AIM-CEA Saclay, France

Signal Processing with Sparse representations

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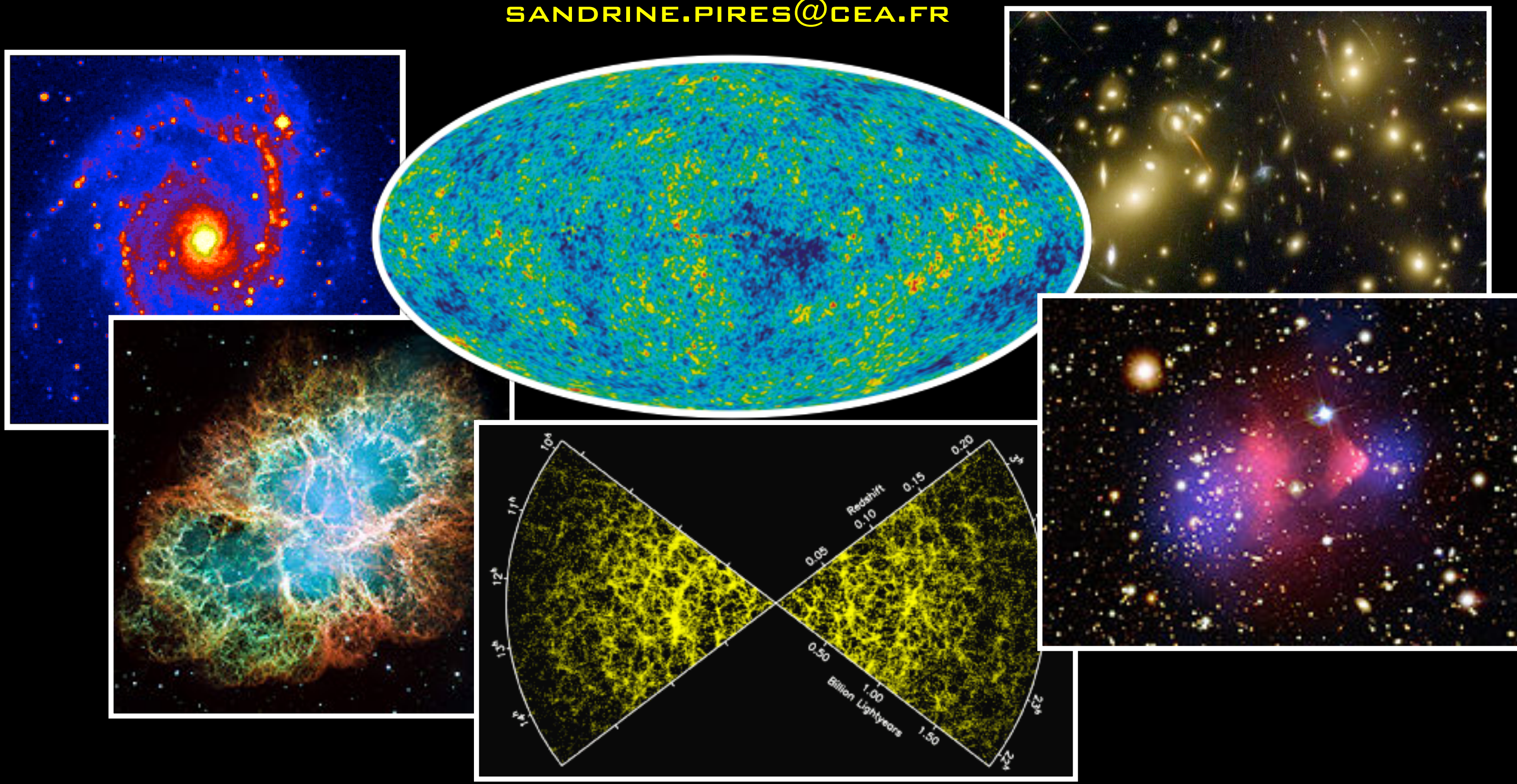


Common Representations

* The Fourier Transform

$$f(k) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi i k x} dx$$

- ✓ Where *f(k)* are the Fourier coefficient of the function *f(x)*
- ✓ The analysing function is $e^{-2\pi i k x}$
- ✓ *k* is the frequence parameter

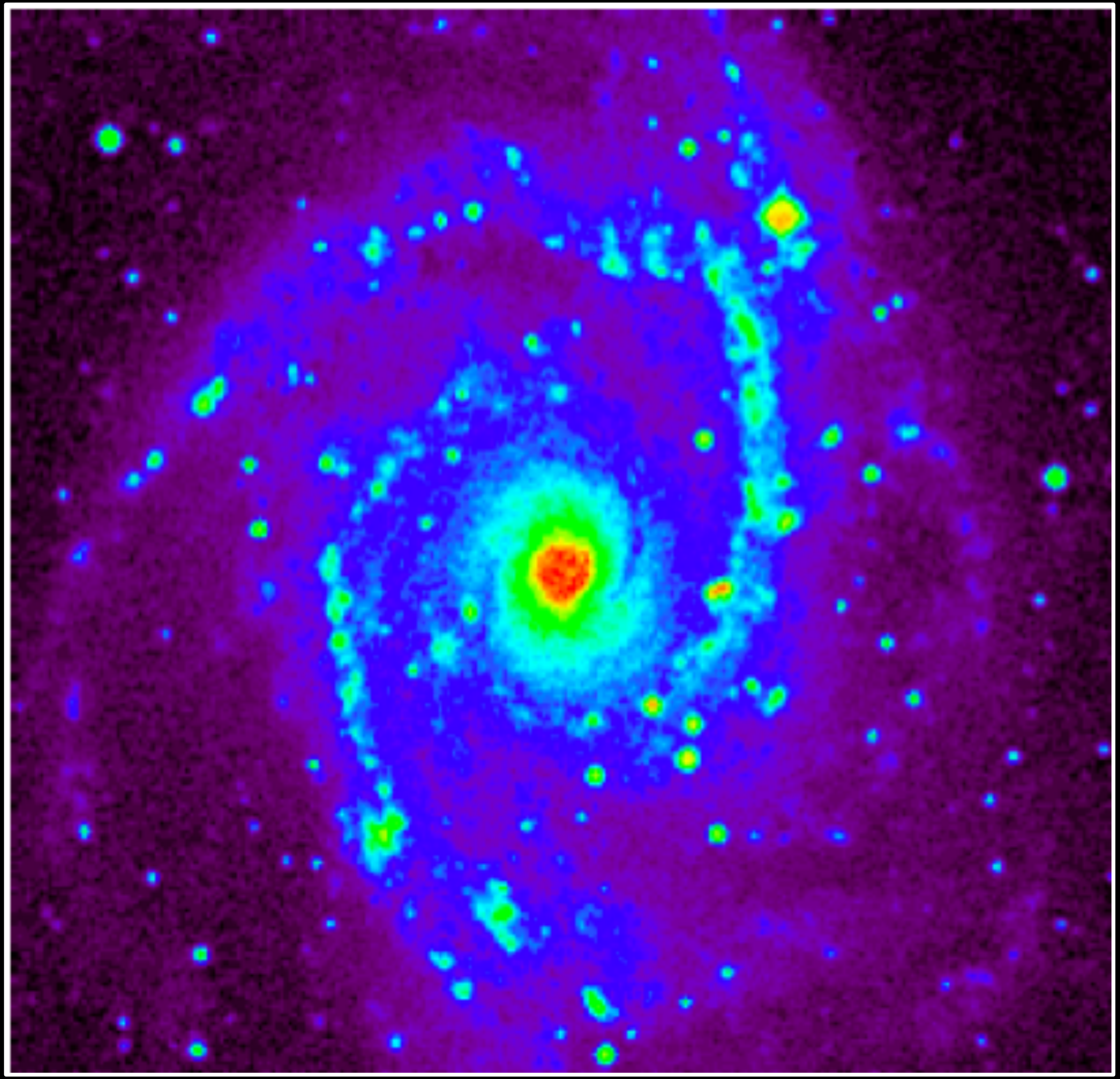
* The Wavelet Transform

$$W(a,b) = K \int_{-\infty}^{+\infty} \psi^*\left(\frac{x-b}{a}\right) f(x) dx$$

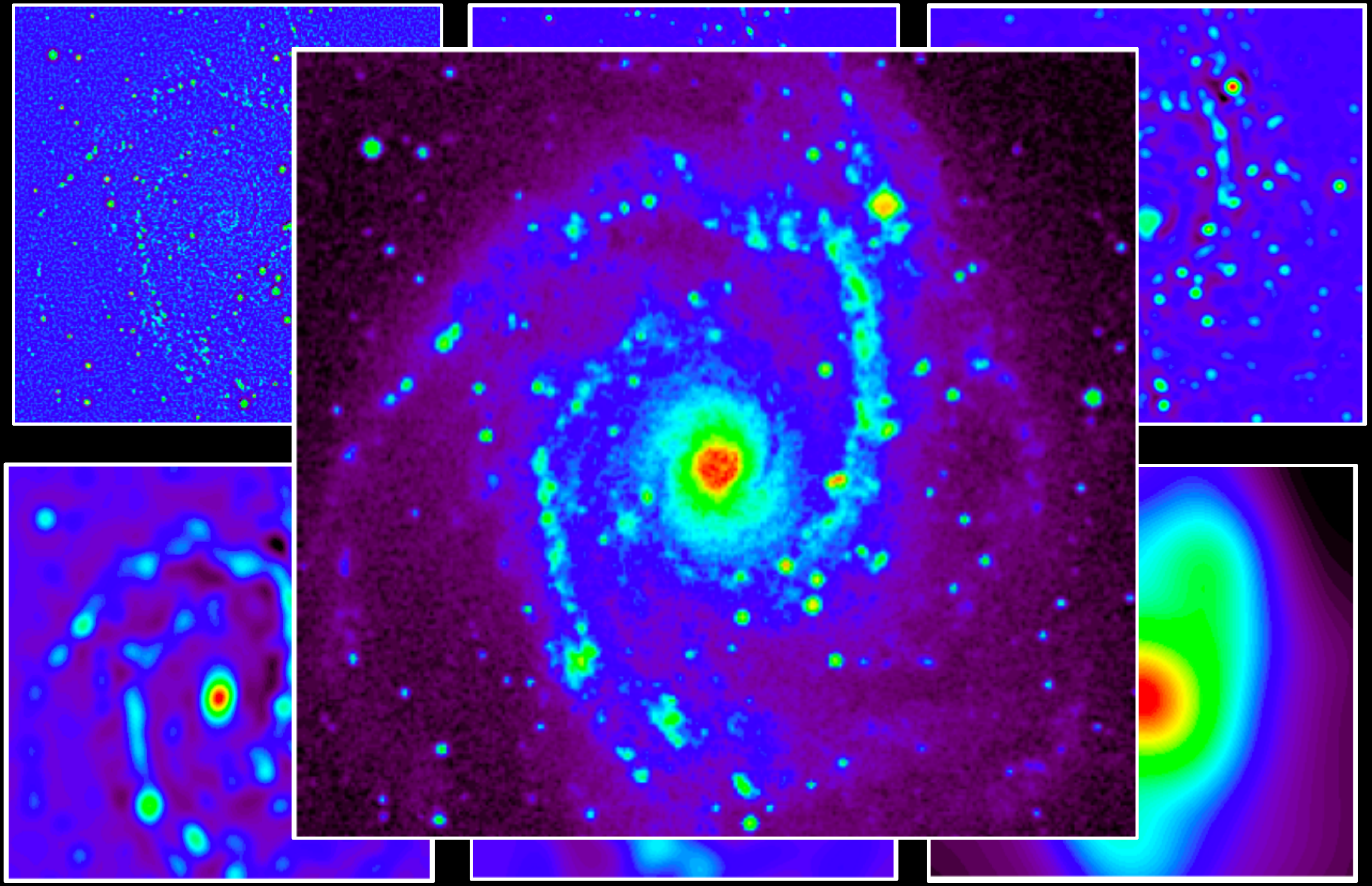
- ✓ Where *W(a,b)* are the Wavelet coefficients of the function *f(x)*
- ✓ The analysing function is $\psi(x)$
- ✓ *a (>0)* is the scale parameter and *b* is the position parameter

When the scale *a* varies the filter $\psi(x)$ is only reduced or dilated while keeping the same pattern.

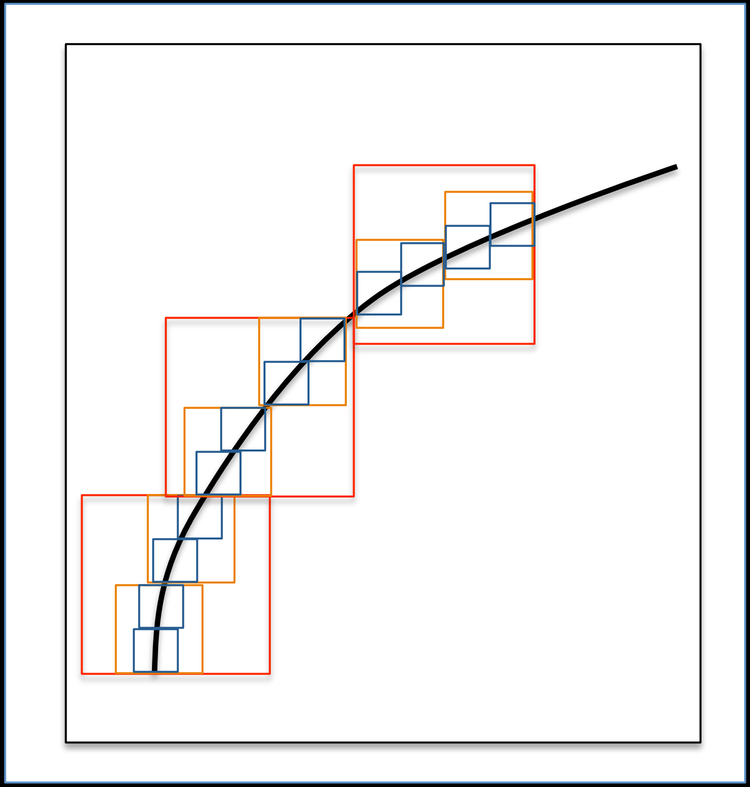
Orthogonal Wavelet Transform



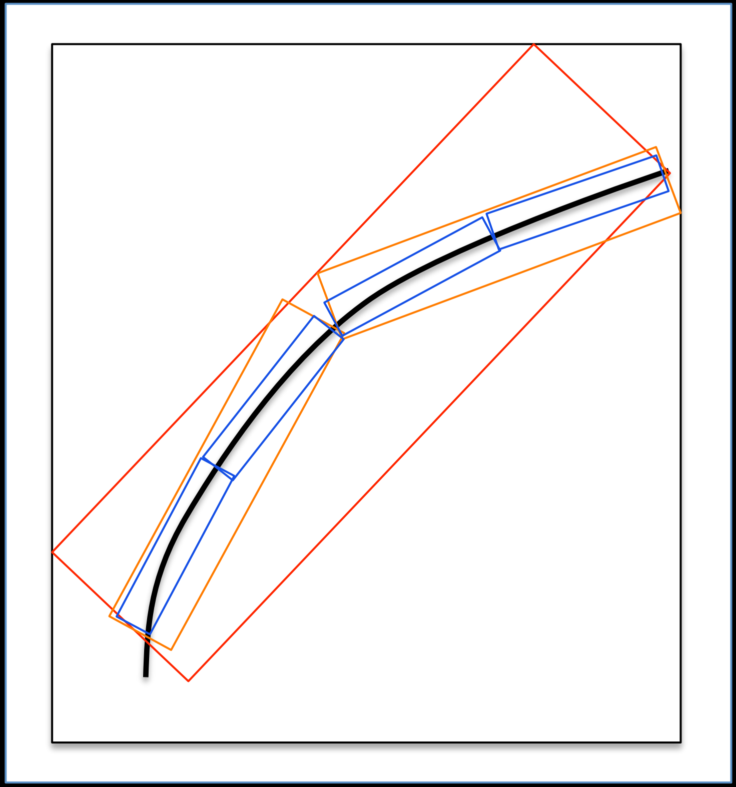
Undecimated Isotropic Wavelet Transform



Other representations

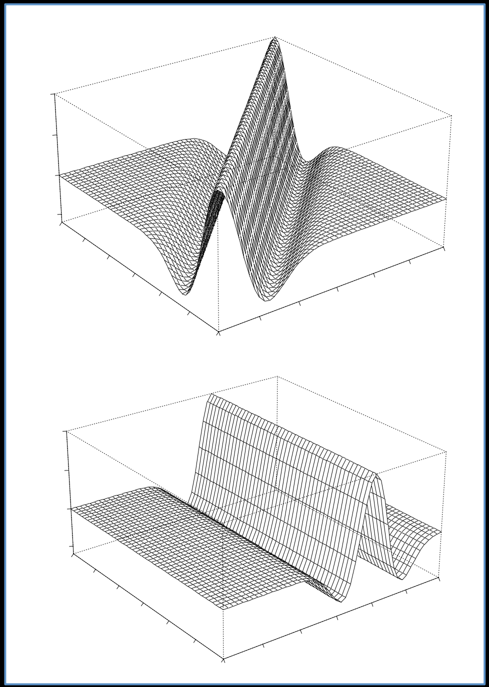


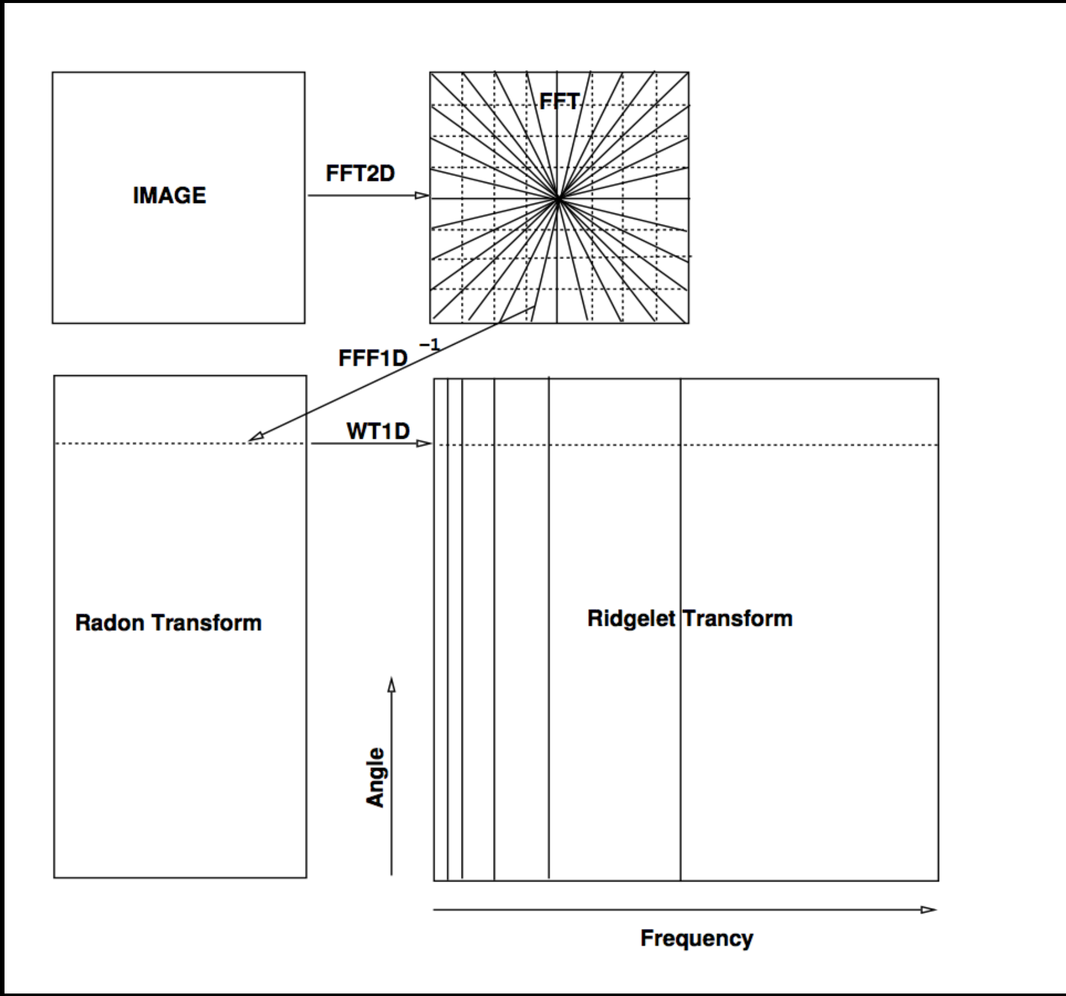
Wavelet representation



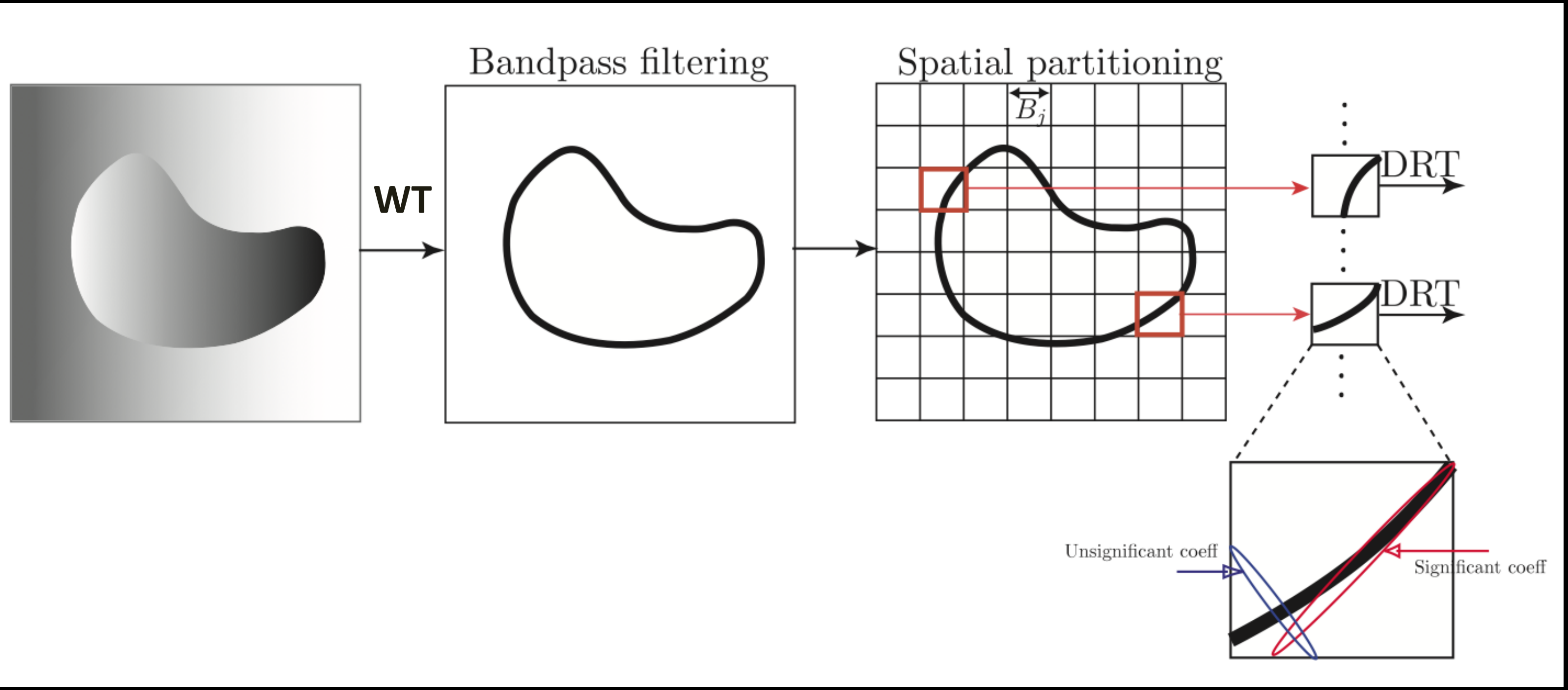
Anisotropic representation

Other transformations : Ridgelet transform





Other transformations : Curvelet transform



What is a good representation for data?

- ✓ We seek representations of the signal (*f*) as linear combination of:
  - ✓ basis elements
  - ✓ frames
  - ✓ dictionary elements

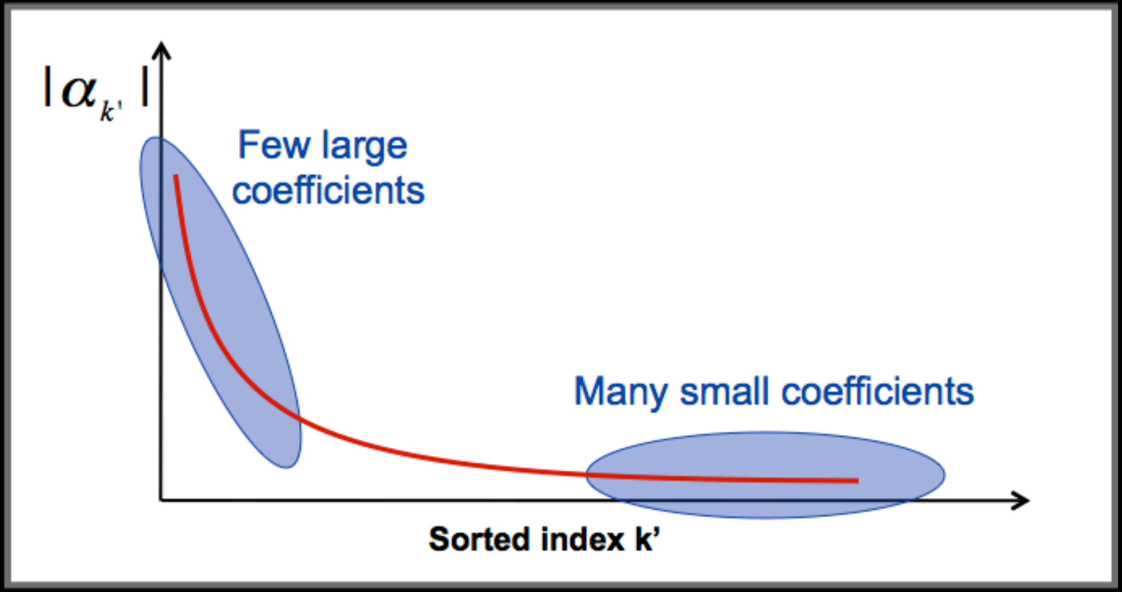
$$f = \sum_k a_k \mathbf{b}_k$$

coefficients basis, frame

- ✓ The analyzing functions should extract the features of interest:
  - ✓ Harmonic features
  - ✓ Isotropic features
  - ✓ Anisotropic feature
- ✓ Recent methods exploit the sparsity of the coefficients

What is sparsity ?

Considering a transform : $\alpha = \Phi^T X$

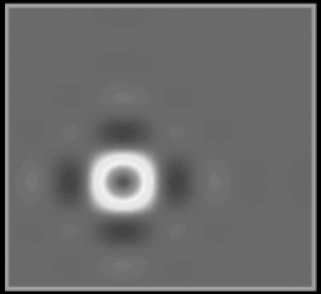


- ✓ Why do you need sparsity:
  - ✓ Data compression
  - ✓ Feature extraction, detection
  - ✓ Image restoration
  - ✓ ….

Signal and image representations

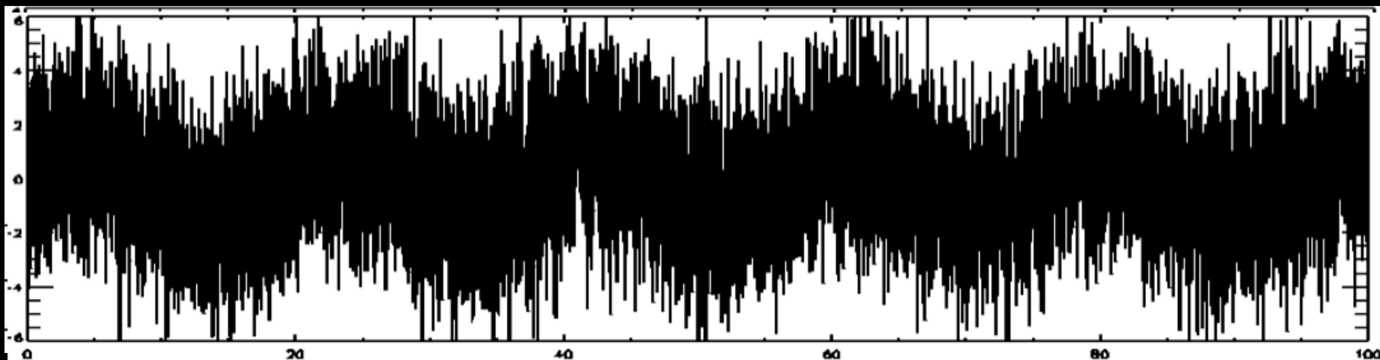
- ✓ Local DCT :
  - ✓ Stationary textures
  - ✓ Locally oscillatory

- ✓ Wavelet Transform
  - ✓ Piecewise smooth
  - ✓ Isotropic structures

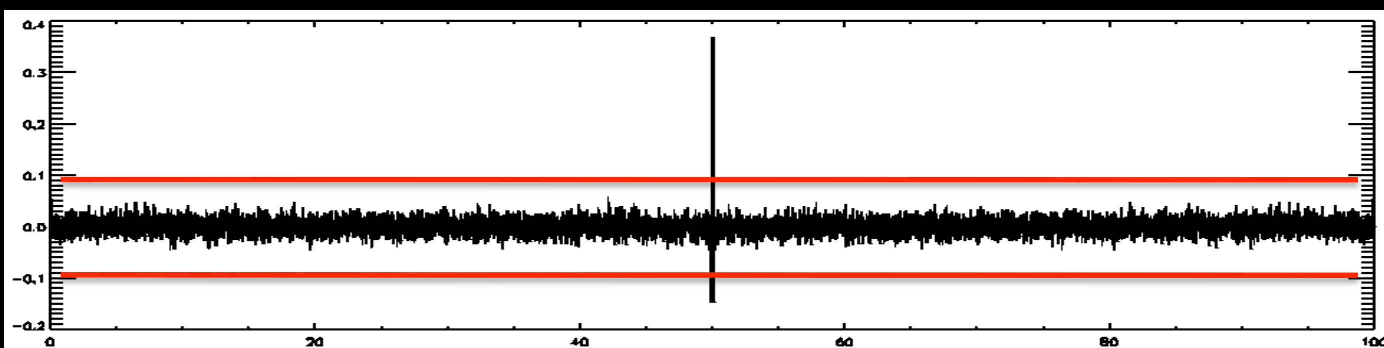


- ✓ Curvelet Transform
  - ✓ Piecewise smooth
  - ✓ Edge structures

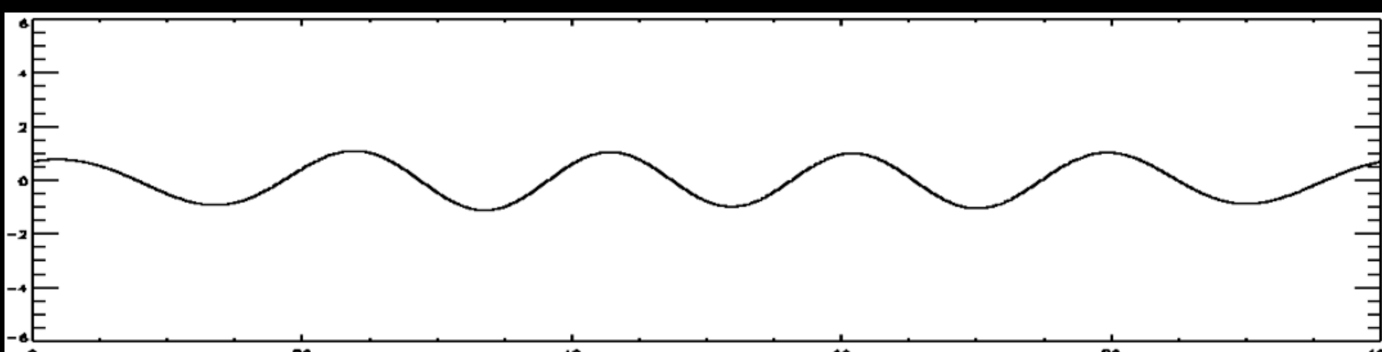
Basic Example



Original signal
Noisy signal

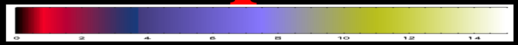


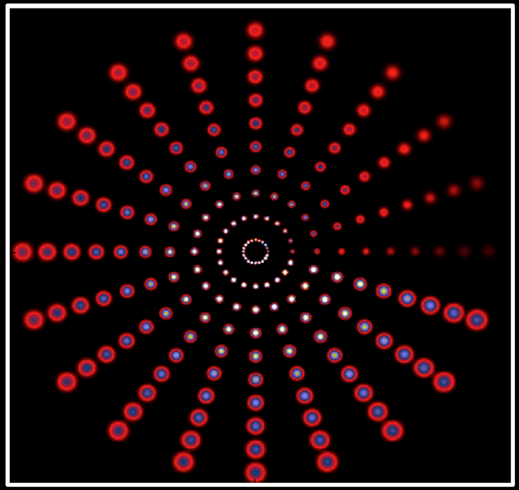
Fourier Transform



Filtered Signal

Adapted Representations





Test image 1

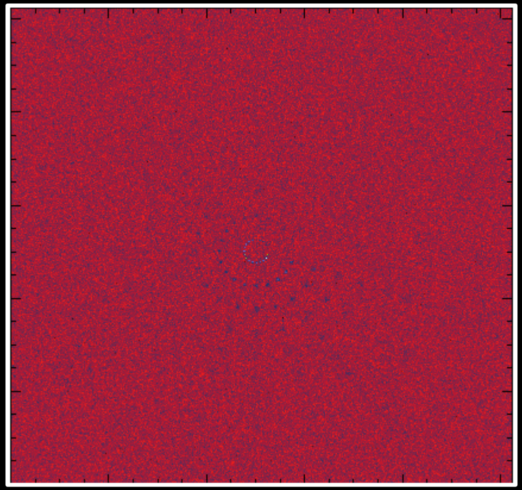
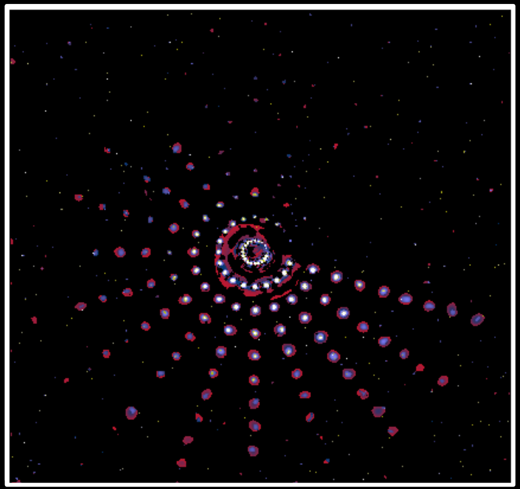
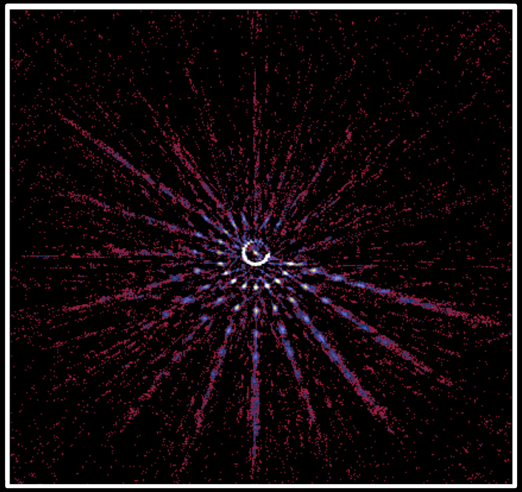


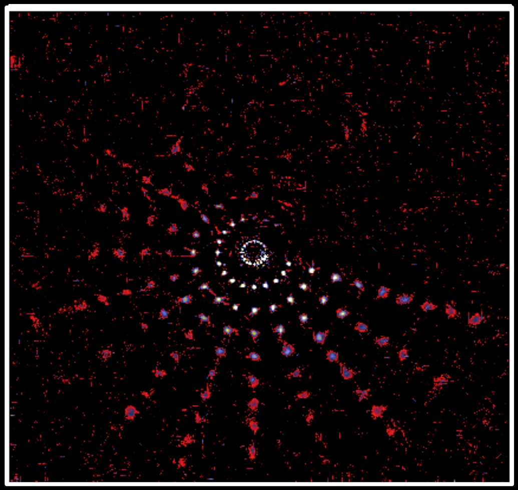
Image test 1 + noise



Wavelet filtering



Ridgelet filtering



Curvelet filtering

Adapted Representations

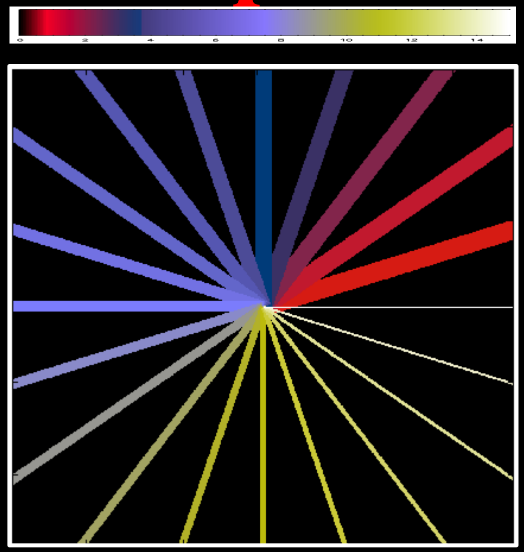


Image test 2

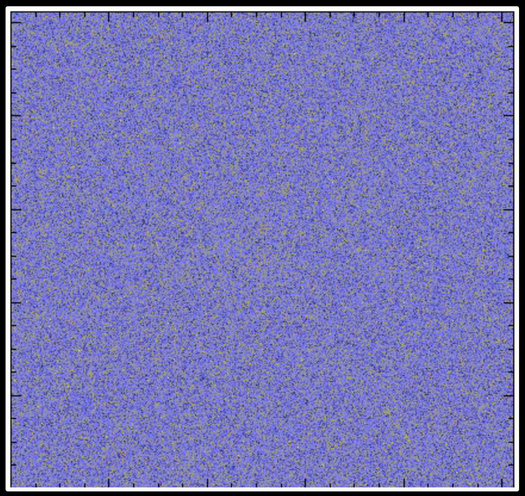
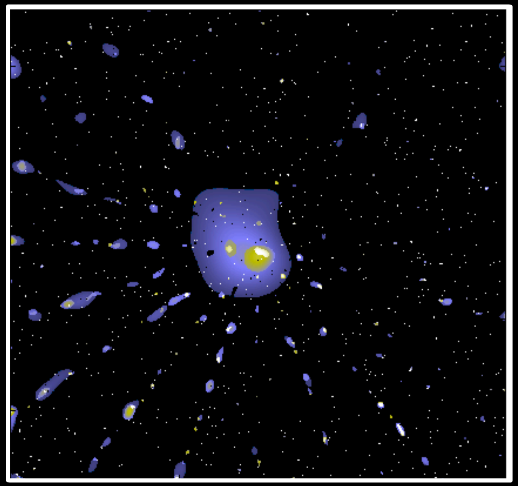
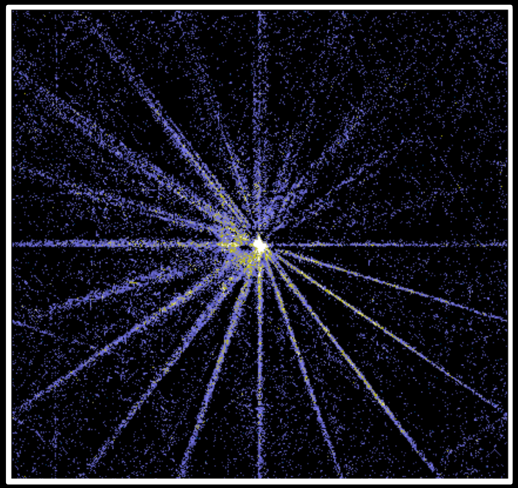


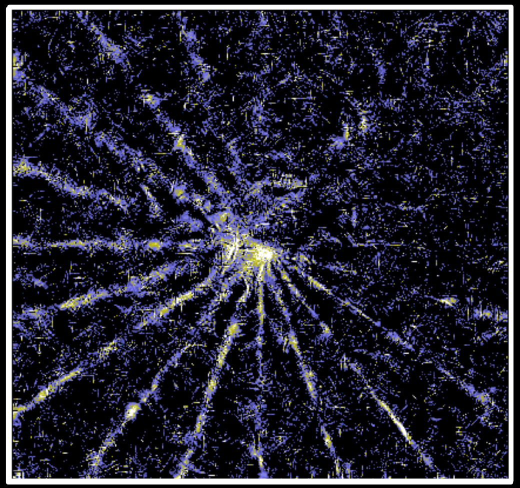
Image test 2 + noise



Wavelet filtering

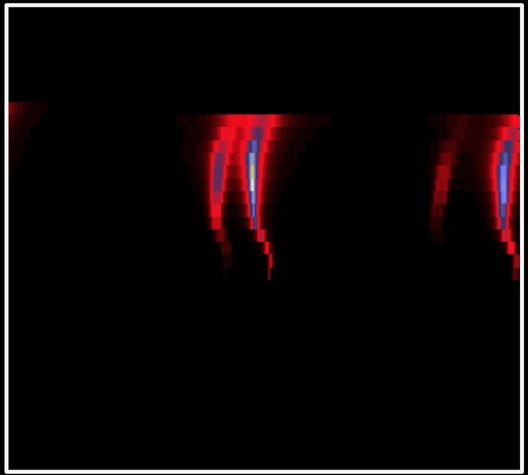


Ridgelet filtering

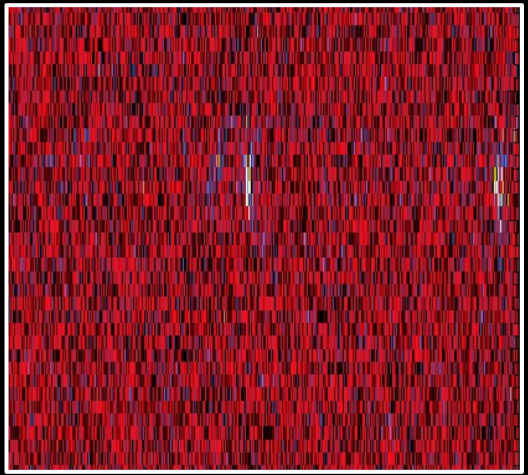


Curvelet filtering

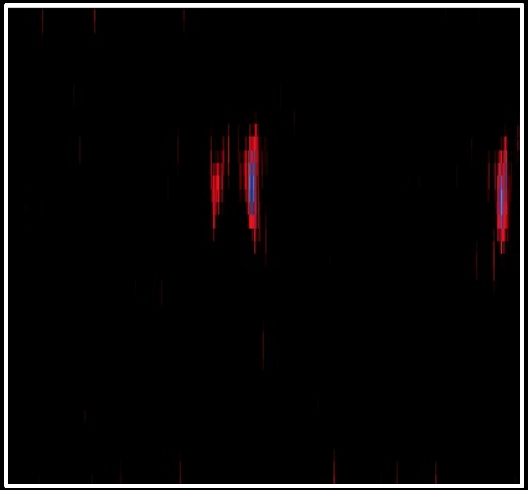
« Echelle » diagram Filtering



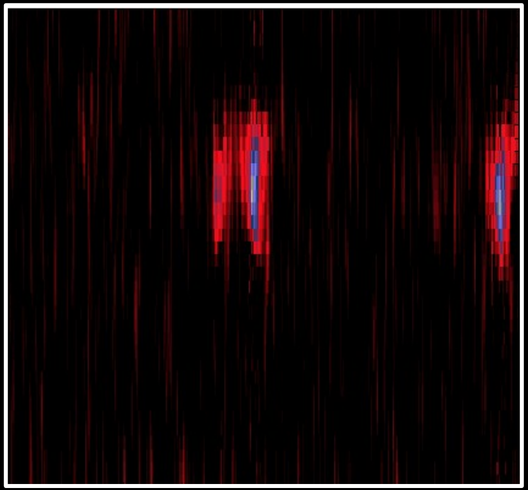
Theoritical "échelle" diagram



Noisy"échelle" diagram



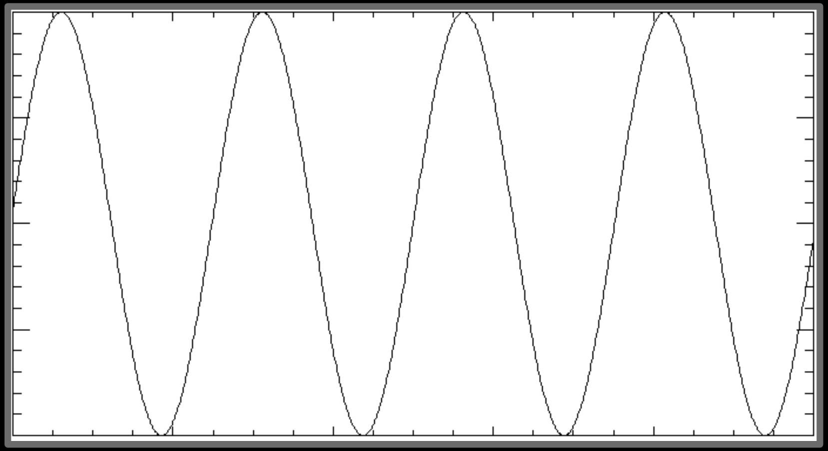
"échelle" diagram filtered by a bi-orthogoanal Wavelet Transform



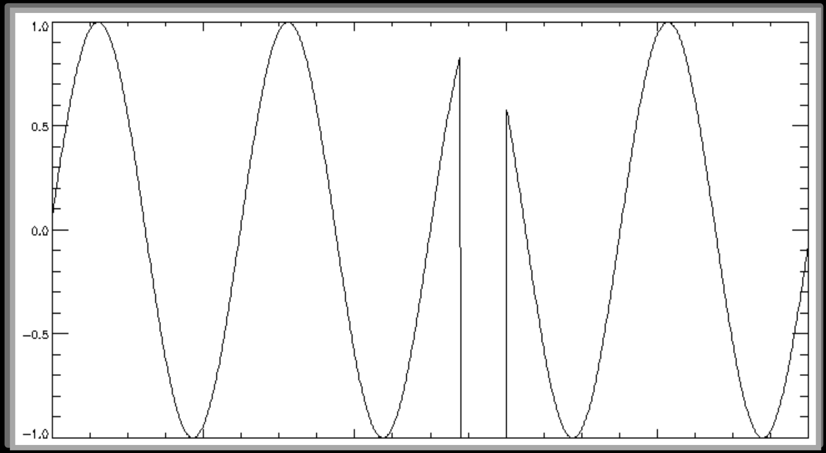
"échelle" diagram filtered by Curvelet Transform

Inpainting based on sparsity

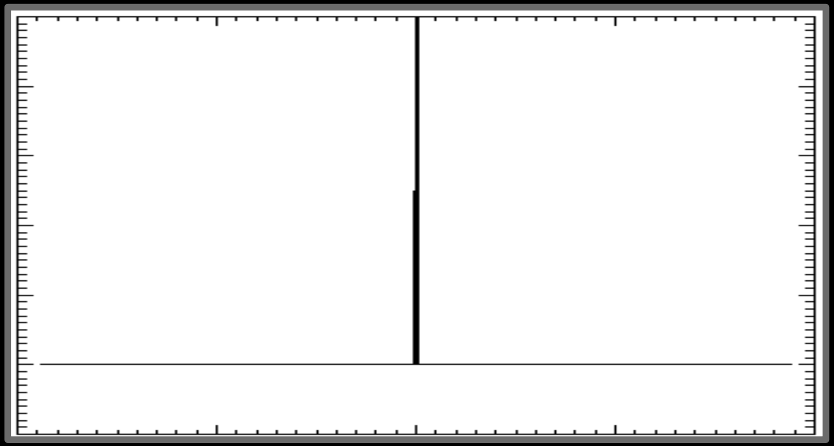
$$\min_{\alpha} ||\alpha||_1 \text{ s.t. } Y = M\Phi\alpha$$



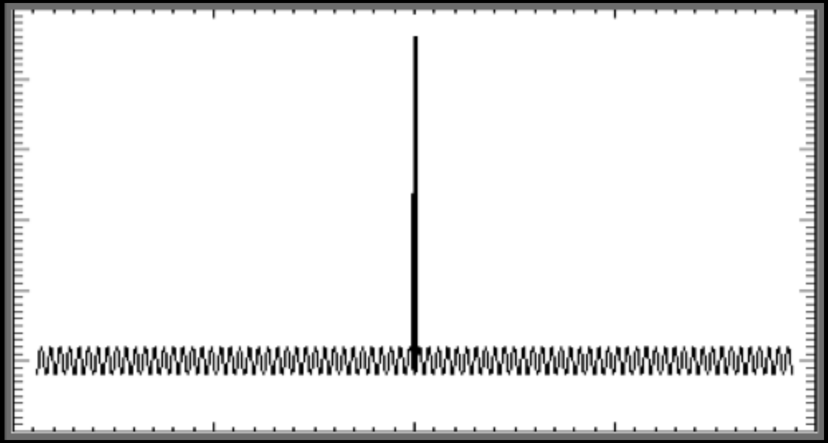
Sine curve



Truncated sine curve



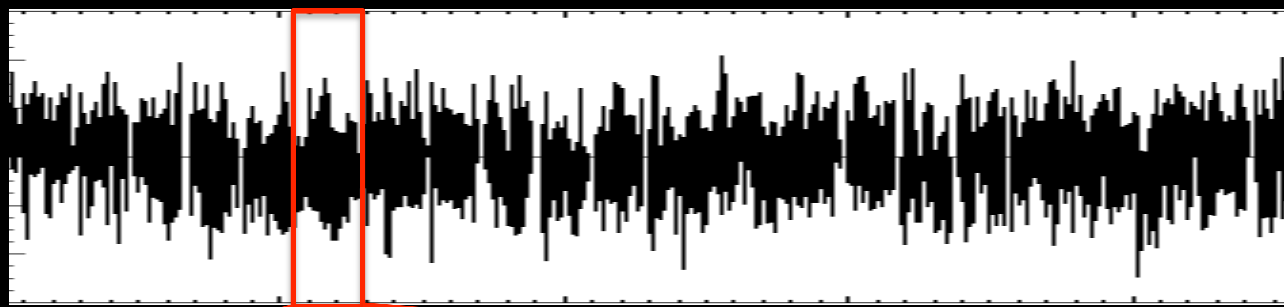
Fourier transform of the sine curve



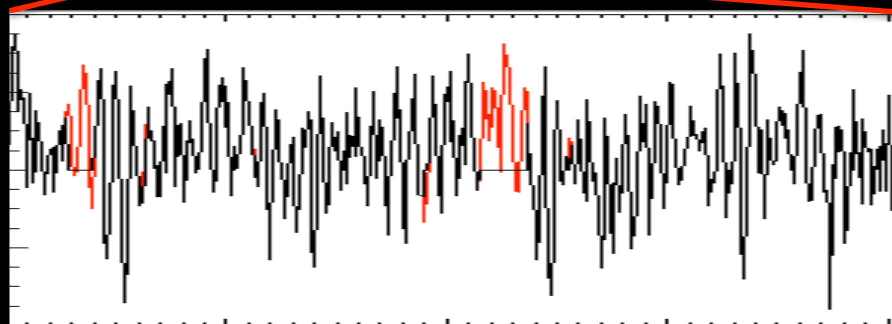
Fourier transform of the truncated sine curve

Inpainting on asteroseismic data

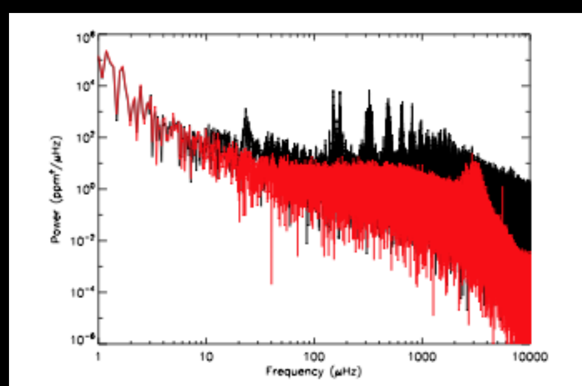
Light curve (time series)



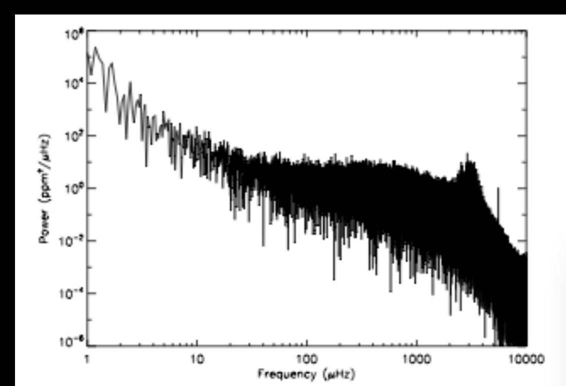
Zoom on the Light curve



Power spectrum

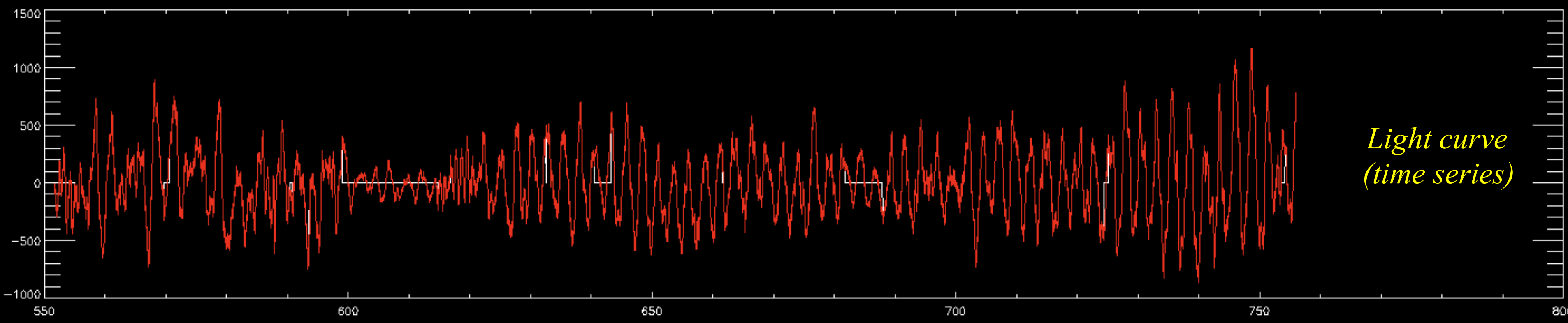


Original (red) and masked (black) data

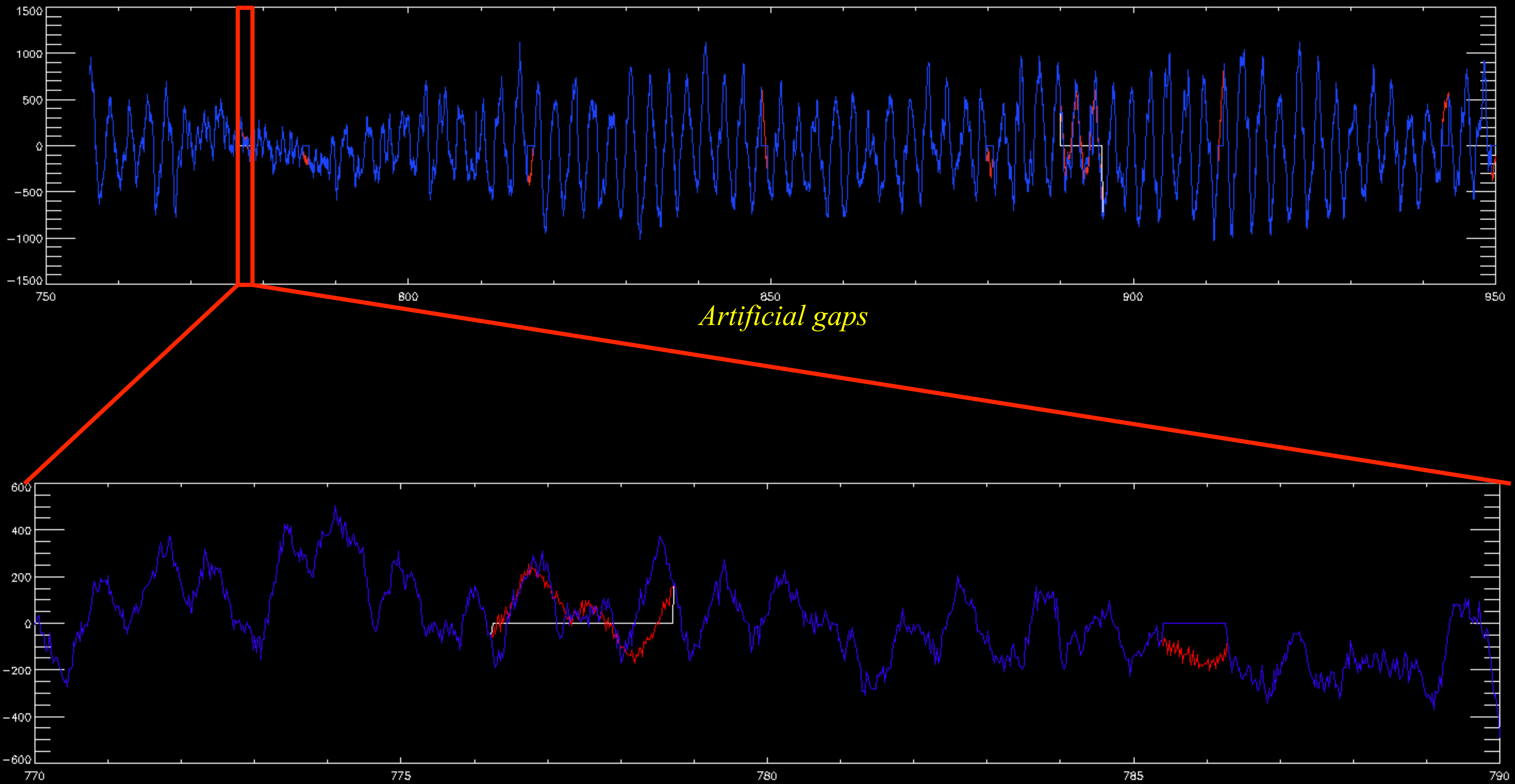


Inpainted data (black)

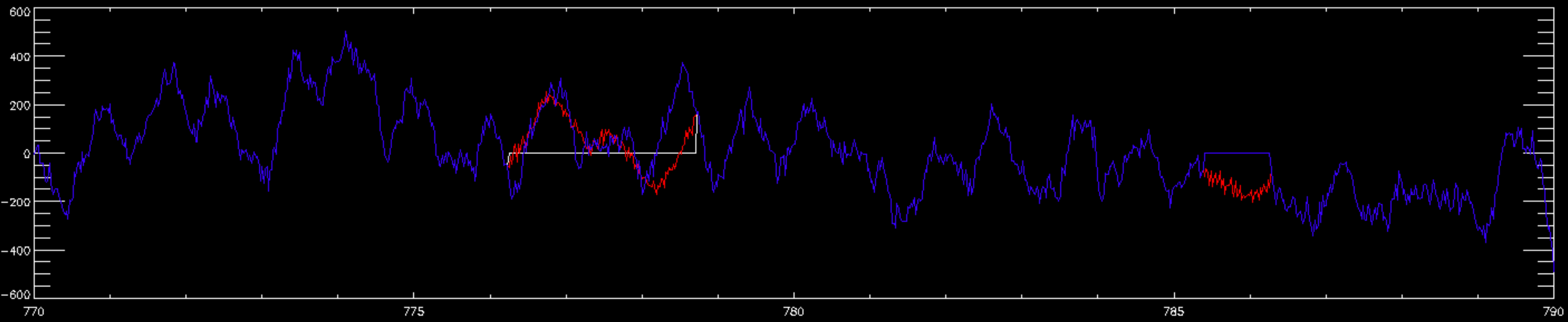
Application to *Kepler* data



Application to *Kepler* data



Application to *Kepler* data



Artificial gaps