INSTITUT DE RECHERCHE
sur les LOIS FONDAMENTALES
de l'UNIVERS

Dark Matter mass map Reconstruction and Analysis using Sparsity

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Collaborators

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- ✓ A. Réfrégier, CEA-Saclay, France
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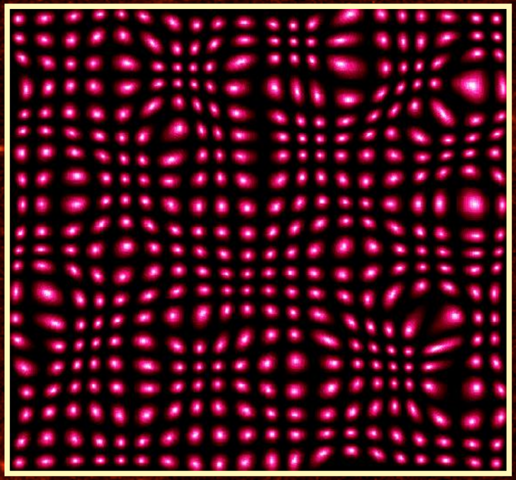
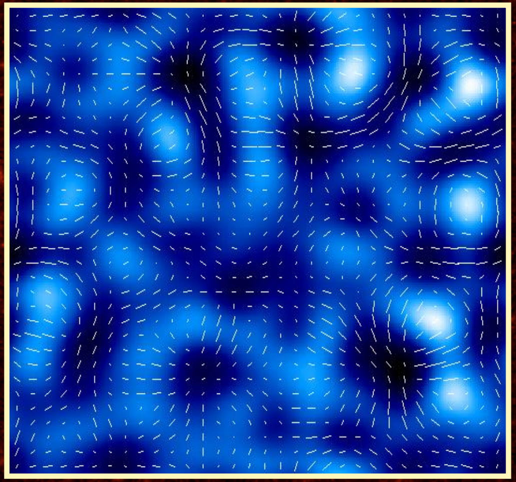
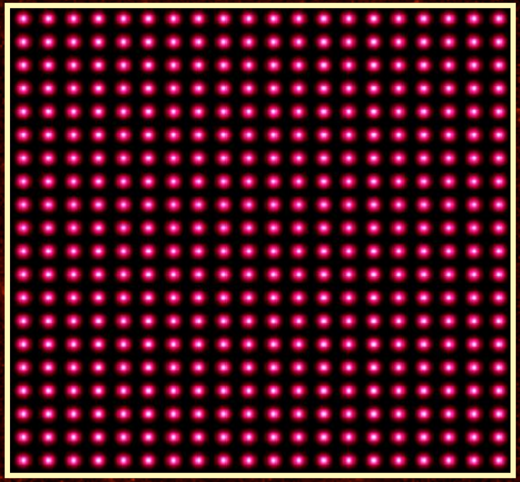
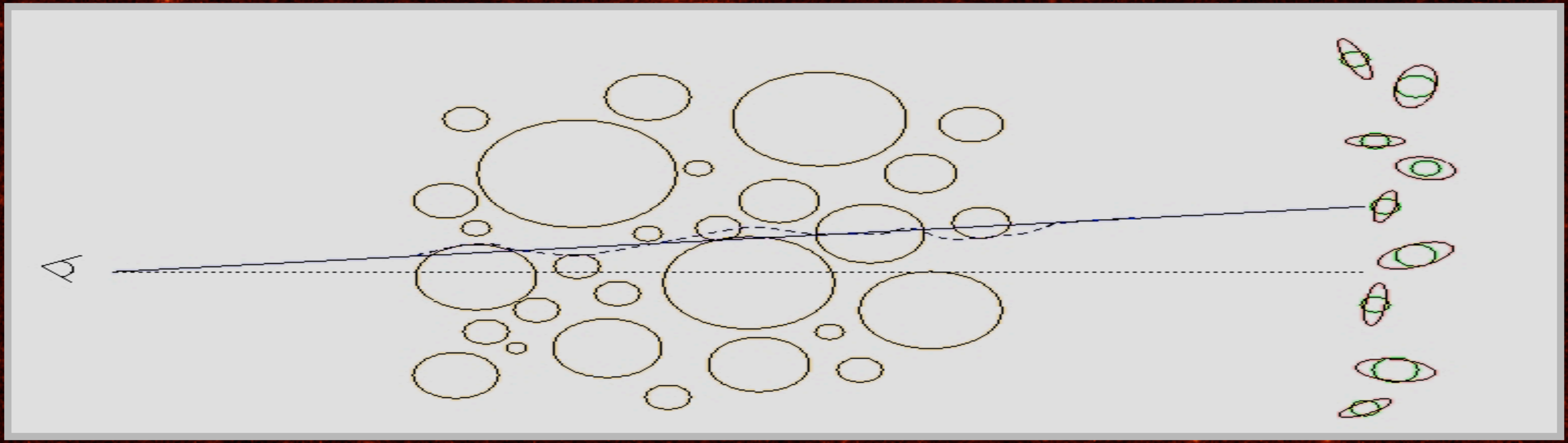
- ✓ R. Massey, California Institute of Technology, USA.
- ✓ R. Teyssier, CEA-Saclay, France
- ✓ J. Fadili, Caen University, France

Weak Gravitational Lensing

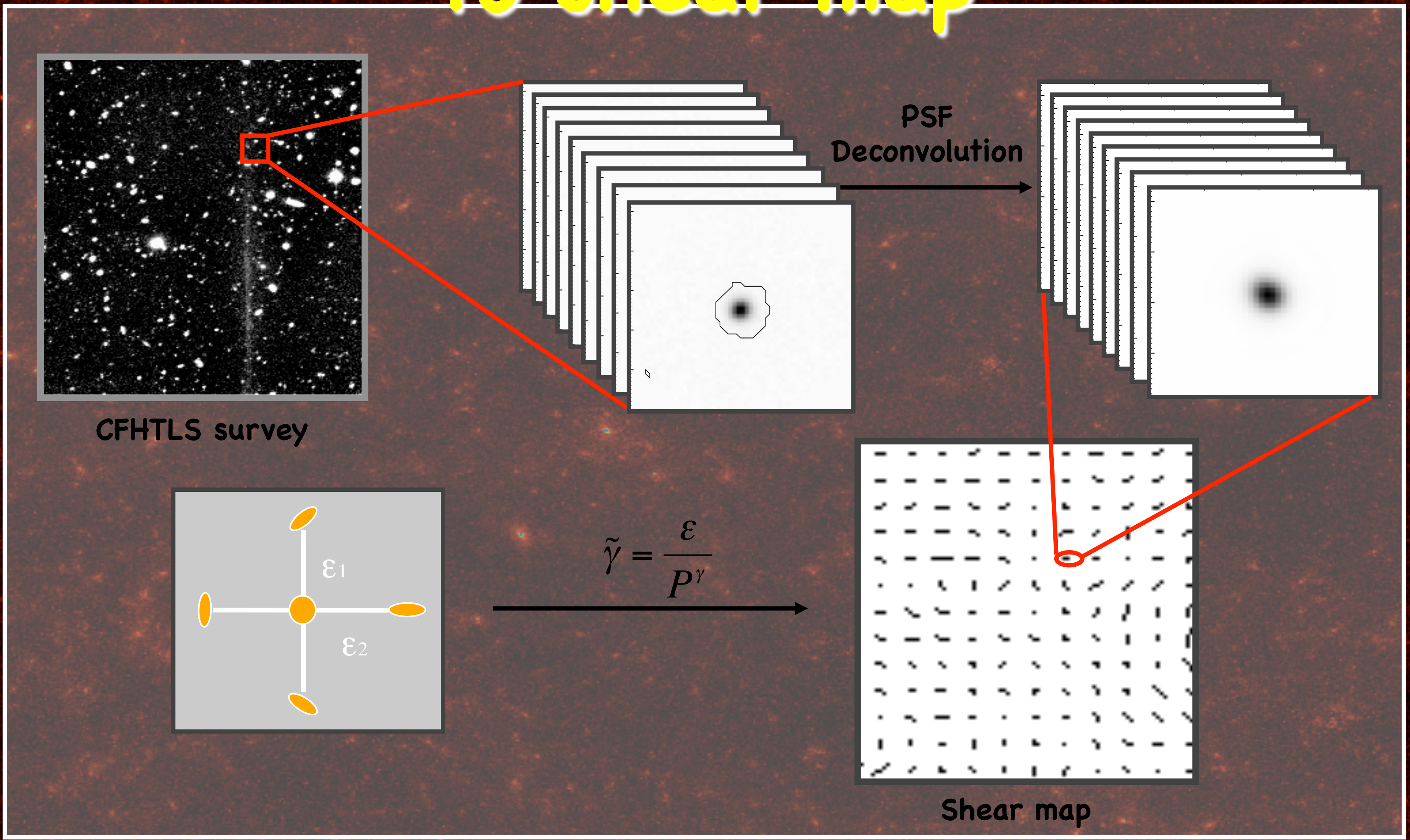

Observer


Gravitational lens

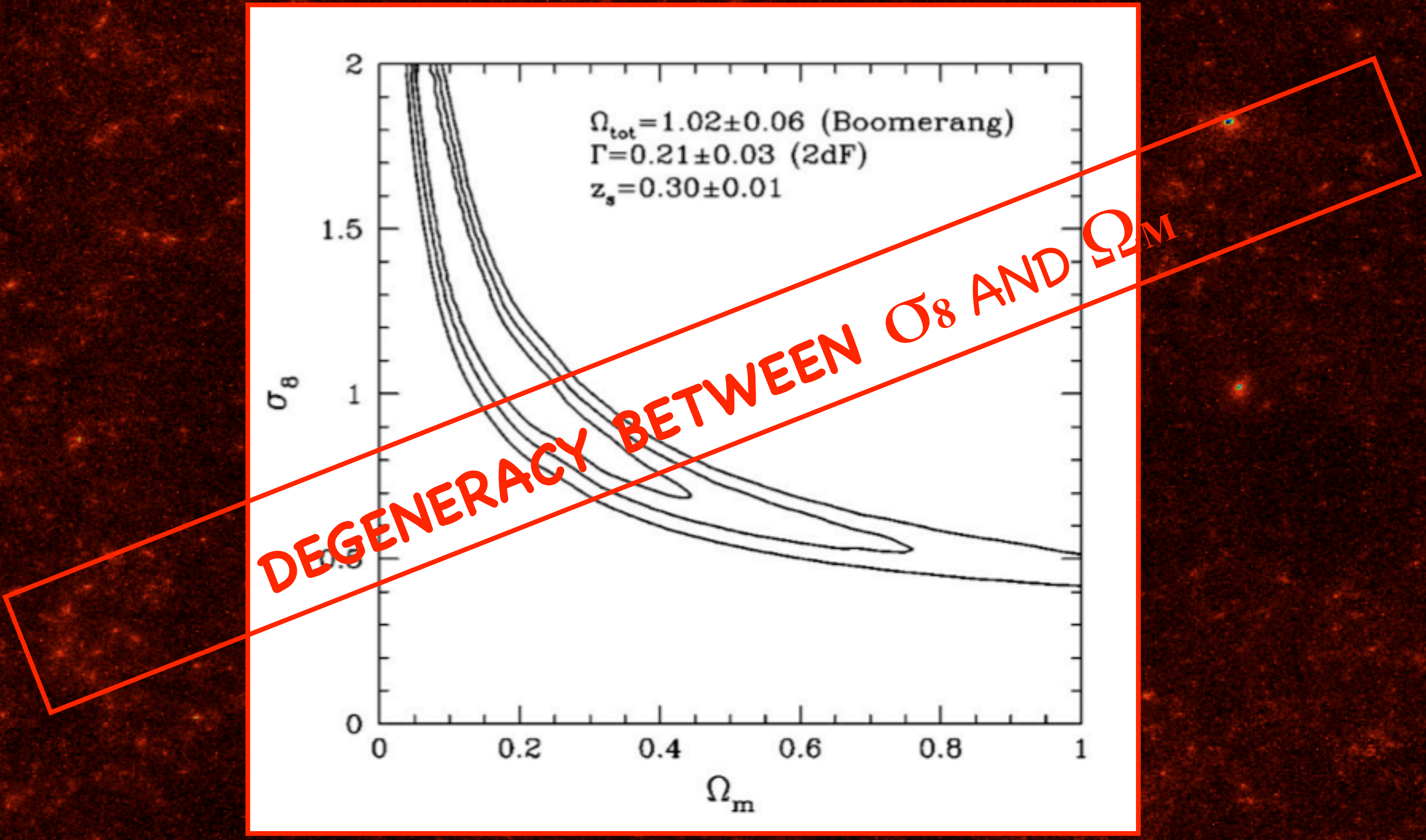

Background galaxies



From shear measurements to shear map



Weak Lensing degeneracy



Mass inversion

**Weak lensing mass map reconstruction using wavelets,
J.L. Starck, S. Pires and A. Réfrégier, A&A, June 2006, Vol. 451, p1139-1150**

$$\gamma_1 = \frac{1}{2}\left(\partial_1^2 - \partial_2^2\right)\psi$$

$$\gamma_2 = \partial_1 \partial_2 \psi$$

$$\kappa = \frac{1}{2}\left(\partial_1^2 + \partial_2^2\right)\psi$$

$$\gamma_i = P_i * \kappa$$

$$\kappa = P_1 * \gamma_1 + P_2 * \gamma_2$$

$$\hat{P}_1(k) = \frac{k_1^2 - k_2^2}{k^2}$$

$$\hat{P}_2(k) = \frac{2k_1 k_2}{k^2}$$

= GOAL =

Constrain cosmological parameters from weak lensing data

☐ Estimation of the PSF from the stars on the field

☐ Estimation of the shear by measuring the ellipticities of the galaxies background and by deconvolution of the PSF

☐ Estimation of the two point correlation function in weak lensing shear maps

☑ Reconstruction of the dark matter mass map from shear maps considering the missing data

☑ Filtering of the noise in weak lensing mass maps to reconstruct the distribution of the dark matter

☑ Statistic analysis of the complete dark matter mass map

Outline

1 - Weak Lensing mass map filtering

- Introduction to the mass map reconstruction problem
- MRLENS filtering
- Results and applications

2 - Mask interpolation using Inpainting

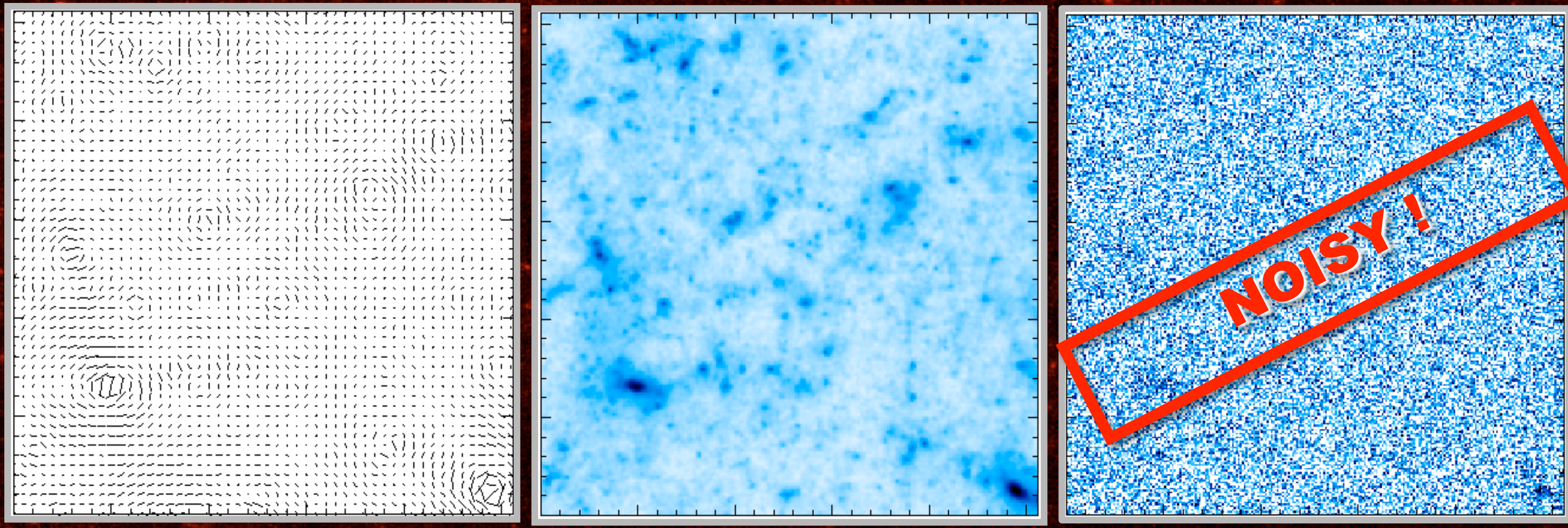
- Introduction to the missing data problem
- Inpainting method to fill-in the gaps (FASTLens)
- Some results

3 - Search of the best statistic to constrain the cosmological model

- Weak Lensing statistics
- Conclusions

Shear map and mass map

(Vale & White, 2003)



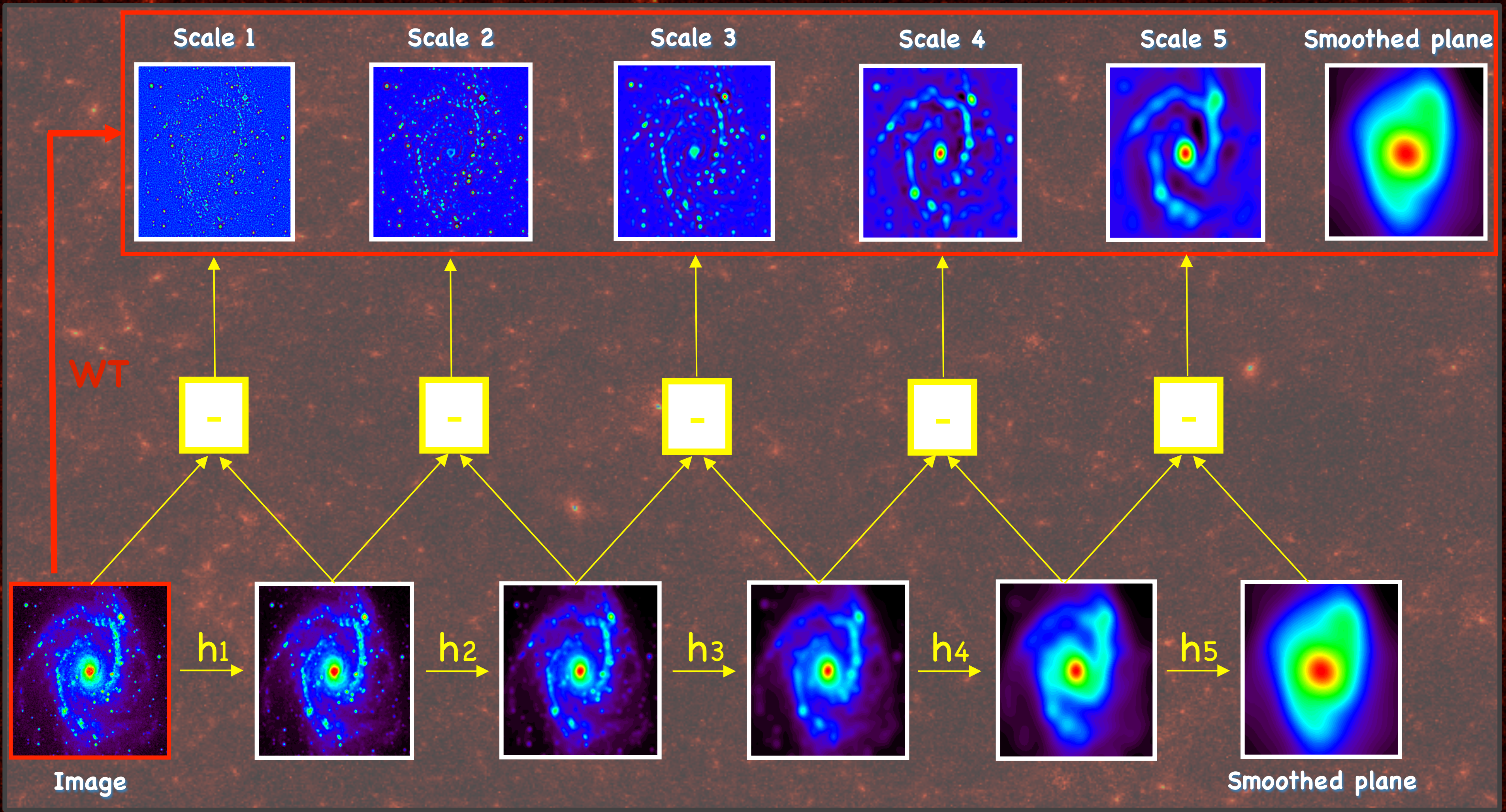
Shear map

Original mass map

Mass map (space observation)

MRLENS : Multi-Resolution for weak LENSing

Weak lensing mass map reconstruction using wavelets,
J.L. Starck, S. Pires and A. Réfrégier, A&A, June 2006, Vol. 451, p1139-1150



MRLENS
False Discovery Rate method (FDR)
(Benjamini et al, 1995)

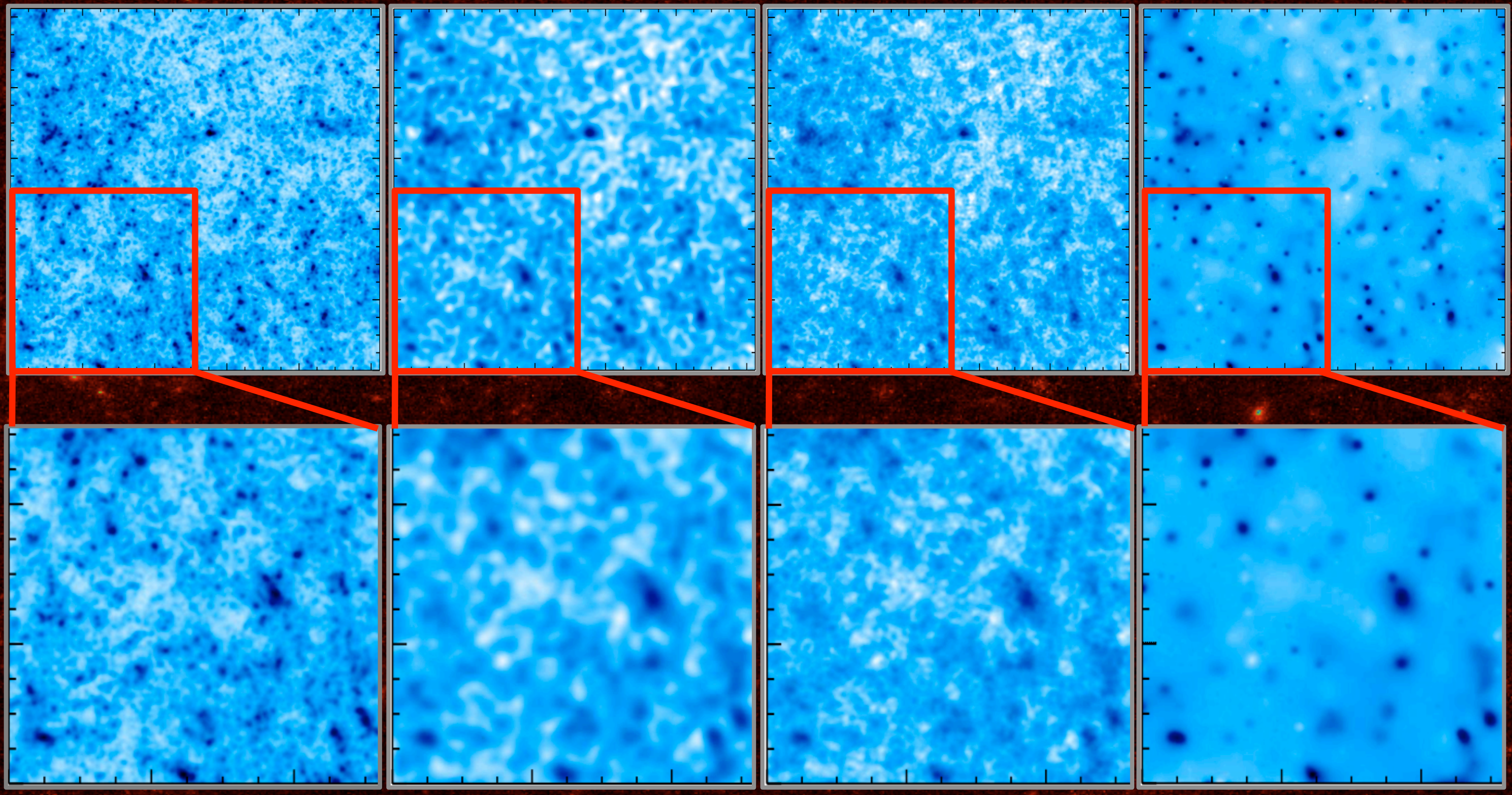
A Thresholding is performed at each wavelet plane:

- ❖ kσ-Threshold : the number of false detections is depending on the number of samples
- ❖ FDR-Threshold : the number of false detections is depending on the number of true detections. The value of the threshold is then function of the level of the noise

MRLENS algorithm

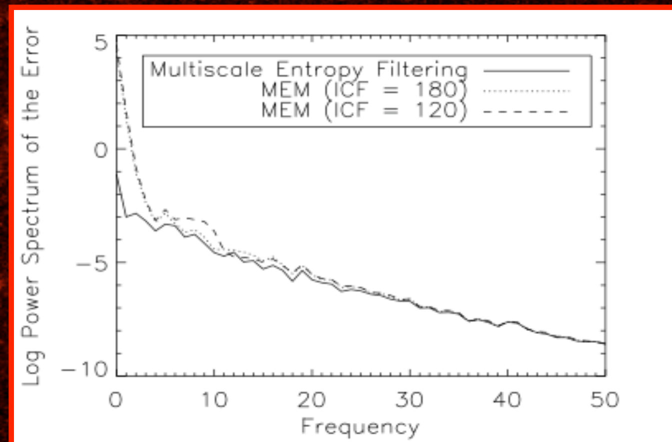
- Wavelet transform
- Estimation of a threshold for each wavelet plane using FDR technique
- Detection of significant coefficients
- Maximum a posteriori method with a multi-scale entropy prior only in non-significant coefficients
- Inverse Wavelet transform using an iterative process

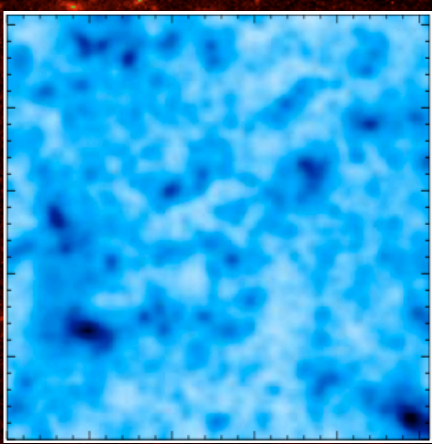
Comparison between Gaussian, Wiener and MRLens filter



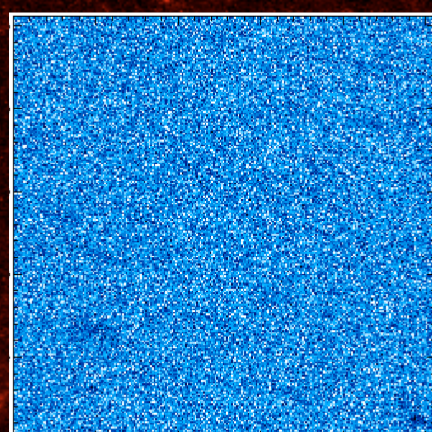
Original map Gaussian filter Wiener filter MRLENS filter

Comparison between MEM and MRLens filter

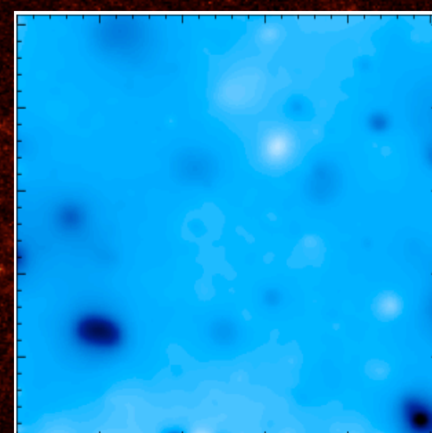




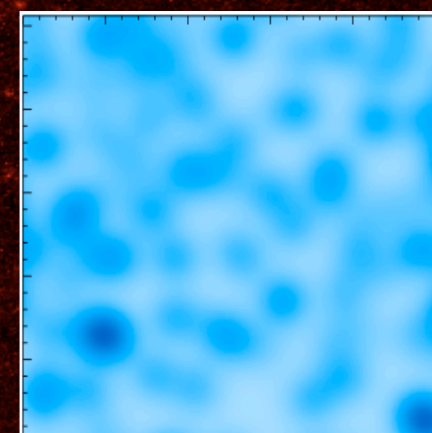
Simulated mass map



Simulated mass map (space observations)



Mass map filtered by MRLENS



Mass map filtered by MEM (Maximum Entropy Method)

MRLENS software

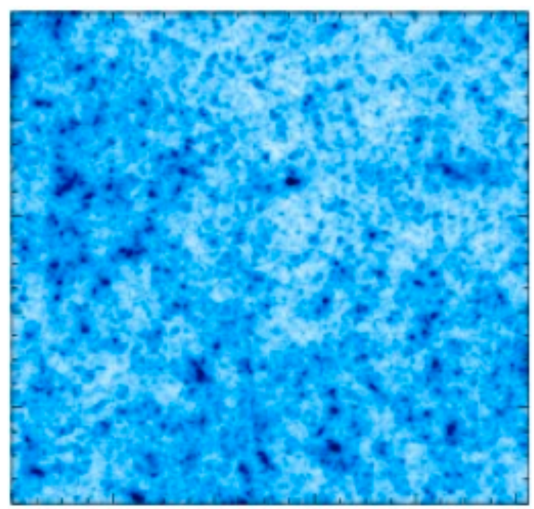
Multi-Resolution methods for gravitational LENSing

http://www-irfu.cea.fr/Ast/mrlens_software.php

Software MRLENS : Multi-Resolution methods for gravitational LENSing

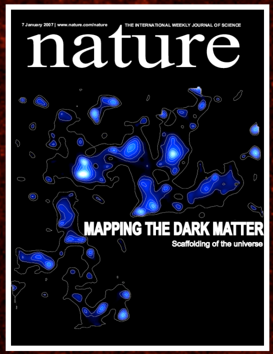
S. Pires, J.L. Starck and A. Réfrégier

Welcome to the MRLENS web page.
This page introduce the MRLENS software (Version 1.0), contains links to our papers and allow you to download a copy of the MRLENS software and its user manual.



Simulated mass map from Vale and White (2003).

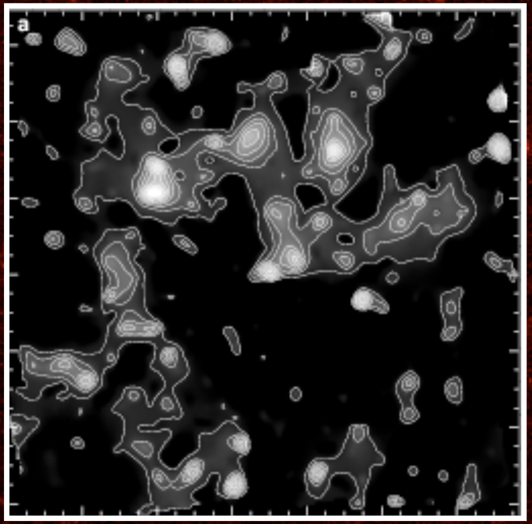
COSMOS data



Maps of the Universe's Dark matter scaffolding,
Massey et al, Nature, Vol. 445, pp. 286-290, 2007

Data characteristics :

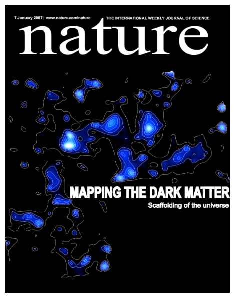
- 575 pointings of the ACS Camera (Wide Field Camera)
- Cover a region of 1.637 square degrees
- 500 000 shapes of distant galaxies have been measured



Dark matter distribution map in the COSMOS field

Main steps on the processing :

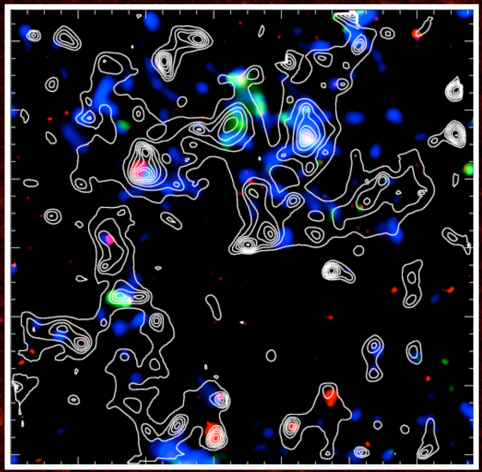
- Raw processing of the HST data
- Making the galaxy catalog (positions and shapes) (R. Massey)
- Production of the 2D mass maps (R.Massey & A. Réfrégier)
- Development of the wavelet filtering technique (S. Pires and J.l. Starck)



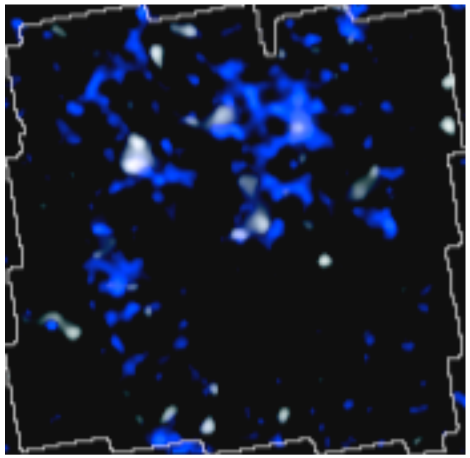
COSMOS data :

Baryonic and non-baryonic matter comparison at large scale

The total projected mass map from WL (dominated by dark matter) is shown as contours in panel a and as a linear greyscale in panels b, c and d. It is compared to 3 independent baryonic tracers : stellar mass (in blue), galaxy number density seen in optical and <u>near-IR</u> light (in green) and the hot gas seen in <u>x-rays</u> (in red).

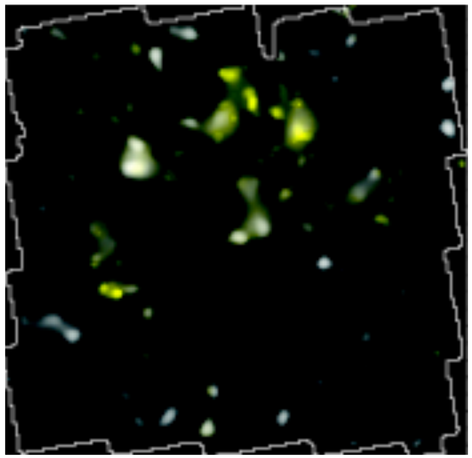


a



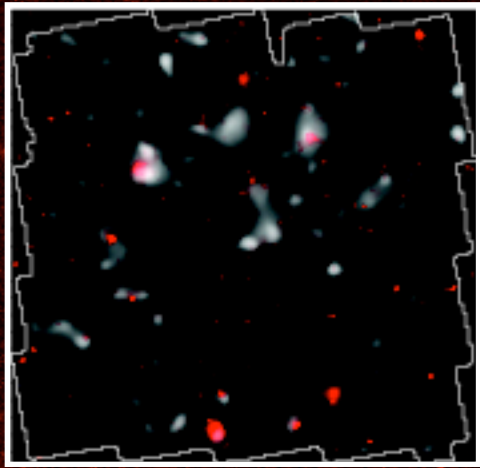
b

Stellar mass and WL



c

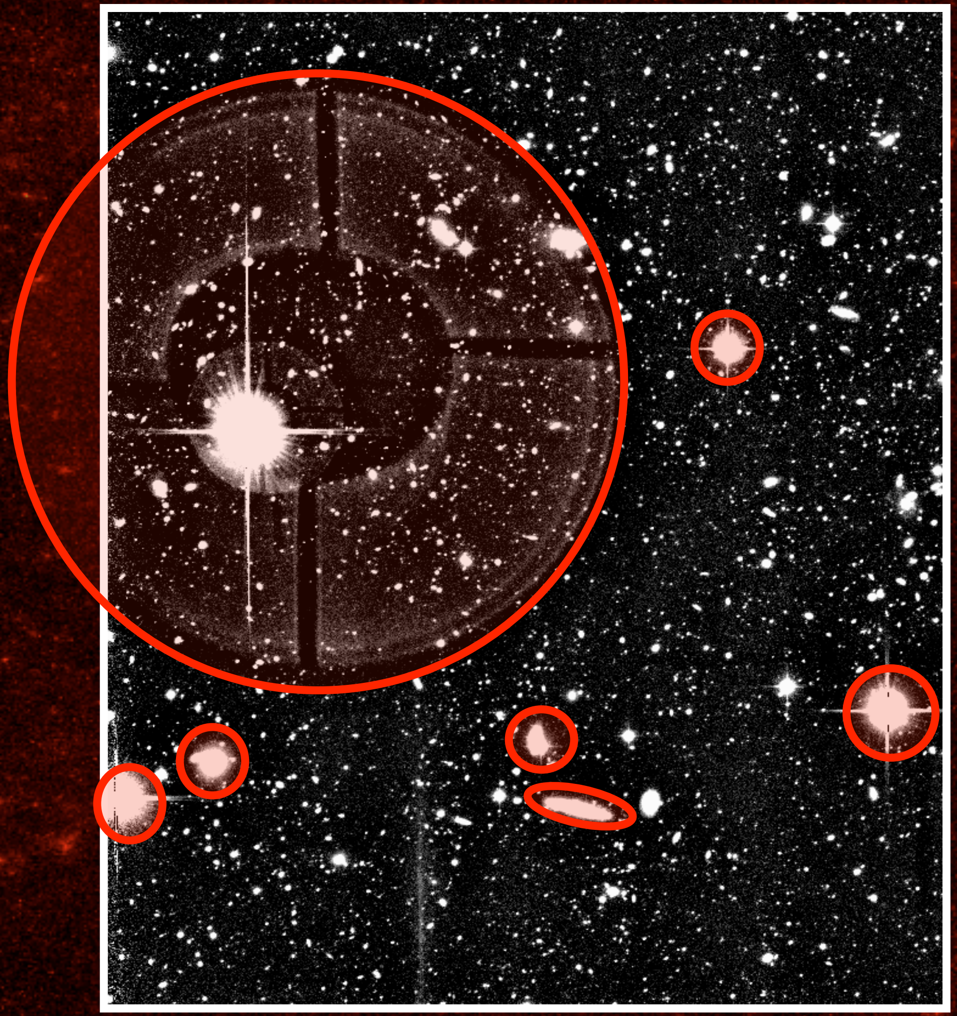
Galaxy nb density and WL



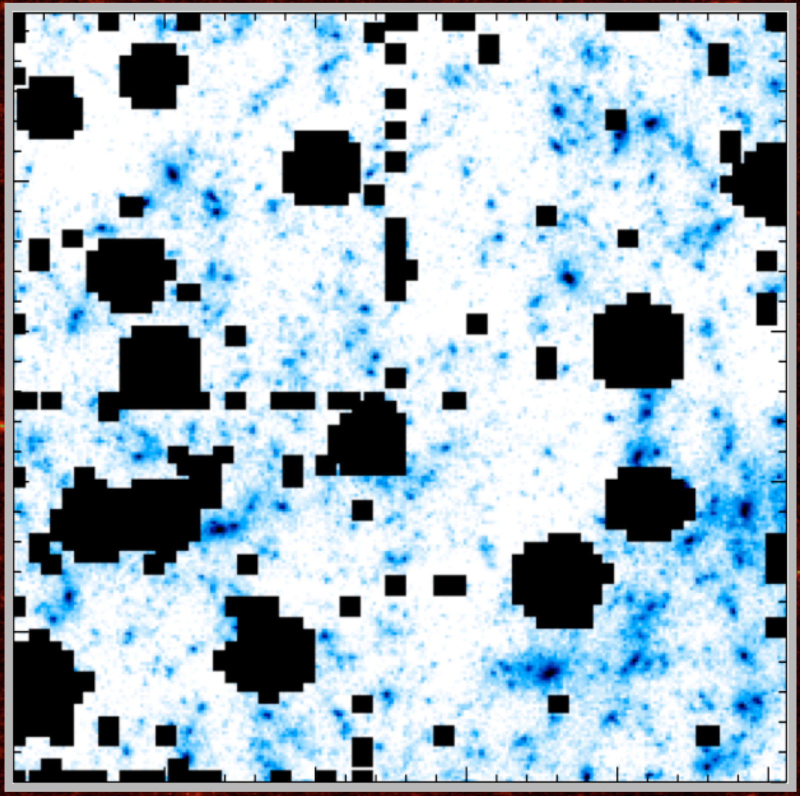
d

Hot gas and WL

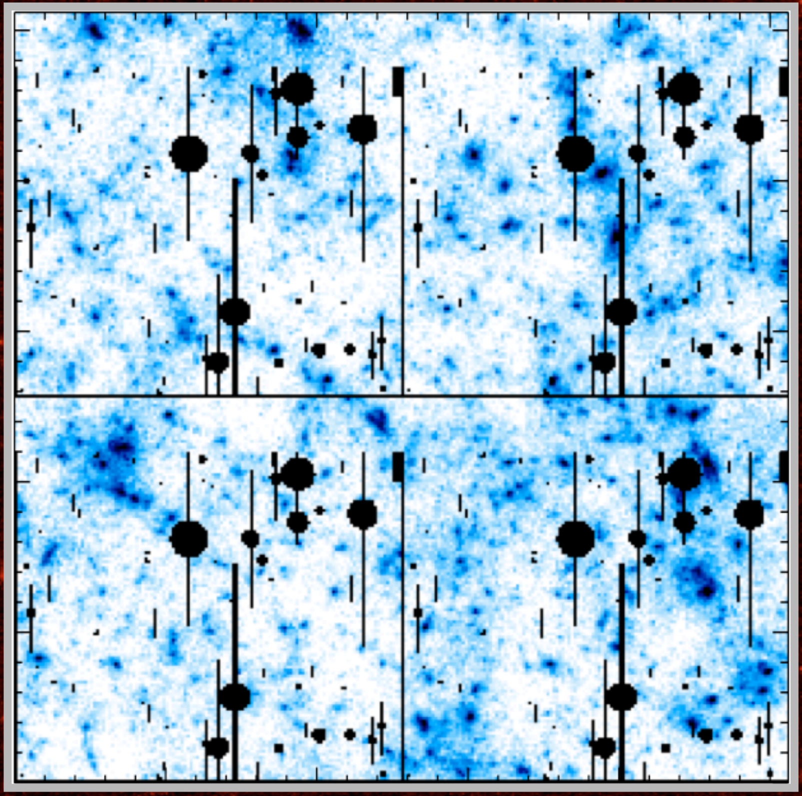
Weak lensing missing data



Masked masks



Mask pattern of CFHTLS survey on 1° x 1° field

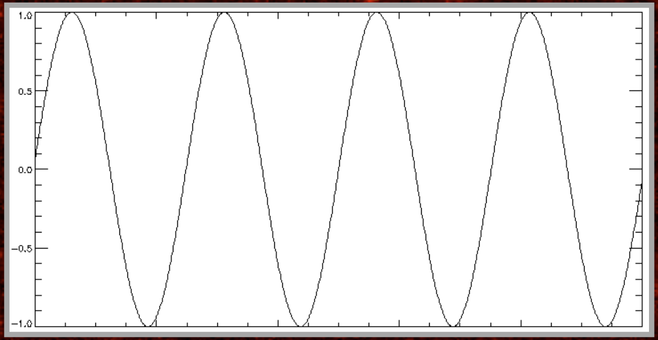


Mask pattern of Subaru survey on 1° x 1° field

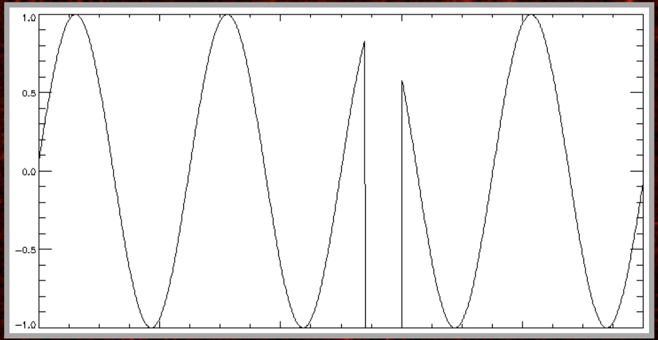
Inpainting based on sparse representation of data

Simultaneous Cartoon and texture Image Inpainting using Morphological Component Analysis,
M. Elad, J.-L Starck, D. Donoho and P. Querre, ACHA, Vol.19, pp.340-358, 2005

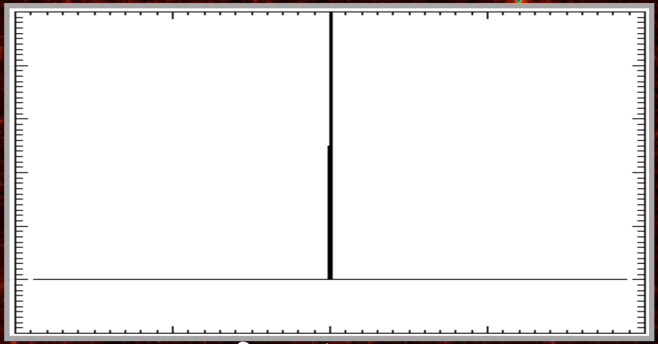
$Y = M.X$ where M is the mask and X the original map



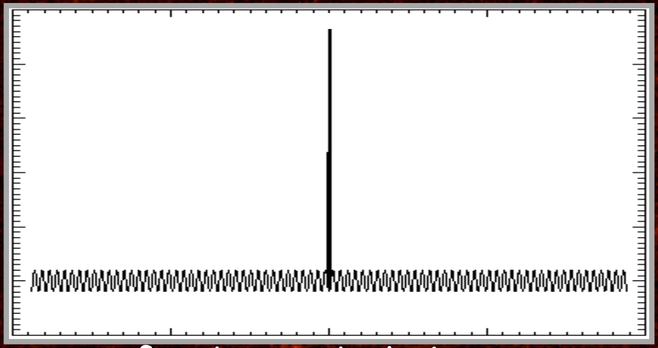
Sine curve



Truncated sine curve



TF of a sine curve



TF of a truncated sine curve

What is sparsity ?

A given signal S (composed of N samples) can be represented by a given dictionary Φ (N × N matrix) :

$$\alpha = \Phi^t S$$

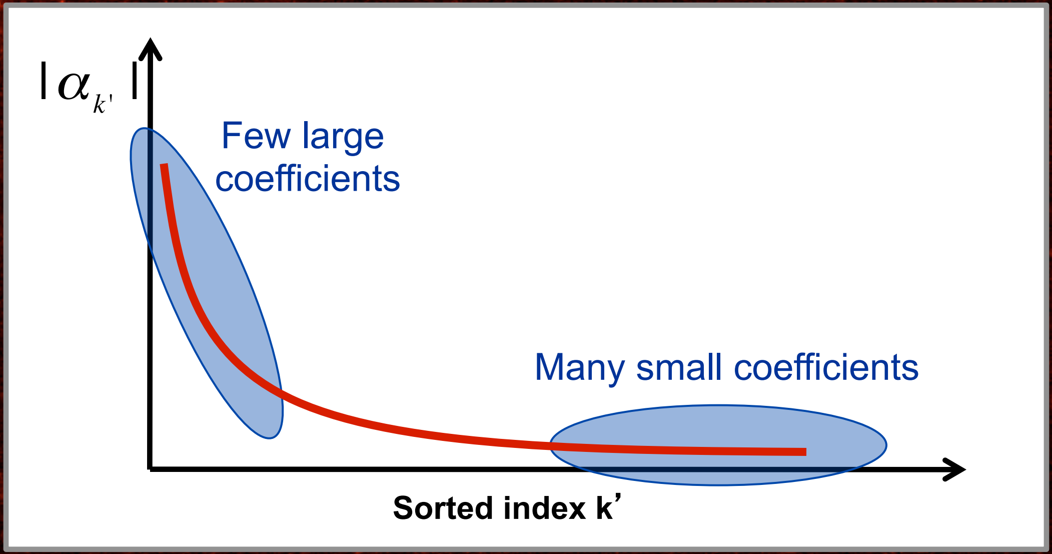
A signal S is sparse in a basis Φ if most of the coefficients α are equal to zero or closed to zero

$$\min_X ||\Phi^t X||_0^2$$

More generally S is sparse in Φ if few α coefficients have significant amplitude.

What is sparsity?

- What is sparsity ?

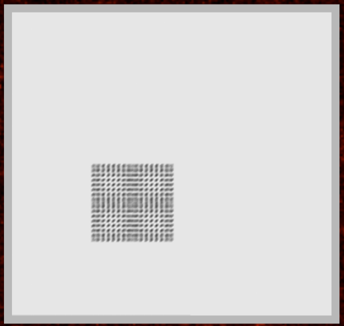


- **Why do we need sparsity?**
  - **Compression of the information**
  - **Clusters detection / extraction**
  - **Image restoration**

Looking for Adapted representations

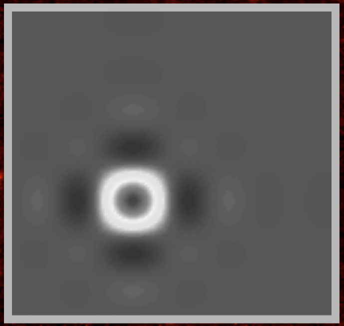
Local DCT :

- Stationary textures
- Locally oscillatory



Wavelet transform :

- Piecewise smooth
- Isotropic structures



Curvelet transform :

- Piecewise smooth
- Edge structures

Inpainting formalism

"Simultaneous Cartoon and Texture Image Inpainting using Morphological Component Analysis (MCA)", M. Elad, J.-L. Starck, D.L. Donoho, P. Querre, ACHA, Vol. 19, pp. 340-358, 2005.

The sparsest representation of an image X is obtained by solving the optimization problem :

$\min ||\alpha||_0$ subject to $\alpha = \Phi^t X$

It has been proposed to replace the l_0-norm by the l_1-norm (Chen, 1995).

The interpolation of missing data is obtained by minimizing :

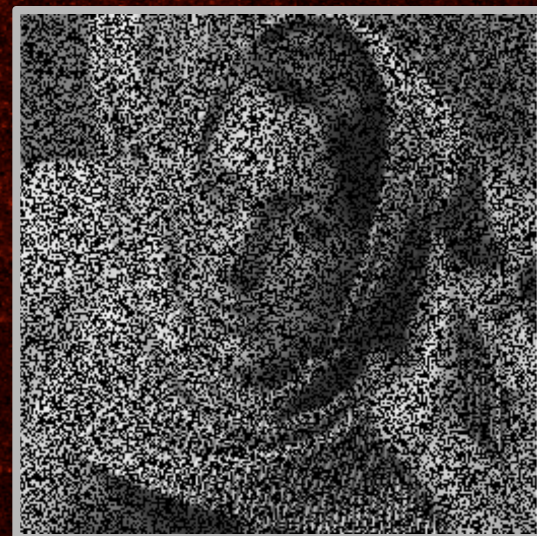
$\min_X ||\Phi^t X||_0$ subject to $||Y - MX||_2^2 < \epsilon$

(where M is the mask $M(i,j) = 0$ for missing data else equal 1)

Inpainting

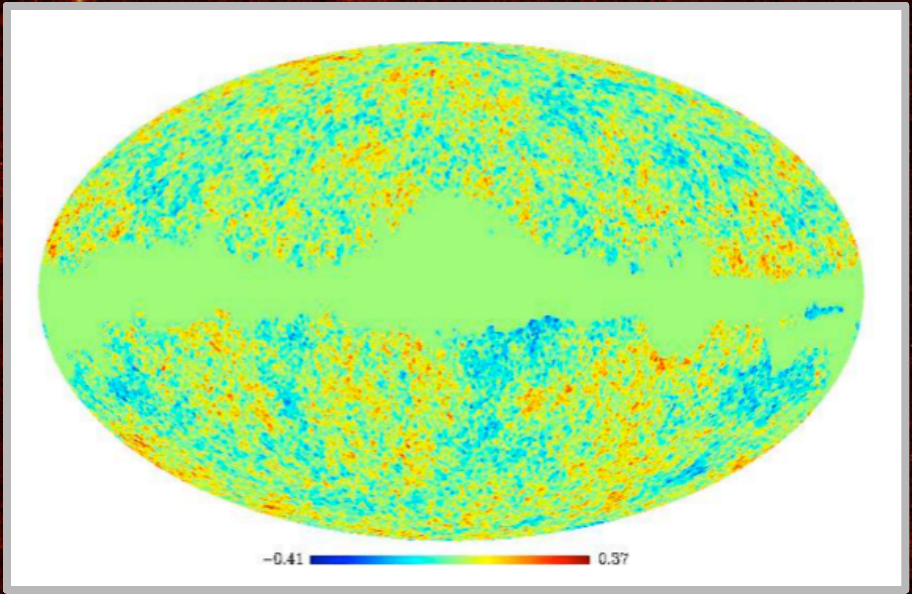
Missing data randomly distributed

50%

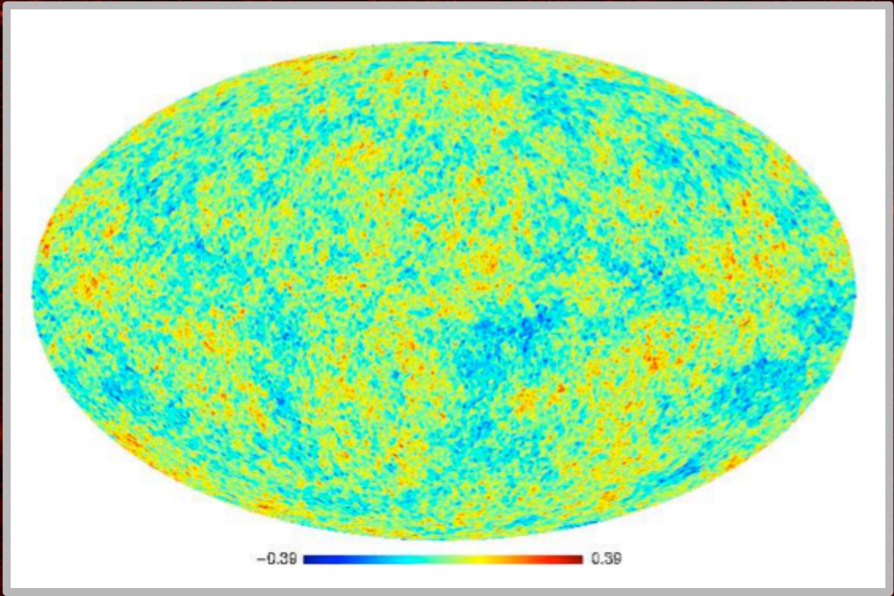


80%

Inpainting on WMAP data

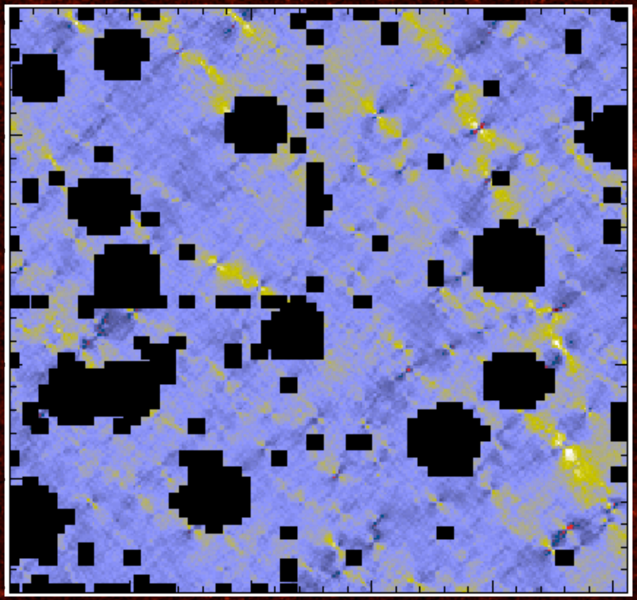


WMAP 3 years

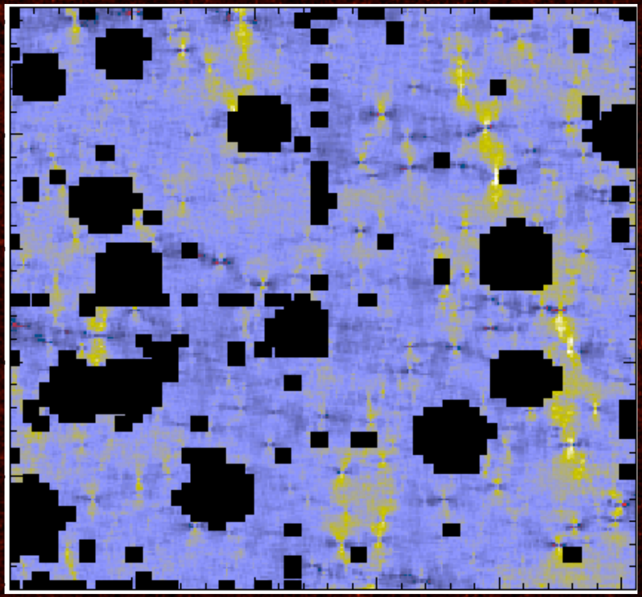


Inpainted map

Inpainting from Shear maps



γ_1



γ_2

$$\hat{\kappa} = \hat{P}_1 \hat{\gamma}_1 + \hat{P}_2 \hat{\gamma}_2$$

Weak lensing inpainting algorithm

$\gamma_i \rightarrow$ $\min_\kappa \| \Phi^t \kappa \|_{l_0}$ subject to $\sum_i \| \gamma_i - M(P_i * \kappa) \|_{l_2}^2 \leq \varepsilon$ $\rightarrow \kappa$

$\downarrow$

Physical priors

$$\gamma_i^{obs} = M.\gamma_i$$

$$\kappa = P_1 * \gamma_1 + P_2 * \gamma_2$$

Φ^t is the DCT

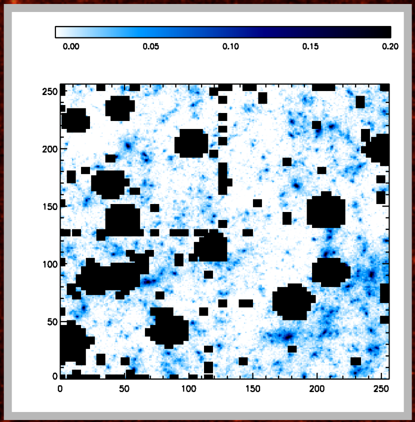
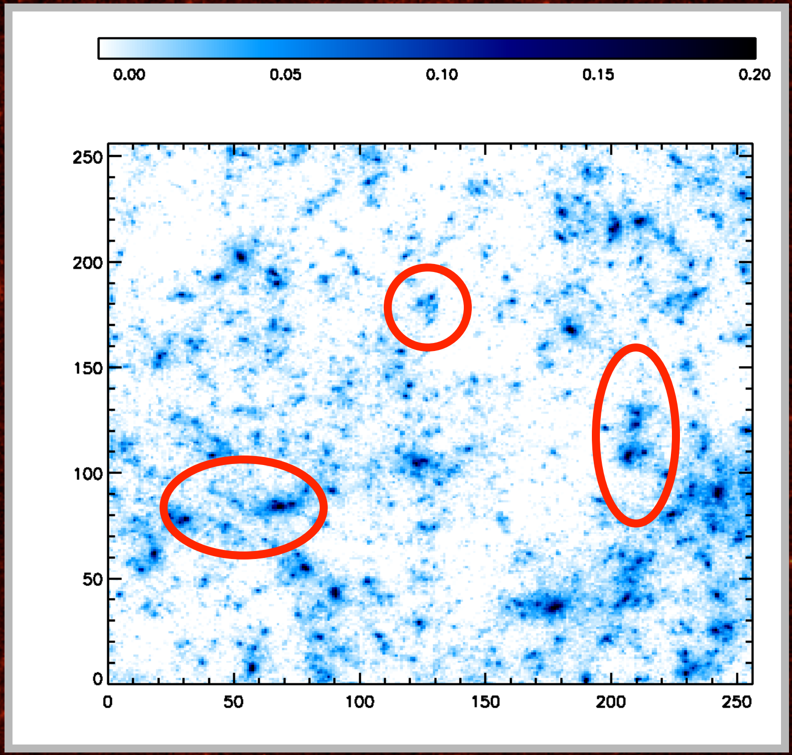
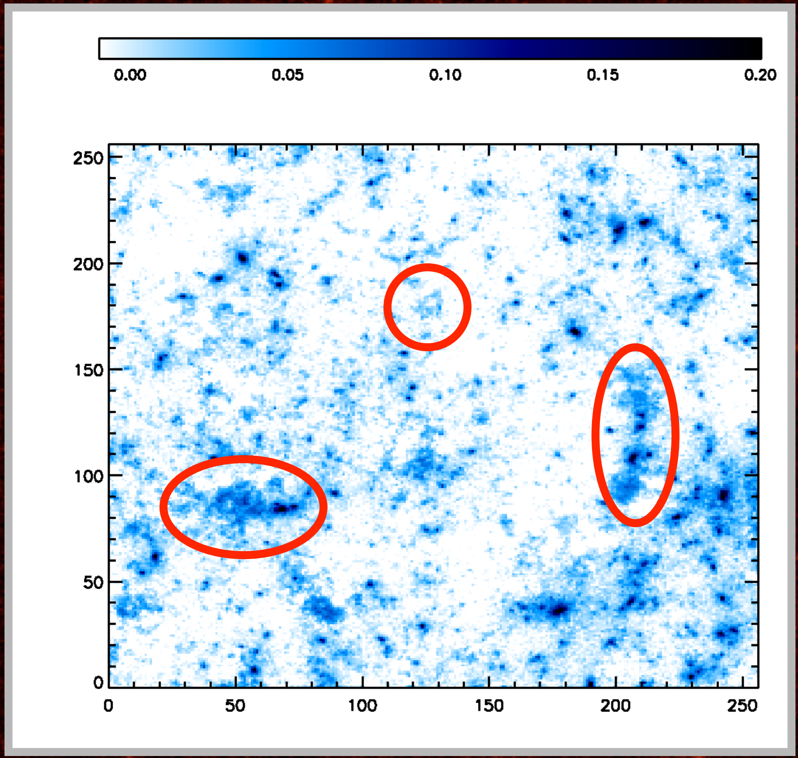


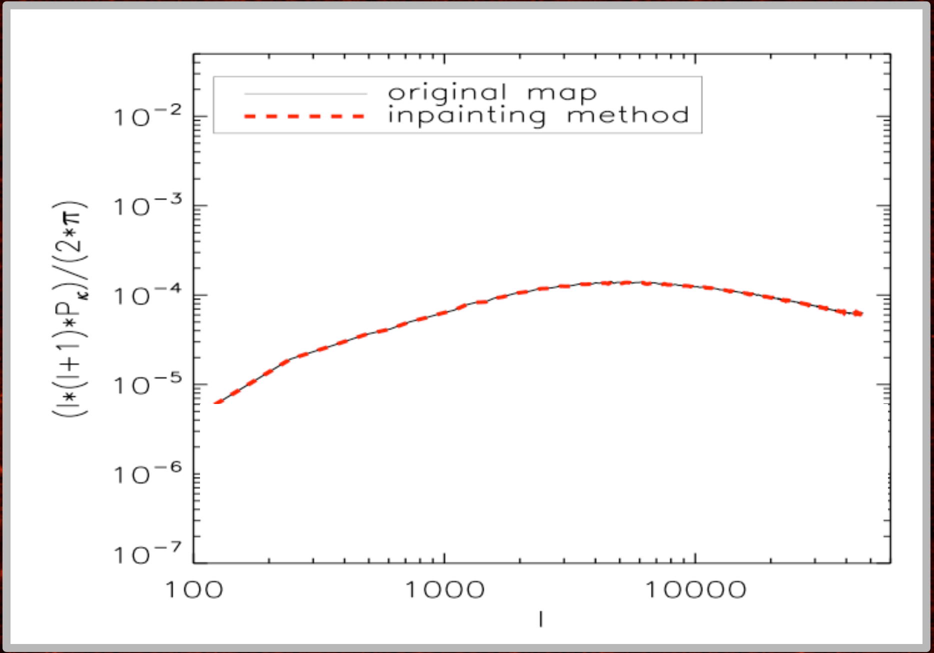
Image reconstruction





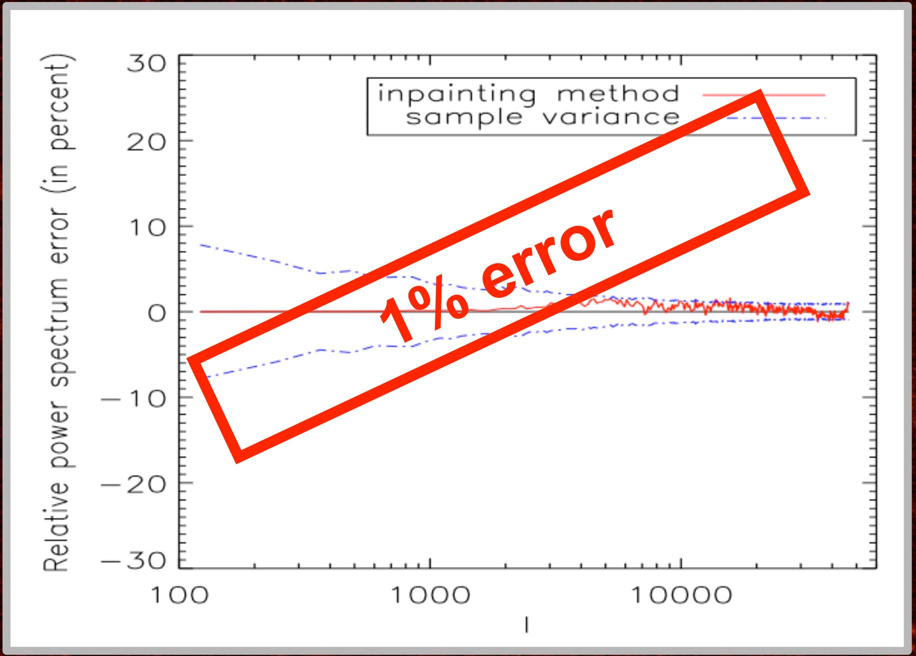
Which image is the original one ?

Power spectrum estimation
= CFHTLS mask =
(arXiv:0804.4068)



Power spectrum recovery from shear maps for CFHTLS masks : The mean power spectrum computed from the 100 complete mass maps (black) and from 100 inpainted maps (red).

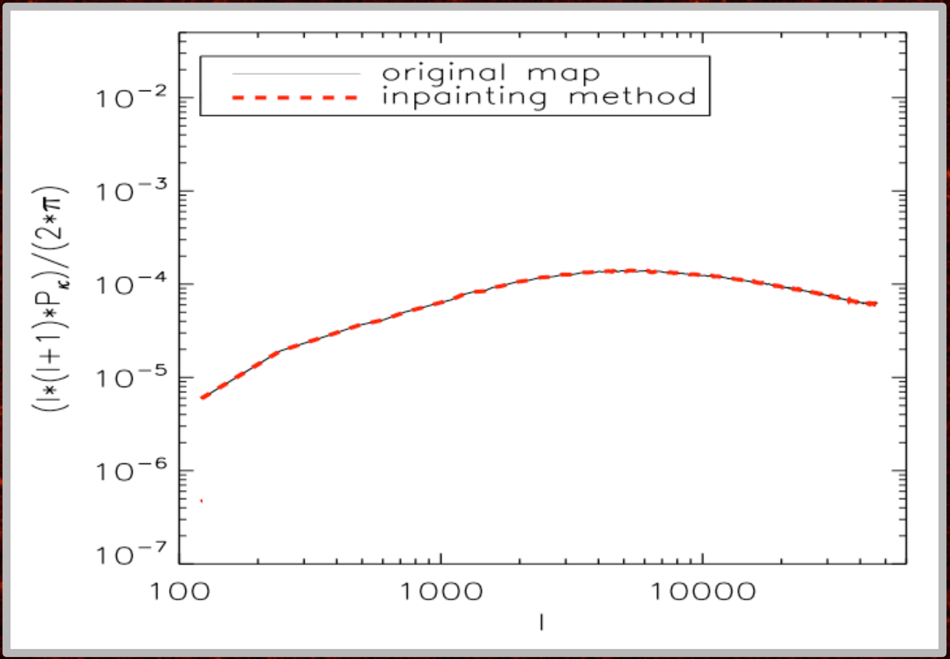
Relative power spectrum error, i.e. the normalized difference between the two upper curves of the left panel. The blue dashed line represents the empirical standard deviation estimated from the 100 complete mass maps.



Power spectrum estimation

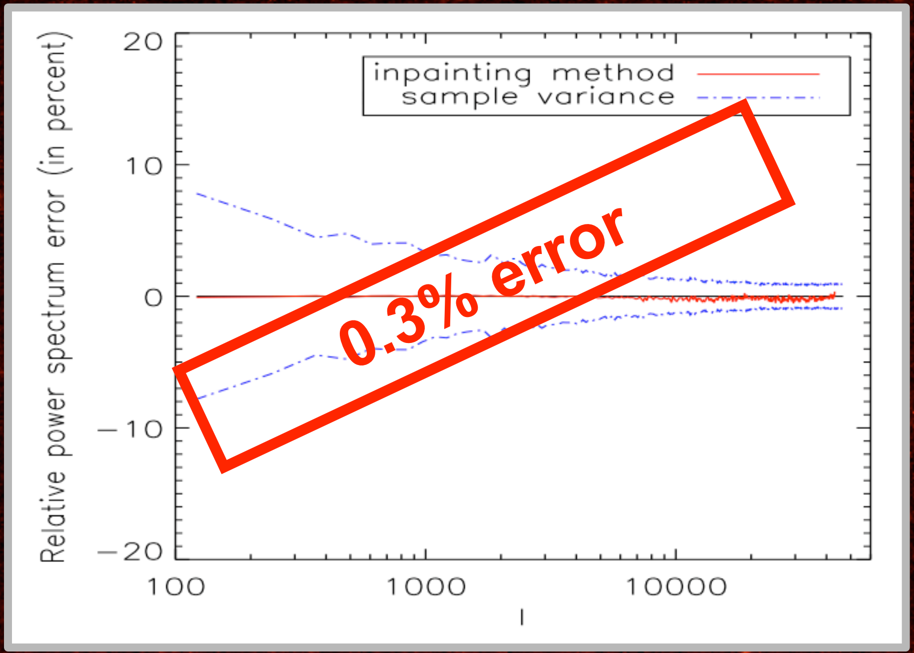
= Subaru mask =

(arXiv:0804.4068)

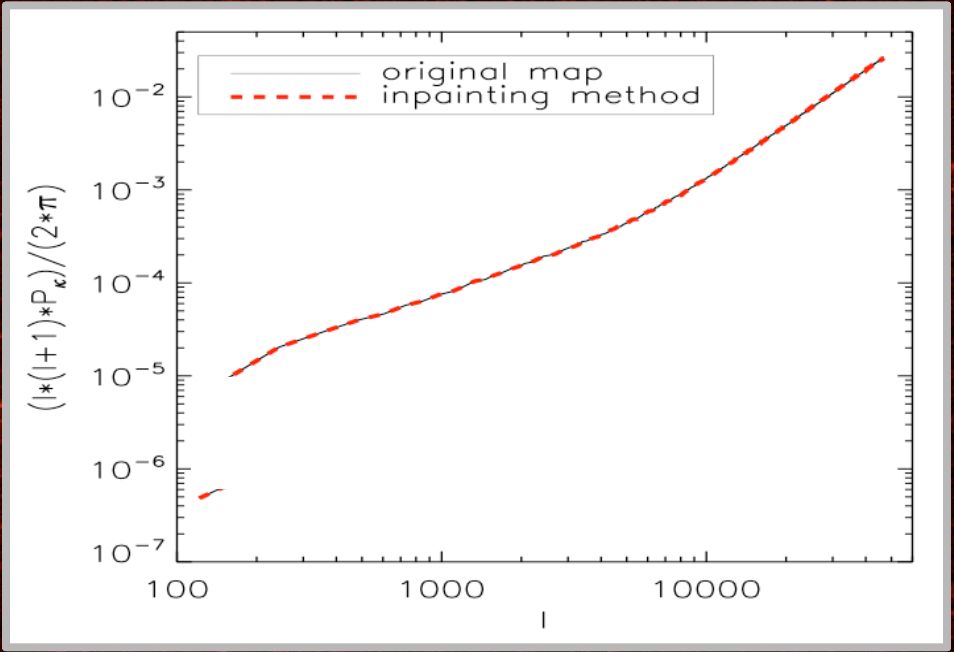


Power spectrum recovery from shear maps for Subaru masks : The mean power spectrum computed from the 100 complete mass maps (black) and from 100 inpainted maps

Relative power spectrum error, i.e. the normalized difference between the two curves of the left panel. The blue dashed line represents the empirical standard deviation estimated from the 100 complete mass maps.

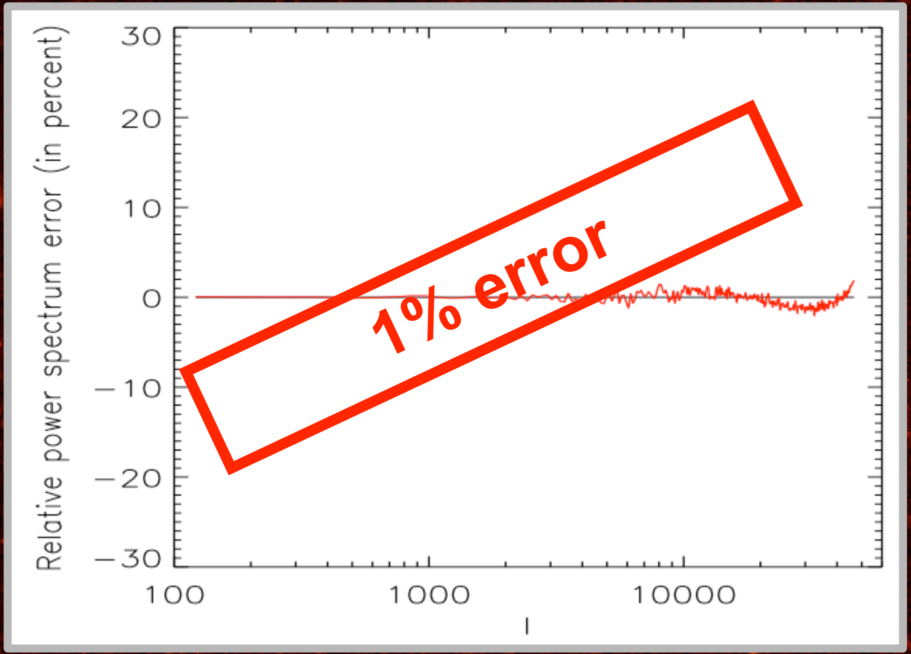


Noisy power spectrum estimation
= CFHTLS mask =
(arXiv:0804.4068)



Noisy power spectrum recovery from shear maps for CFHTLS masks. The mean noisy power spectrum computed from the 100 complete mass maps (black) and the inpainted reconstructed maps from 100 masked shear maps (red).

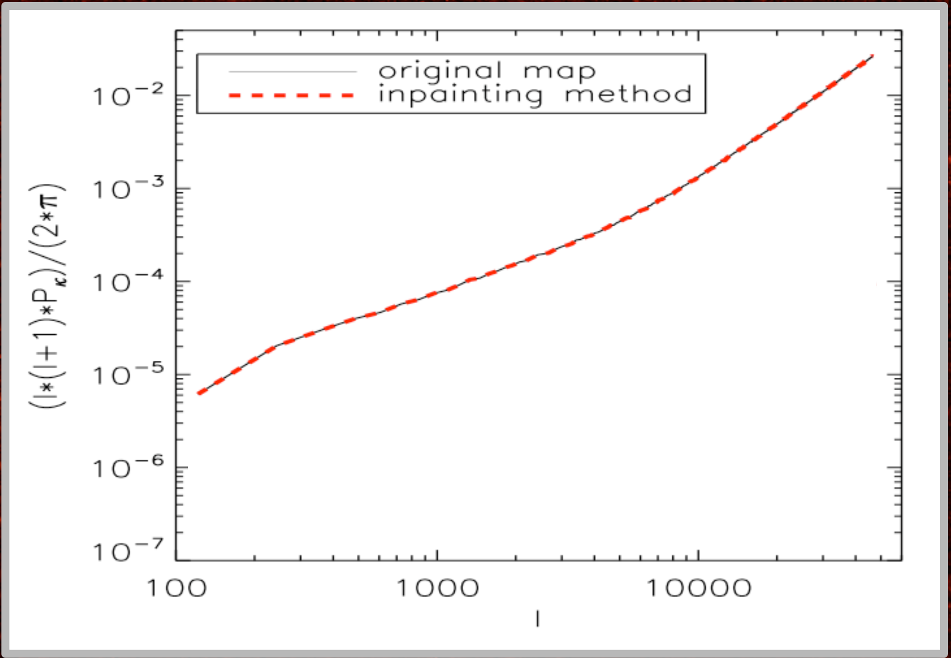
Relative noisy power spectrum error, i.e. the normalized difference between the two curves of the left panel.



Noisy power spectrum estimation

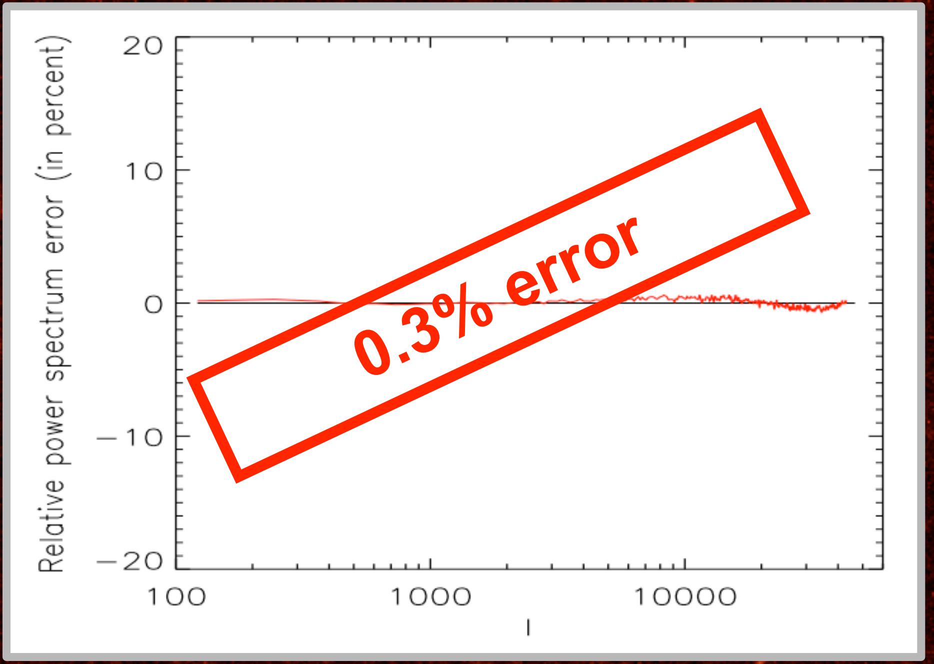
= Subaru mask =

(arXiv:0804.4068)



Noisy power spectrum recovery from shear maps for Subaru masks. The mean noisy power spectrum computed from the 100 complete mass maps (black) and the inpainted reconstructed maps from 100 masked shear maps (red).

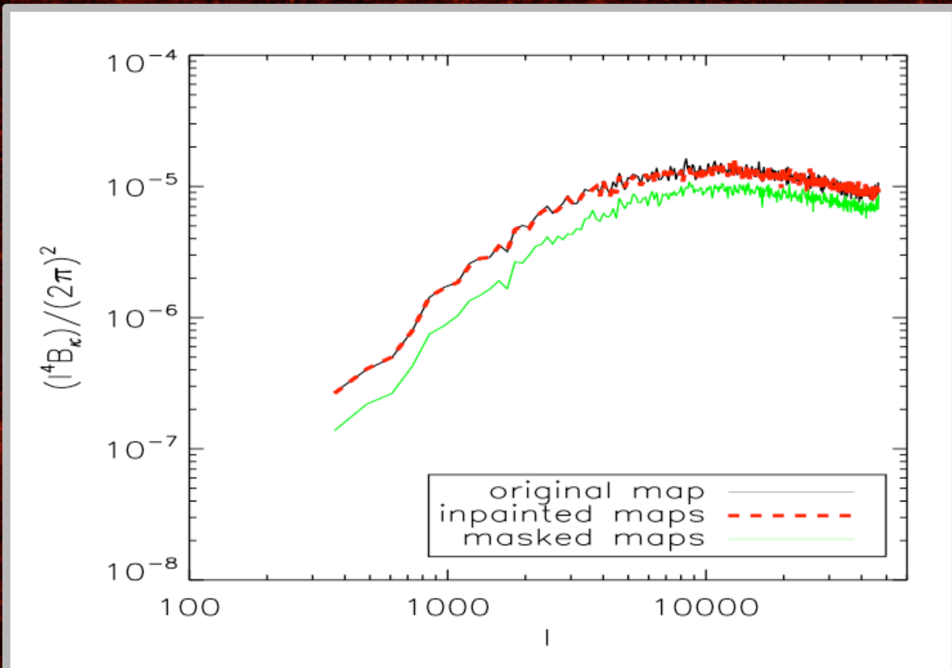
Relative noisy power spectrum error, i.e. the normalized difference between the two curves of the left panel.



Equilateral bispectrum estimation

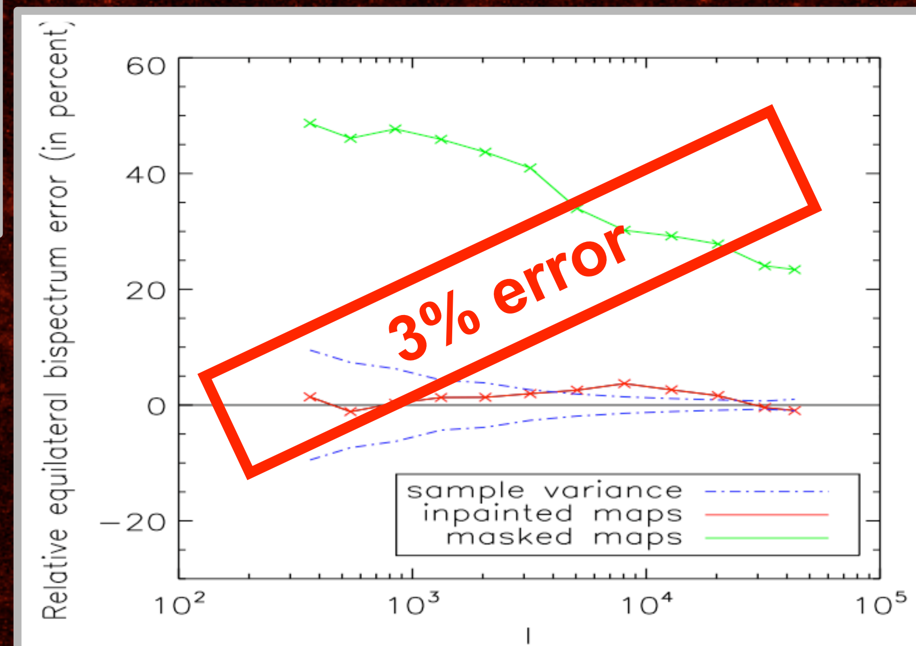
= CFHTLS mask =

(arXiv:0804.4068)



Bispectrum recovery from shear maps for CFHTLS masks : The mean bispectrum computed from the 100 complete mass maps (black), from 100 inpainted reconstructed maps (red) and from 100 incomplete mass maps (green) .

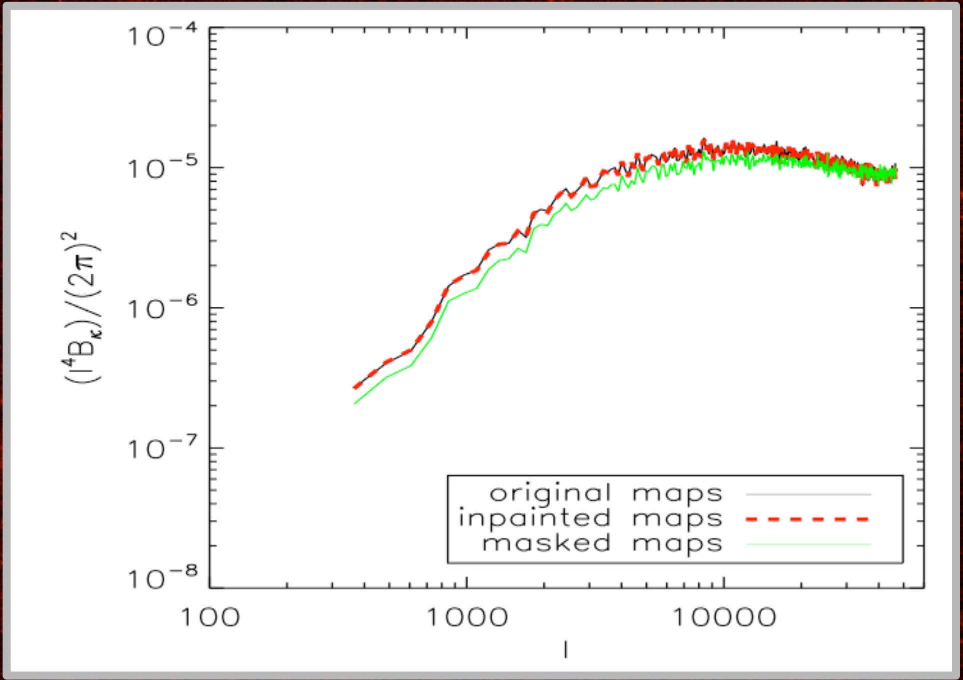
Relative bispectrum error, i.e. the normalized difference between the two upper curves of the left panel. The blue dashed line represents the empirical standard deviation estimated from the 100 complete mass maps.



Equilateral bispectrum estimation

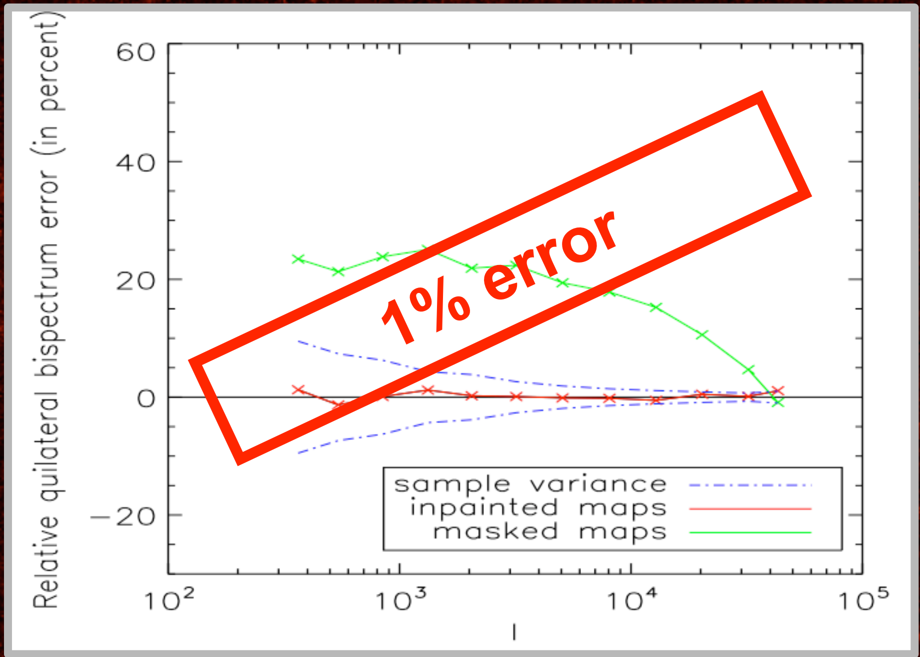
= Subaru mask =

(arXiv:0804.4068)



Relative bispectrum error, i.e. the normalized difference between the two curves of the left panel. The blue dashed line represents the empirical standard deviation estimated from the 100 complete mass maps.

Bispectrum recovery from shear maps for Subaru masks : The mean bispectrum computed from the 100 complete mass maps (black), from 100 inpainted reconstructed maps (red) and from 100 incomplete mass maps (green) .

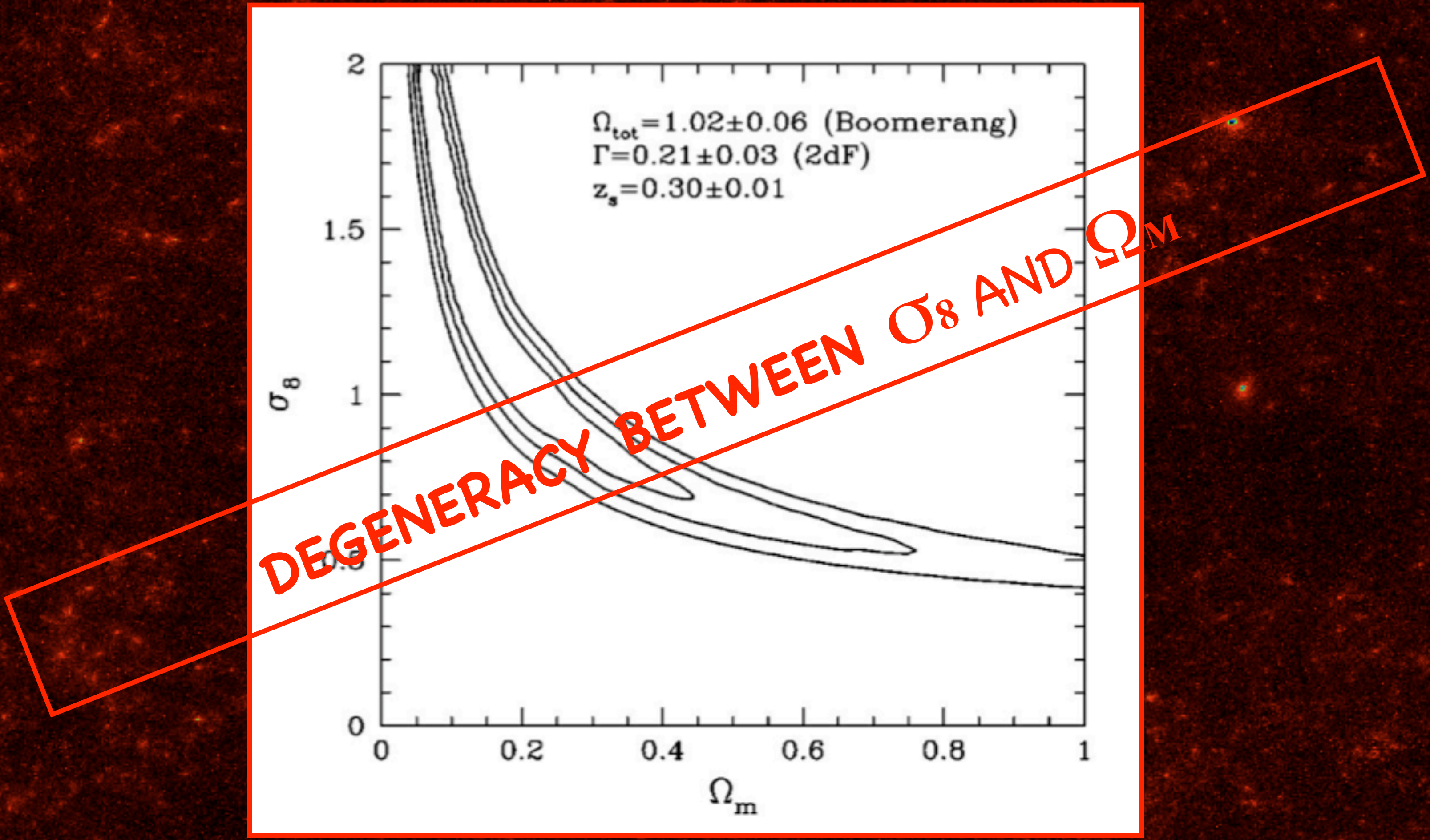


INPAINTING

Reconstruction of a complete dark matter mass map

- FAST ESTIMATION OF ANY STATISTICS :
  - THE MAXIMUM ERROR ON THE POWER SPECTRUM ESTIMATION IS 1%
  - THE MAXIMUM ERROR ON THE BISPECTRUM ESTIMATION IS 3%
- CLUSTER STATISTIC ANALYSIS
- DARK MATTER DISTRIBUTION STUDY

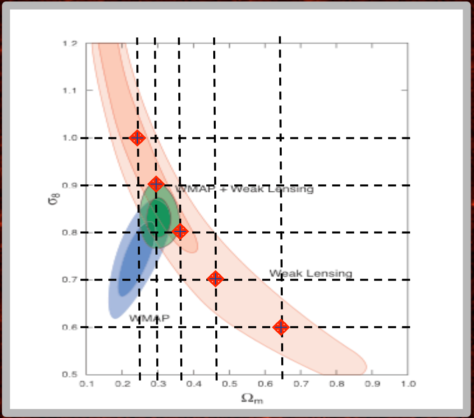
Weak Lensing degeneracy



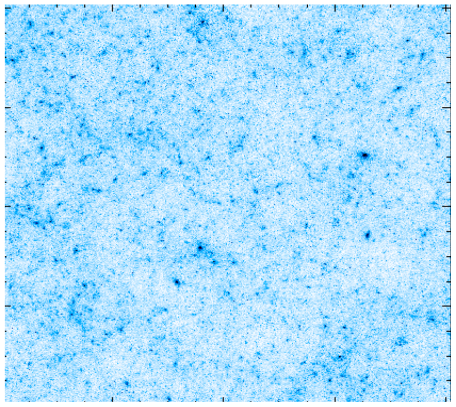
Statistic Candidates

1. Skewness (third-order moment) estimated on a Direct, Fourier, Wavelet, Ridgelet and Curvelet representation
2. Kurtosis (fourth-order moment) estimated on a Direct, Fourier, Wavelet, Ridgelet and Curvelet representation
3. Higher Criticism (Donoho & Jin, 2004) estimated on a Direct, Fourier, Wavelet, Ridgelet and Curvelet representation
4. Peak counting estimated on a direct and wavelet representation
5. Bispectrum (arXiv:0804.4068)

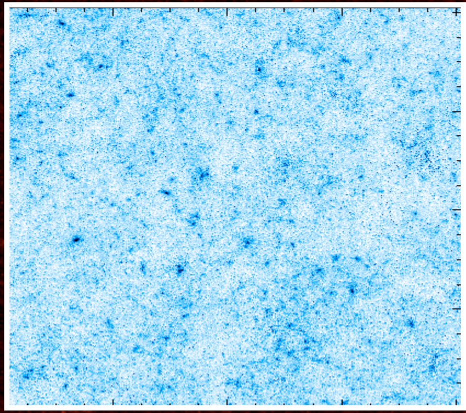
Cosmological model simulations



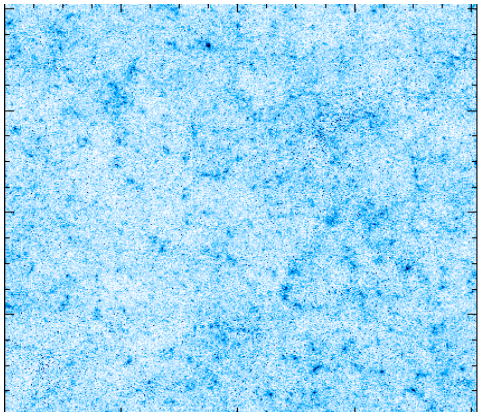
Model1 (σ_8=1, Ω_m=0.23)



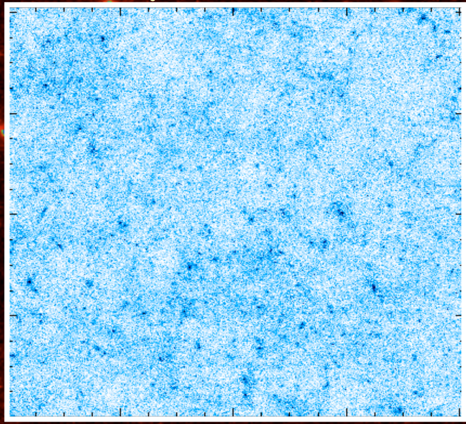
Model2 (σ_8=0.9, Ω_m=0.3)



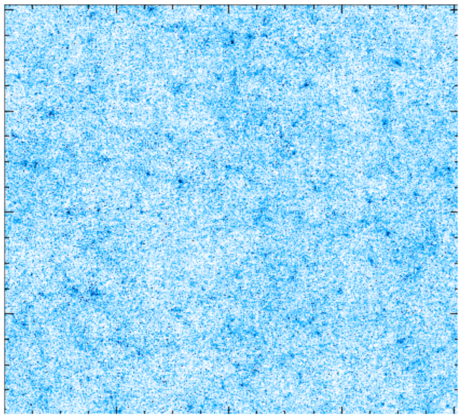
Model3 (σ_8=0.8, Ω_m=0.36)



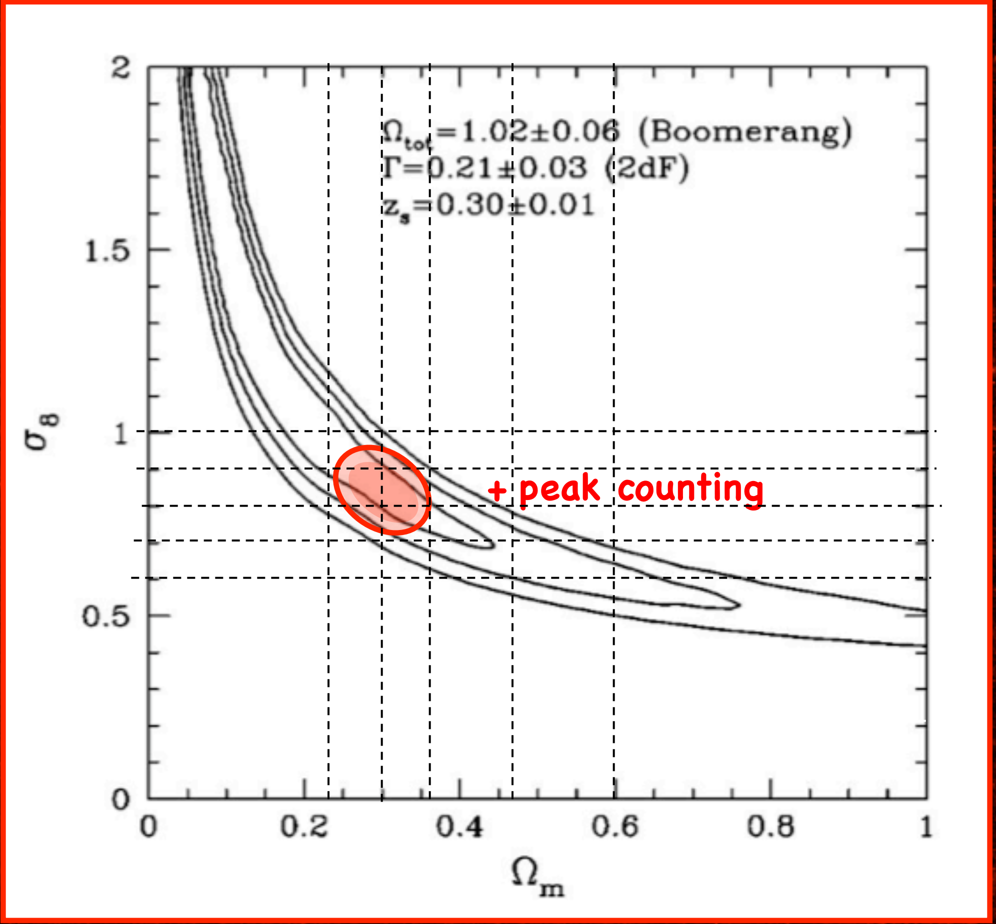
Model4 (σ_8=0.7, Ω_m=0.47)



Model5 (σ_8=0.6, Ω_m=0.64)



Peak counting breaks the degeneracy



Conclusions

- A method for filtering the noise of Weak Lensing dark matter mass map has been developed (MRLens)
  - ✓ Outperforms existing methods
  - ✓ Applied to real data (COSMOS field)
  - ✓ MRLens is freely available on the web (google mrlens)
- A method to reconstruct full Weak Lensing mass map from incomplete shear maps has been developed (FASTLens)
  - ✓ The maximum error in the estimation of the power spectrum is 1%
  - ✓ The maximum error in the estimation of the bispectrum is 3%
  - ✓ FASTLens will be soon available including a method to estimate the equilateral bispectrum
- We have studied the best way to constrain the cosmological model
  - ✓ A preliminary result seems to show that the better statistic is the peak counting

Perspectives

- Extension of the MRLens filter to the processing of data on the sphere (Euclid project)
- Extension of the MRLens filter to the processing of 3D Weak Lensing data.
- Developement of a new method to estimate the shear in the GREAT08 projet (using sparsity)
- Application of the method MRLens and FASTLens to CFHTLS data