, D. N., and Wandelt, B. D.: 2005, *ApJ* **634**, 14

[12] Opennheim, A. V. and Schafer, R.: 1975, *Digital Signal Processing*, pp 548–554

[13] Oroian, T.-C., Vizitiu, C.-I., and Serban, F.: 2008, *Transaction on signal processing* 4

[14] Pires, S., Starck, J., Amara, A., Teyssier, R., Réfrégier, A., and Fadili, J.: 2009, *MNRAS* **395**, 1265

[15] Welch, P.: 1967, *IEEE Transactions on Audio Electro acoustics* pp 70–73

[16] Wembing, Y., Tianren, Y., and Tao, H.: 2002, *Journal of electronics* 19(2)

[17] White, M., Carlstrom, J. E., Dragovan, M., and Holzapfel, W. L.: 1999, *ApJ* **514**, 12

***Revision Notes**

Dear referees,

Thank you for your reports. I have proceeded to the modifications you have addressed us. The modifications are in boldface in the text.

Response to Reviewer #1:

Major points :

1- First, the introduction contains some misleading statements about the current state of the art of inflation models and computations of non-Gaussianity. After introducing the local ansatz in Eq.(1), the authors state that "...most of models of inflation are only doing predictions for this form of non-Gaussianities". In fact, it is well understood which particular models give rise to other bispectra, and many detailed calculations of other forms have been carried out. It is true that many papers quote a single number, often labeled f_{NL}, characterizing the amplitude of the bispectrum, but this contains at least a crude accounting for shape. As the authors say later, it is well understood that the local ansatz with an amplitude that can be observed by Planck must be generated by a multi-field scenario, and that non-Gaussianity from slow-roll inflation resembles the local type (although with a tiny amplitude). The authors would be more correct to point out that the primary reason the local model is the most simulated (and so used to test observational techniques) is that it is easy: it is complete (all higher moments are determined in terms of a single parameter f_{NL}), while for other models one can try to reproduce a particular bispectrum but must allow for variation in the higher order correlations.

> The text has been modified in this way: « The local model is the more currently used because of its simplicity and because all the higher-order moments are determined in terms of this f_NL parameter. Most of models of inflation are only doing predictions for this form of non-Gaussianity. However, other models have been investigated given rise to different types of deviation from Gaussianity and detailed calculations of other form of non-Gaussianity have been carried out (see for example [1, 5]) . »

2- Still, the authors correctly point out that directly measuring the bispectrum present in the CMB is currently too hard, and so at the moment estimators for a few important shapes that can analyzed quickly are individually constrained. Based on comments at the end of Section 1, I imagine that what the authors mean by ``full bispectrum'' is the bispectrum actually present in the CMB, but even this is not entirely clear in the actual analysis. The bulk of the results in the paper demonstrate using their method for the power spectrum, optimizing over several possible sources of error. When the authors move to apply this method to the actual bispectrum, they mention that they will compare to the analytic prediction given in their reference [9] for optimization, but do not repeat the analytic prediction or say where it comes from. A look at reference [9] shows that it is dealing with non-Gaussianity either from slow-roll inflation (which is too small to be measured even with Planck) or with non-Gaussianity from secondary (non-primordial) sources, or from the local ansatz. It is not clear which of these bispectra the authors are using to optimize their method, although since the figure captions indicate maps with $f_{NL}=100$, I assume they mean local type. But, they go on to say that they will focus on only the equilateral configurations of that model, which further confuses the issue since this doesn't seem like the best thing to do for local type non-Gaussianity. Also, it doesn't do much to convince the reader that this method is more powerful than what is currently done, since the implication was that they could measure the bispectrum rather than focusing on specific configurations.

> The following text has been added to the subsection 3.1 to introduce this limitation of the study: « The second limitation is that there exists no code publicly available to compute the theoretical full bispectrum for a given level of non-Gaussianity, certainly because it is extremely complex to write a user friendly algorithm. The comparison with theory is required to be sure that the

processing is not introducing important errors in the mean bispectrum. An analytic prediction of the bispectrum has been given in [10] for non-Gaussianities of the local-type but only for the equilateral configuration. In this paper, the comparison with the theoretical bispectrum has been barely done on the equilateral configuration. However, even focusing on a particular configuration like the equilateral configuration, our method still remains more powerful than f_NL estimation methods because all modes of the equilateral bispectrum are reconstructed compared to a single parameter. »

The following sentence has been added to the 3.3 section :
« This study has been limited to the equilateral configuration of the bispectrum. As soon as a code to compute analytical prediction for the full CMB bispectrum will be released, the same study could be extended to all the configurations of the bispectrum. »

3- In the presentation of the results, there is no comparison with measuring non-Gaussianity by other techniques, so it is very difficult to assess what is gained by the authors methods. Finally, it isn't at all clear that an optimization that works for one type of non-Gaussianity (eg, where the strongest couplings are in the squeezed configuration) will also work for a very different type (say with couplings largest in the equilateral shape). This seems like a critically important point for their method, since if the optimization changes significantly with shape this may not actually be a more efficient technique.
In short, I find the physical discussion of non-Gaussianity a bit confused, and the utility of the technique not convincing. The second part seems particularly important since it is the authors' primary purpose.

The approach presented in this paper enables the reconstruction of the whole bispectrum directly from the observational data although the optimization has only been carried out on the equilateral bispectrum. The full bispectrum estimation has been moved to a new section to empasize the primary goal of the paper.
The goal of this method is, of course, to measure the level of non-Gaussiannity but above all to verify if the main features of the bispectrum are in agreement with the local model or another model. For this reason, the comparaison of our method with a single parameter f_NL is not very useful. The comparison should be done with analytical predictions to verify if the features of the bispectrum are well recovered.

Response to Reviewer #2:

Major points :

1- The main revision necessary is to be clear in the abstract and introduction about what has been accomplished. This paper doesn't really present a full bispectrum estimator, but rather lays the groundwork for one, by fairly thoroughly exploring the patch approach for the correlation function, anddoing a *preliminary* exploration of it for a *restricted* bispectrum (forequilateral triangles). It was not clear to me (as an outsider to this field) whether the study stopped where it did simply because the work is in progress, or because the necessary simulations or computations are beyond current community capabilities. This should be clarified. The above revisions alone will manage the expectations of readers and go a long way toward making the paper leave the reader with a good impression by the end.

To be clear about what have been accomplished, the abstract has been modified as follows: « This paper sets the grounds for a full bispectrum estimator based on the decomposition of the sphere into projected patches. » and the following two sentences have been added to the introduction:

« This paper introduces a promising approach for accelerating the calculation of the fully general bispectrum. » and « This paper sets the grounds of the method but to go further and fully optimize the estimator, we need more accurate non-Gaussian simulations that are not yet available and we need also a code to compute the analytical predictions of the full bispectrum for a given level of non-Gaussianity. ». Finally, section 3.1 that presents the limitations of the study has been added.

The conclusion has also been fully rewritten: « The goal of this paper is to compute quickly the bispectrum of the full-sky CMB map. It should be pointed out that it is currently too hard to directly measure this full CMB bispectrum. This paper introduces a promising approach for accelerating the estimation of the full bispectrum on the sphere. This method involves the decomposition of the HEALPix map into small projected Cartesian maps. A mean full bispectrum can then be estimated by combining results from all the projected maps. First, this approach has been used to estimate the power spectrum of a full- sky CMB map. A number of optimizations have been done to obtain the best power spectrum estimator. An interesting application of the method in order to test the quality of the CMB component separation on the galactic region has then been presented. The approach has then be applied to the full bispectrum estimation to accelerate its computation. The approach presented in this paper enables a fast reconstruction of the whole bispectrum directly from the observational data. A number of optimizations have been performed to improve the quality of the bispectral estimation. However, these optimizations has been only tested on the equilateral configuration of the bispectrum because of the lack of analytical pre- dictions for the full bispectrum. Anyway, this study could be easily extended to other configurations as soon as a code to compute analytical predictions for the full CMB bispectrum will be released. Another limitation of the study comes from the resolution of the non-Gaussian CMB simulations used for the analysis. This is a preliminary result that should be improved as earlier as is feasible. However, the equilateral bispectrum that is computed from the non-Gaussian CMB simulations using this approach is in very good agreement with the analytical predictions. Indeed, the features expected by the theory are present despite the poor resolution of the estimated bispectrum. Thus, this approach appears very promising to constrain the CMB non-Gaussianity in a model-independent way. »

2- The introduction should also present a more fair account of current approaches for modeling, detecting and measuring non-Gaussian structure in the CMB, which includes several approaches that look beyond simple scalar summaries like f_NL, even though they might present an f_NL calculation to tie the statistics to the local model ansatz.

The text has been modified in this way:
« The local model is the more currently used because of its simplicity and because all the higher-order moments are determined in terms of this f_NL parameter. Most of models of inflation are only doing predictions for this form of non-Gaussianity. However, other models have been investigated given rise to different types of deviation from Gaussianity and detailed calculations of other form of non-Gaussianity have been carried out (see for example [1, 5]) . »

3- A main task in the paper is comparing estimates with different choices of patch size and window. To tell whether results of different estimators are significantly different, the uncertainties of the estimates should be considered. But there is no discussion or calculation of uncertainties in the estimates except for Fig 6. This seems to me to be a major omission. In several plots, the various estimates are so close that they are hard to distinguish. Plots of differences or residuals, with error bars, would be more illuminating (perhaps in the common form of showing plots with two panes with a shared abscissa, the top one showing the true and estimated function, the lower one showing residuals with error bars). The bottom panel of Fig 6 supplies some of this information; it would be helpful for other cases, esp. bispectrum estimates, which are the main goal of the paper. For example, the uncertainty probably varies with latitude, but Fig 7 doesn't really reveal this. Also, the standard deviations in Fig 6 are simulation standard deviations; how would one estimate uncertainties for an analysis of a single real data set? This may be too much to address in this publication, but at least some discussion of uncertainties in the estimates is needed.

Error bars have been added to all the plots (standard deviations have been obtained from simulations). A zoom of the power spectrum has also been added for clarity.

Minor points :

Typographic issues:

* Use $\langle STUFF \rangle$ for angle brackets (e.g. eq 1), not greater/lesser signs.
* Use roman type for subscripts or superscripts that are syllables or words (rather than math symbols), and possibly also for acronymns, e.g., $f_{\rm NL}^{\rm local}$.
* There should be no spaces before colons.
* Use LaTeX markup to prevent periods in abbreviations from being treated as periods ending a sentence, e.g., "Fig.~5" or "Fig.\ 5" (the former would be preferred for a figure, so there can be no line break before the number).

The typographic issues have been corrected.

The title

The title should be changed. "Non-Gaussian estimator" doesn't really make sense; it is not the estimator that is non-Gaussian, it's the process whose bispectrum is being estimated that (possibly) is non-Gaussian. Also, it is not clear what is meant by "full" bispectrum, at least not in the title; and the paper itself never implements a fully general bispectrum calculation. Finally, relatively little of the paper actually focuses on the bispectrum; most of the paper addresses fast correlation function estimation, as a sensible prelude to the harder, related task of bispectrum estimation. A possible revised title that emphasizes the purpose and preliminary nature of the results might be:
Towards a fast, model-independent cosmic microwave background bispectrum estimator ("CMB" should be avoided in the title, since the StatMet audience is largely statisticians.)

Following the referee comments, the title has been change to : « Towards a fast, model-independent cosmic microwave background bispectrum estimator »

Two issues in the abstract:

* "Any measurement of primordial non-Gaussianity in the observed temperature fluctuations will be a signature of the model of inflation." This wording is somewhat ambiguous and can be taken different ways (some erroneous). I think what is meant is: "Measurement of non-Gaussianity in the observed fluctuations can constrain proposed models of inflation and structure formation, or potentially point the way to new models."
* "The bispectrum is the most promising tool to measure this non-Gaussianity."

This is a matter of opinion. The bispectrum is a nonparametric measure of non-Gaussianity. No nonparametric measure can have high power for detecting all possible departures from the null hypothesis (of Gaussianity); this is a theorem in hypothesis testing, not just an opinion. If one has a specific model of interest, a model-dependent statistic will almost surely have greater power. Probably the strongest statement that is justified is along the lines of: "The bispectrum is a natural and widely-studied tool for measuring non-Gaussianity in a model-agnostic way."

The abstract has been fully rewritten: « The measurement of the statistical properties of the Cosmic Microwave Background (CMB) fluctuations enables to probe the physics of the very early

Universe especially at the inflation epoch. A particular interest lays on the detection of non-Gaussianity in the CMB because it can be used to constrain proposed models of inflation and structure formation, or possibly point out new models. The bispectrum is a natural and widely studied tool for measuring non-Gaussianity in a model-independent way. Most studies to measure non-Gaussian signatures in the CMB are highly model-dependent, focusing on the measurement of a single parameter f_NL characterizing the amplitude of the bispectrum. This paper sets the grounds for a full bispectrum estimator based on the decomposition of the sphere into projected patches. The mean bispectrum estimated this way can be calculated quickly and is model-independent. This approach is very flexible, allowing exclusion of some patches in the processing or consideration of just a specific region of the sphere. »

Introduction

After eqn 1:

is the non-linear coupling function -> is the non-linear coupling constant

It has been corrected.

After eqn 4:

"The bispectrum is the most promising statistic..." See the comment above, for the abstract. Any statement like this must be appropriately qualified. There is no single, best nonparametric statistic for anything.

The sentence has been replaced by : « The bispectrum is a natural model-independent statistics to probe the small departure from Gaussianity that could originate during inflation »

The CMB angular bispectrum consists of the product of three... -> The CMB angular bispectrum may be calculated from products of three...
wigner -> Wigner

It has been corrected.

Sxn 2 title:

"Decomposition of the sphere into 2D maps"
The celestial sphere is inherently two-dimensional, so this title doesn't really say anything. I think what is meant is something along the lines of:
Decomposition of the sphere into rectangular maps
Decomposition of the sphere into planar maps
Decomposition of the sphere into Cartesian maps
Decomposition of the sphere into rectangular Cartesian maps

The title has been modified to « Decomposition of the sphere into rectangular Cartesian maps »

If I understand it correctly, the main issue is that the authors want to use 2D FFTs. Unless complex boundary corrections are implemented, this requires them to map the celestial sphere into rectangular patches with Cartesian coordinates. So the FFT aspect should probably be mentioned in P1 to motivate the focus on rectangular patches.

The following sentence has been added : « To facilitate the post-processing especially the FFT required to do spectral analysis, the patches ought to be rectangular and have preferably the same orientation »

Then later in the text "2D map" should be changed to "rectangular map" or "rectangular projection" or "projected patch" or whatever. Perhaps an acronym like RCP (Rectangular Cartesian Patch) should be used.

The modifications have been done.

Sxn 2.1:

Breaking a time series into segments, or a 2D region into patches, is a simple example of the idea underlying multitapering. The authors briefly cite an astronomy paper using multitapering at the end of sxn 2.2.2, but the notion should probably be mentioned here, since this is the modern way of handling variance reduction in power spectra, and it is a generalization of Welch's approach, cited here. Multitapering probably deserves more attention (in a sequel!); it is probably what a statistician or signal processing expert would naturally consider for this.

The text has been modified in the following way : « In this section, we will describe a method to speed up the spectral analysis that consists in decomposing the sphere into patches.Such method is similar to the Welch's method [15, 12] that is commonly used to reduce the variance in spectral analysis of 1D data. For a sphere, the size and the repartition of the patches have to be decided. Then, the pixels of the patches have to be projected into rectangular Cartesian maps. A spectral estimator is then obtained by averaging the result of the spectral analysis on each rectangular Cartesian map. Such methods have a bias due to the limited size of the projected patches. In a future paper, we will consider multi-taper techniques [2] that consist in averaging over different tapers using the full data. In these methods, the bias is smaller since the data length is not shortened. In this section, we will follow the basic approach that consists in decomposing the sphere into patches. »

Sxn 2.2.1, P2:

Regarding substantial overlap at the pole, "this effect is negligible... because of isotropy." But the aim here is to detect non-Gaussianity, and some types of non-Gaussianity can produce localized hot spots or other structure, in which case an estimator with anisotropic properties may be compromised. This just amplifies the point that there can be no single, model-independent measure of non-Gaussianity that will be optimal for all types of non-Gaussianity.

The text has been modified in the following way : « This decomposition introduces a substantial overlapping at the pole. For power spectrum estimation, this effect can be neglected by assuming the CMB field is isotropic. But this decomposition should not be used to detect non-Gaussianity because some types of non-Gaussianity can produce localized hot spots or other structures. »

p 10, last P:

S1 says for full-sky data the full-sky method is faster than using patches. Is this really true? Aren't they comparable in speed (or the patch method logarithmically faster)? Here or earlier, for statistician readers, there should be a sentence or two explaining why full-sky maps are not available, even from all-sky survey missions.

The following sentences have been added to explain why full-sky method is faster and why full-sky maps are not availbale: « It could be thought, the per patch method is not recommended for power spectrum estimation because the estimation of the power spectrum of a full-sky CMB map from the spherical harmonics is quite fast and on the other hand, the decomposition of the sphere into patches takes some time. But in practice, we never have access to a full-sky CMB map because of the contamination by residual foregrounds. » and « As said previously, in practice, we never have access to a full-sky CMB map. Indeed, the CMB map obtained by a method of component separation is always partially masked to discard pixels where the level of contamination by residual foregrounds is expected to be important. About 15-20 % of the most contaminated data is removed mostly in the galactic plane. »

p 11 P1:

"C_l are not equal to C_k = |dT/T|" That last equation does not make any sense; presumably some moment of dT/T is what is meant.

This has been removed from the text.

p 12, end of sxn 3.1:

The authors suggest study of distortion in the estimates "can possibly help to define the region to be masked." They should elaborate on this, if only briefly. How could this be accomplished for real data, where the underlying true correlation function being distorted is not known?

The following sentence has been added: « Obviously, this only could be accomplished on simulated data for which the true power spectrum is known. »

Sxn 3.2.2, P1:

The last S defers further optimization of choices for bispectrum estimation until non-Gaussian simulations with better resolution are available. Is this straightforward, or is this something that is an open issue for the community as a whole? If the latter, it probably should be mentioned early (in the introduction) as a reason for limiting the goals of the present paper.

The following sentence has been added to the introduction: « This paper sets the grounds of the method but to go further and fully optimize the estimator, we need more accurate non-Gaussian simulations that are not yet available » and the text of Sxn 3.1 has been modified as follows: « The first limitation is the resolution of the non-Gaussian simulations of CMB currently available. The bispectrum of a Gaussian field being null, non-Gaussian CMB simulations have to be used to test the validity of the method. For that purpose, we have used the CMB non-Gaussian simulations of local type provided at the following address: http://planck.mpa-garching.mpg.de/cmb/fnl-simulations. A non-Gaussian CMB temperature map can easily be computed with any desired level of non-Gaussianity f_ NL by linear combination of the a_lm provided. More details about the simulations can be found in [3]. A major problem in our study is that the HEALPix resolution parameter of these simulations is n_side=512 which is quite low compared to Planck data (n_side=2048). This will seriously limit our study. But, there exists no other non-Gaussian simulations of CMB publicly available with a better resolution. »

Best Regards,
S.Pires et al