2D Mass Mapping

OU-LE3 Weak Lensing - Lausanne 2015

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Introduction

Main issues in Weak-Lensing mass-mapping:

- Missing data (mask and limited number densities)
- Shape noise
- Reduced shear

$\Longrightarrow$ Most approaches avoid these issues by **smoothing**

Our goal

Write the mass-mapping as a single optimization problem with a **multi-scale sparsity prior** addressing all these issues.

Optimization problem

$$\min_{\kappa} \frac{1}{2} \parallel (1-\kappa)g - \mathbf{P}\kappa \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^t \kappa \parallel_1$$

- g : reduced shear
- λ : regularization parameter
- $\mathbf{P}$: lensing operator
- $\mathbf{\Phi}$: wavelet dictionary

A few remarks:

- Recovers the convergence from the reduced shear
- $\mathbf{P}$ can be defined with and without binning the shear
- $\mathbf{P}$ can be ill-posed in case of missing data
- Sparse regularization of noise and missing data
- We use isotropic wavelets for $\mathbf{\Phi}$
  $\Longrightarrow$ Adapted to the recovery of cluster signal

Optimization problem

$$\min_{\kappa} \frac{1}{2} \underbrace{\| (1-\kappa)g - \mathbf{P}\kappa \|_2^2}_{\text{Data fidelity term}} + \lambda \| \mathbf{\Phi}^t \kappa \|_1$$

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Proximal minimization algorithm

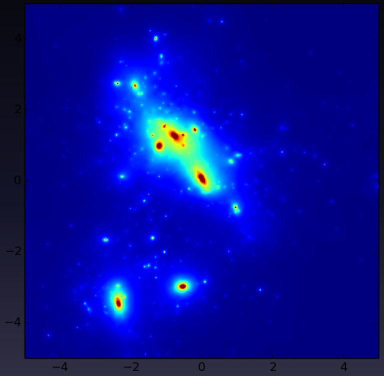
Primal-dual splitting:

$$\begin{cases} \kappa^{(n+1)} & = & \kappa^{(n)} + \tau \left(\nabla F(\kappa^{(n)}) + \mathbf{\Phi} \alpha^{(n)} \right) \\ \alpha^{(n+1)} & = & (\mathrm{Id} - \mathrm{ST}_\lambda) \left(\alpha^{(n+1)} + \mathbf{\Phi}^t (2\kappa^{(n+1)} - \kappa^{(n)}) \right) \end{cases}$$

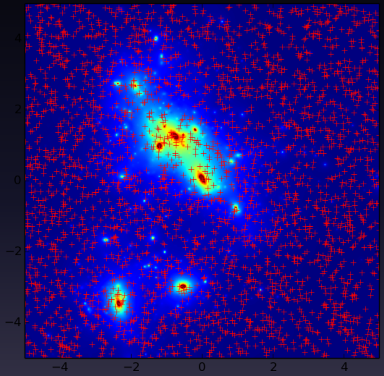
adapted from Vu (2013)

- Fast and flexible algorithm
- Sparsity constraint λ estimated locally by noise simulations
$\Longrightarrow$ Accounts for **survey geometry**, **varying noise levels**
- Same algorithm used for all the problems presented here

Handling missing data:

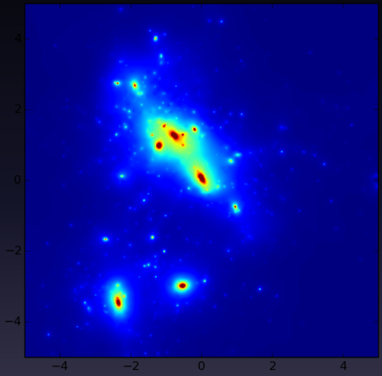


(a) Input



(b) Galaxy catalogue with 30 gal/arcmin^2

Handling missing data:



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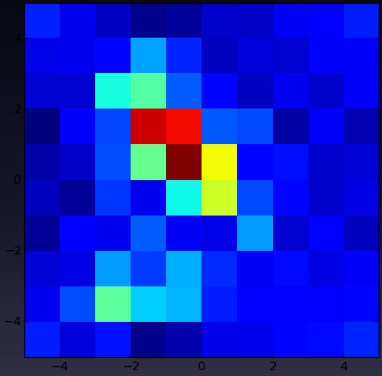
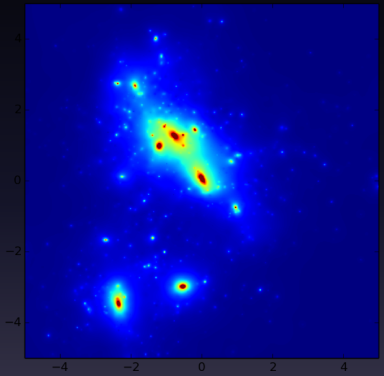


(b) Kaiser-Squires with 1’ bins

Handling missing data:



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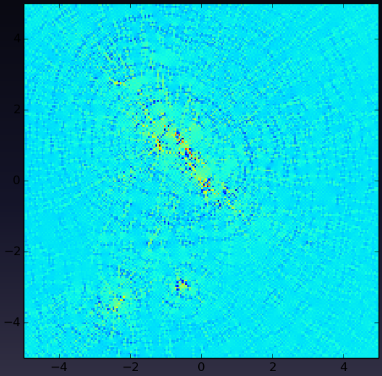
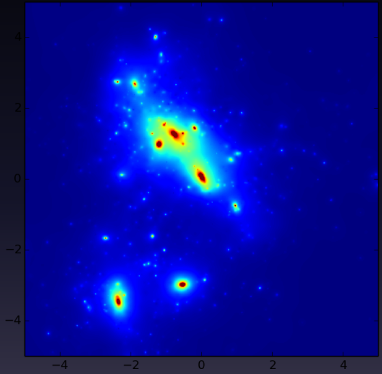


(b) Kaiser-Squires with 0.05’ bins

Handling missing data:



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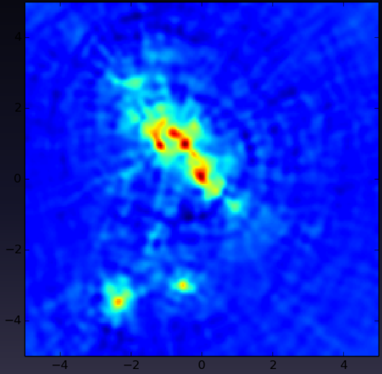
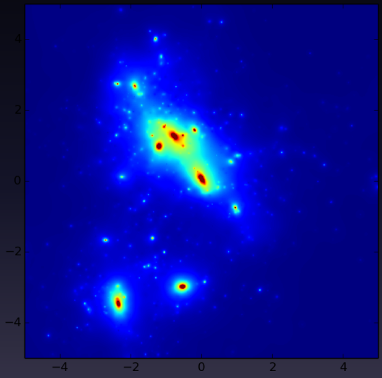


(b) Kaiser-Squires with 0.05’ bins + 0.1’ smoothing

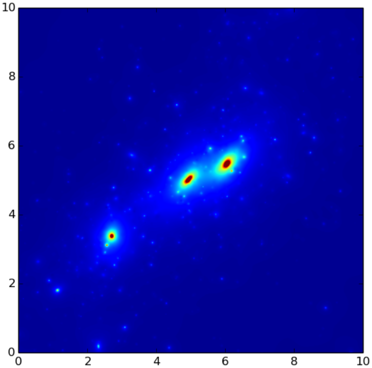
Handling missing data:



(a) Input

(b) Glimpse 2D

Handling shape noise:



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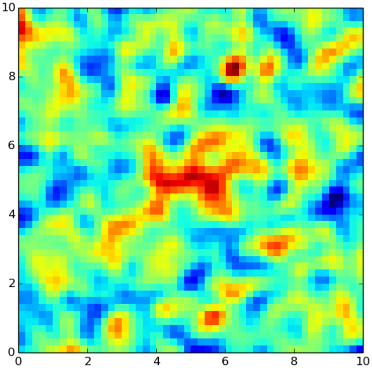

(b) Kaiser-Squires + 0.5’ smoothing

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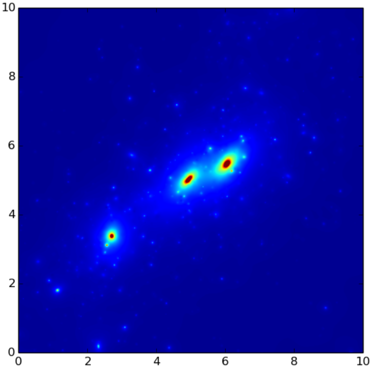

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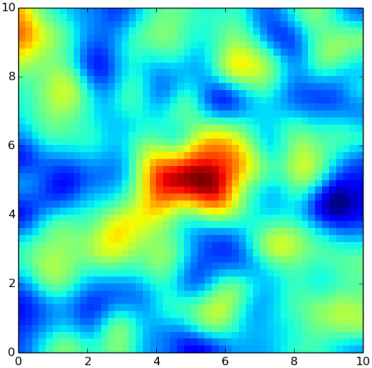
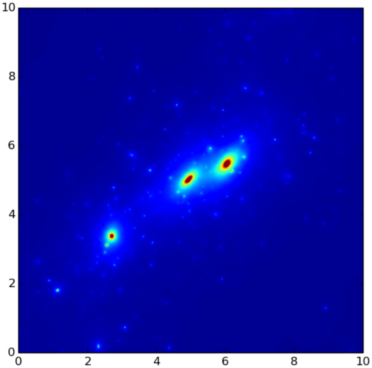


(b) Kaiser-Squires + 1.0' smoothing

Handling shape noise:



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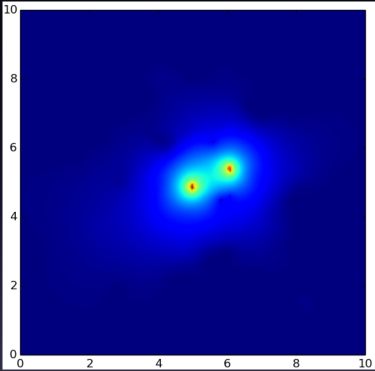

(b) Glimpse 2D

- So far, we have only considered mapping from shear alone.
- The framework can be expanded to include additional information, in particular **redshifts**.

$\Longrightarrow$ Allows **cluster density mapping**[1] (assuming knowledge of lens and sources redshifts)

[1] Lanusse F., Starck J.-L., Leonard A., and Pires S. (2015), *High Resolution Weak Lensing Mass Mapping combining Shear and Flexion* , in prep.

Our method is particularly well suited as it doesn't need to bin the data.

Why avoid binning the shear catalogue ?

$\Longrightarrow$ Allows to include **individual galaxy redshifts** :

$$\min_{\kappa_\infty} \frac{1}{2} \parallel (1 - \quad \kappa_\infty) g - \quad \mathbf{P} \kappa_\infty \parallel_2^2 + \lambda \parallel \Phi^t \kappa_\infty \parallel_1$$

with $\mathbf{Z} = \Sigma_{critic}^{\infty} / \Sigma_{critic}(z_i)$ and $\Sigma_{lens} = \kappa_\infty \Sigma_{critic}^{\infty}$

Individual redshifts have two benefits:

- Directly **map the surface mass density** of the lens
- Mitigate the **mass-sheet degeneracy** when κ becomes significant (Bradac et al. 2004)

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Simulation set:

- Density maps from Bolshoi simulations (Kyplin et al. 2011)
- Virial masses [$10^{14}\ h^{-1}$M$_\odot$]: 10.9 - 3.02 - 2.70
- Cluster redshift: $z_{clus} = 0.30$
- Field size: 10×10 arcmin^2 $\sim 2 \times 2\ h^{-2}$Mpc^2

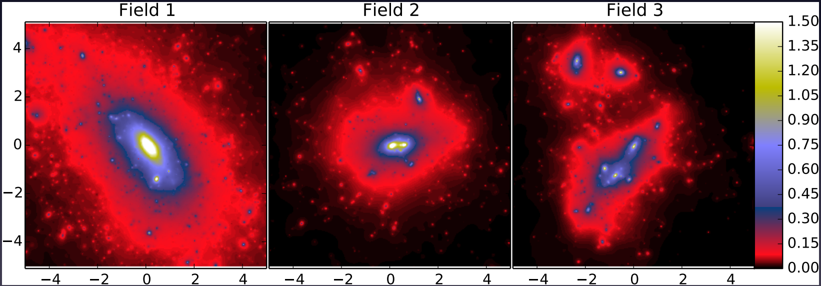
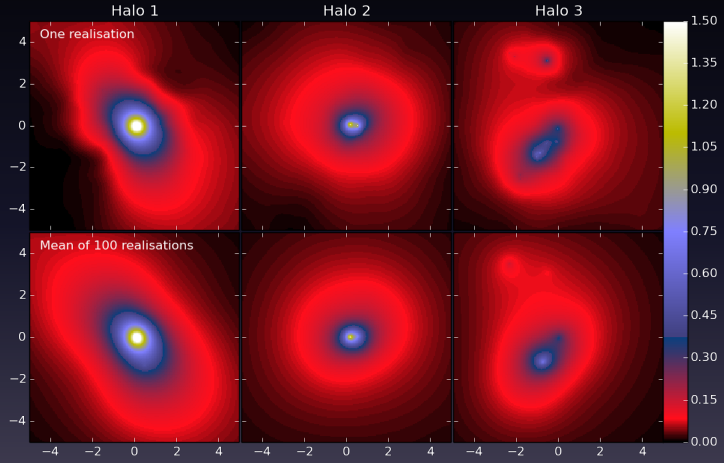


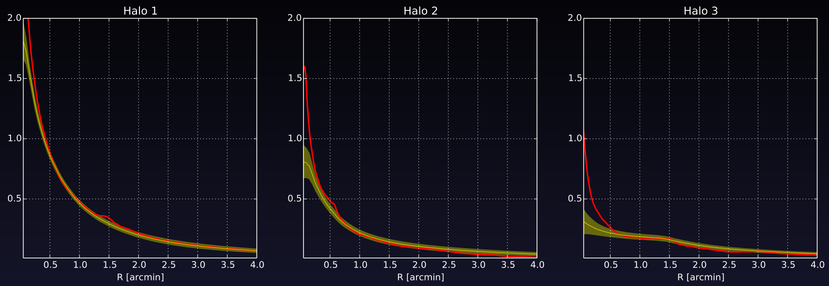
Figure : Convergence maps for sources at $z \to \infty$

Simulated deep space-based galaxy catalogue (HST/ACS):

- Uniform angular distribution with $n_{gal} = 80$ gal/arcmin^2
- Redshift distribution $z_{med} = 1.75$
- Strongly lensed sources ($|g| > 1$) excluded
- Gaussian photo-z errors $\sigma_z = 0.05(1 + z)$
- Gaussian shape noise $\sigma_\epsilon = 0.30$

$\Longrightarrow$ We generate 100 independent noise and galaxy distribution realisations


Halo 1
Halo 2
Halo 3
One realisation
Mean of 100 realisations
4
2
0
−2
−4
1.50
1.35
1.20
1.05
0.90
0.75
0.60
0.45
0.30
0.15
0.00



Field	Input $M_{\rm ap}$ $10^{14}\ h^{-1}$M$_\odot$	Measured $M_{\rm ap}$ $10^{14}\ h^{-1}$M$_\odot$
1	4.08	3.91 ± 0.18
2	1.88	1.90 ± 0.20
3	1.59	1.60 ± 0.18

Table : Aperture mass in the central 2′ of each field.

- Shear is noise dominated on small scales
  $\Longrightarrow$ **Substructure is lost** when mapping from shear alone.
- Small-scale substructure can be recovered from strong lensing when available.
- Gravitational **Flexion** is useful in the intermediate regime.

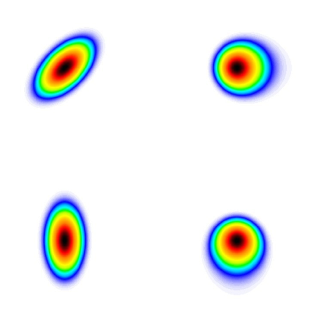


Figure : Shear (left) and first flexion (right) (Bartelmann 2010)

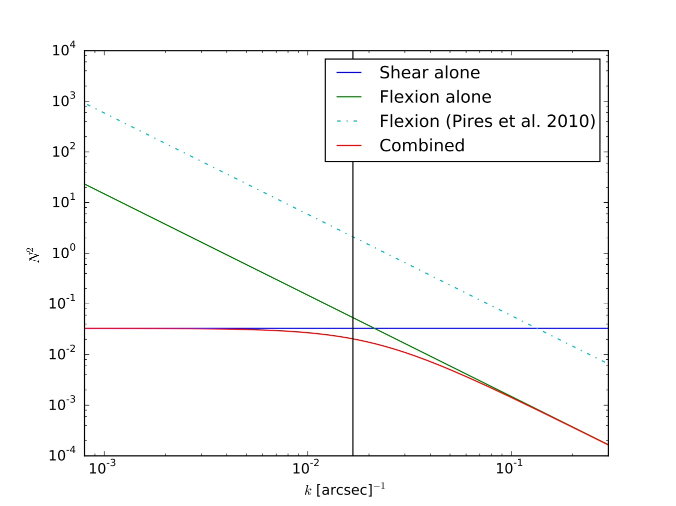


Figure : Noise power spectra for convergence estimator from shear and flexion

- Recent forward fitting method developed by Cain et al. (2011) and flexion mapping paper on substructure recovery Cain et al. (2015).

- We can integrate flexion in our reconstruction framework
$\Longrightarrow$ Jointly fit shear and flexion

$$\min_{\kappa} \frac{1}{2} \parallel (1-\kappa) \begin{bmatrix} g \\ F \end{bmatrix} - \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} \kappa \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^t \kappa \parallel_1$$

Adding flexion to our simulation set:

- Flexion noise $\sigma_F = 0.029$ arcsec^{-1}
$\Longrightarrow$ Reported by Cain et al. (2011) for Abell 1689

- Simulate **reduced flexion**, strongly flexed galaxies excluded ($|F| > 1.0$ arcsec^{-1})

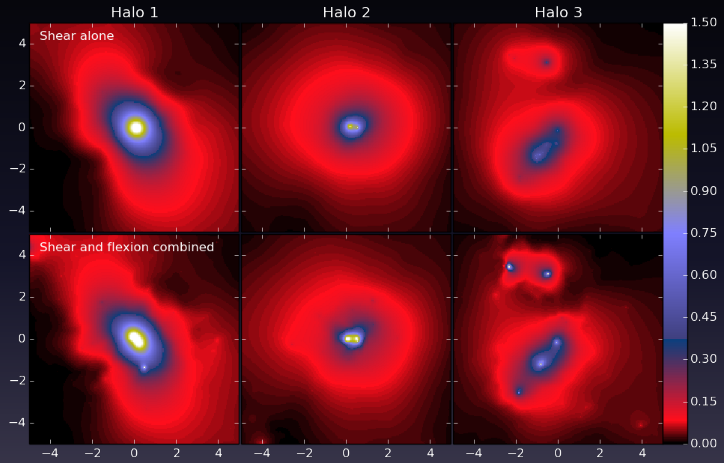


Figure : Reconstruction from one realisation

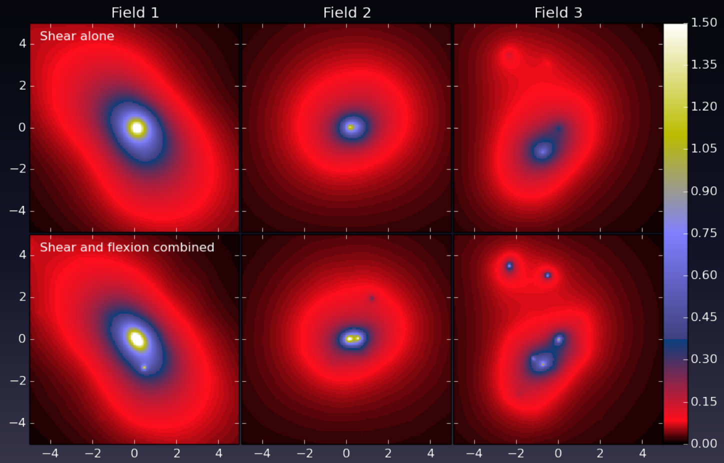


Figure : Mean of 100 reconstructions

Benefits of adding flexion:

- Improvement on the recovered profiles below 0.5 arcmin
- Recovery of small-scale substructure at the 10 arcsec scale

Conclusion:

- New method to recover mass maps accounting for
  - Reduced shear
  - Missing data
  - Shape noise
- Particularly efficient to recover small scale information (useful to detect and map galaxy clusters)
- Can be extended by including individual redshifts and flexion for high resolution cluster density mapping
- Can be trivially extended to 3D mapping (work in progress)