Application of multiscale methods to Weak Lensing: Reconstruction and Analysis of Dark Matter mass maps

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Outline

0- Introduction

- Introduction to weak lensing
- Weak lensing data processing line

1- Mask interpolation using Inpainting

- Introduction to the missing data problem
- Inpainting method to fill-in the gaps (FASTLens)
- Some results

2 - Weak Lensing mass map filtering

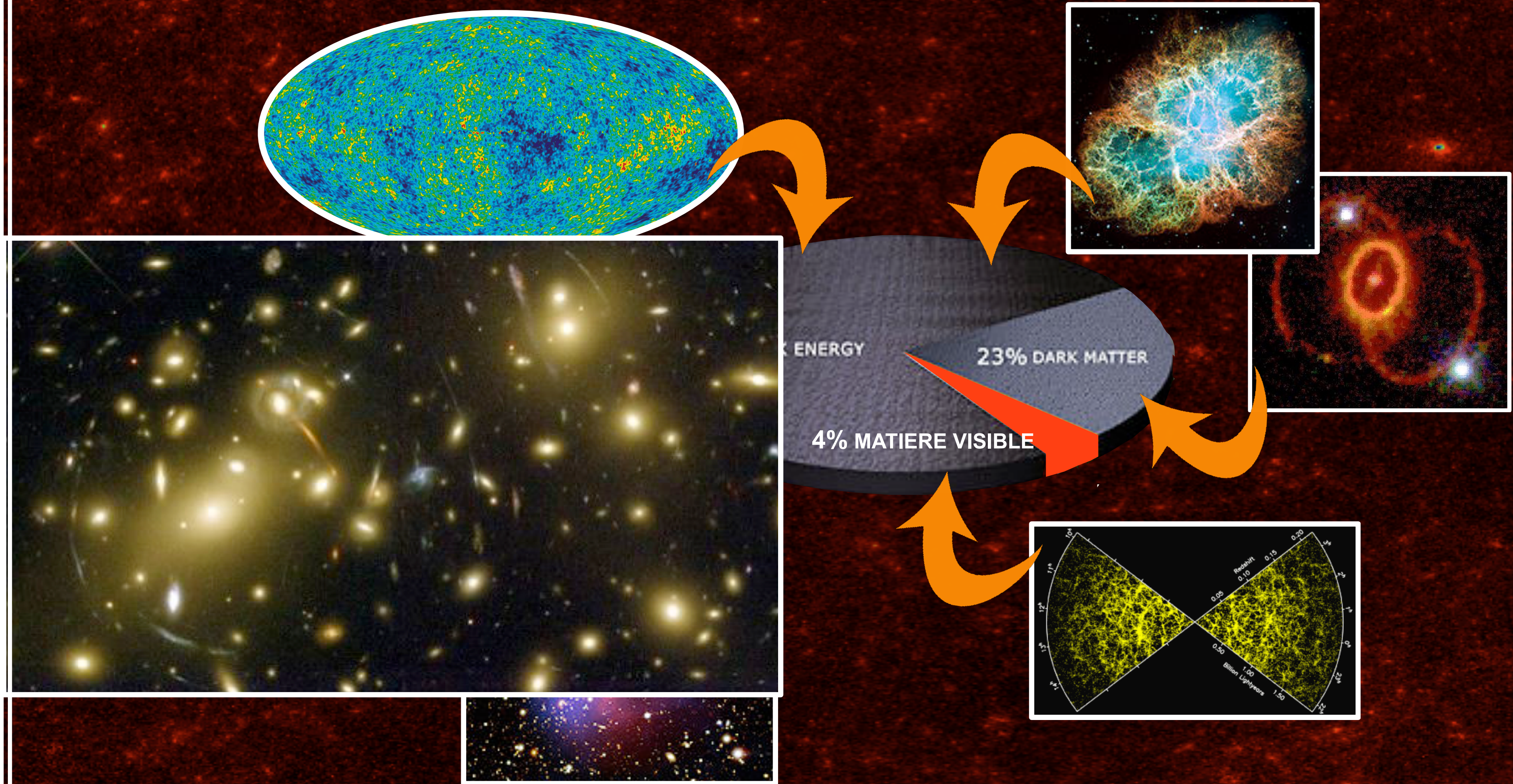
- Introduction to the mass map reconstruction problem
- MRLENS filtering
- Results and applications

3 – Cosmological model constraints with Weak Lensing

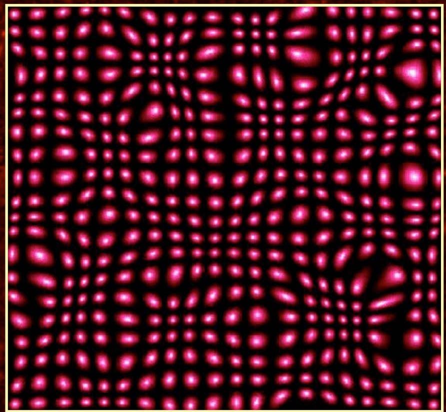
- Weak Lensing statistics
- Conclusions

From observations to cosmological model

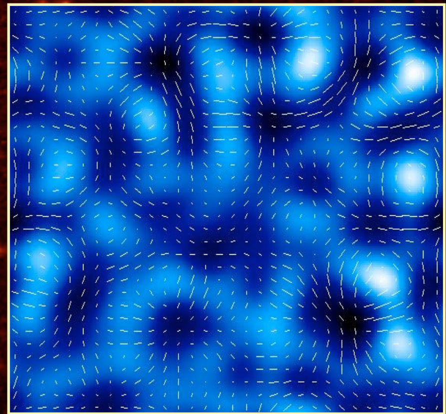
$$\mathcal{M}(\Omega_M, \Omega_\Lambda, \Omega_b, \sigma_8, \ldots)$$



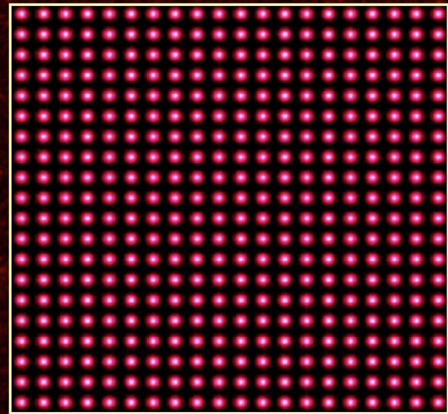
Weak Gravitational Lensing



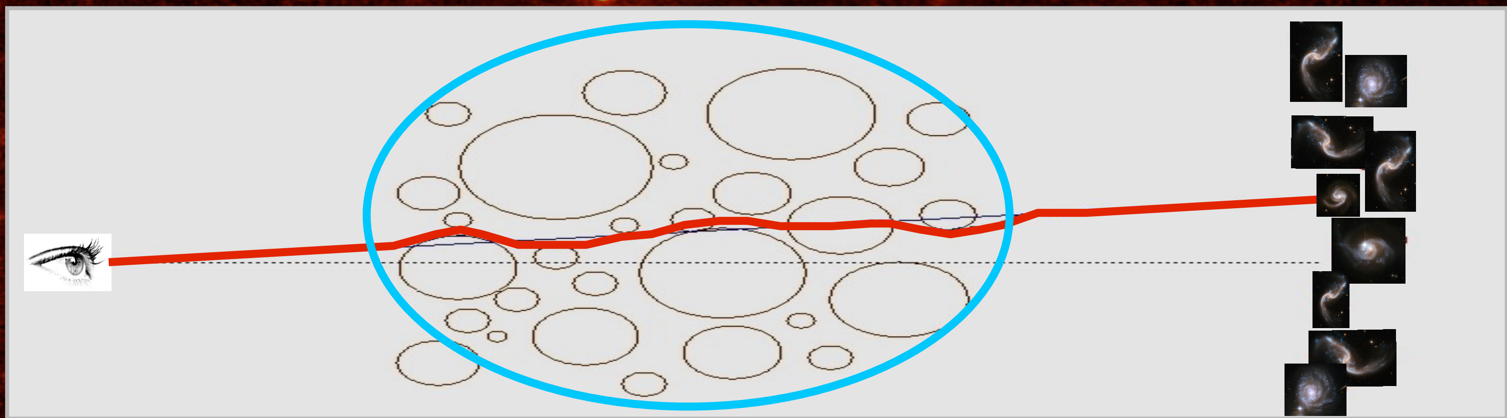
OBSERVER



GRAVITATIONAL LENS



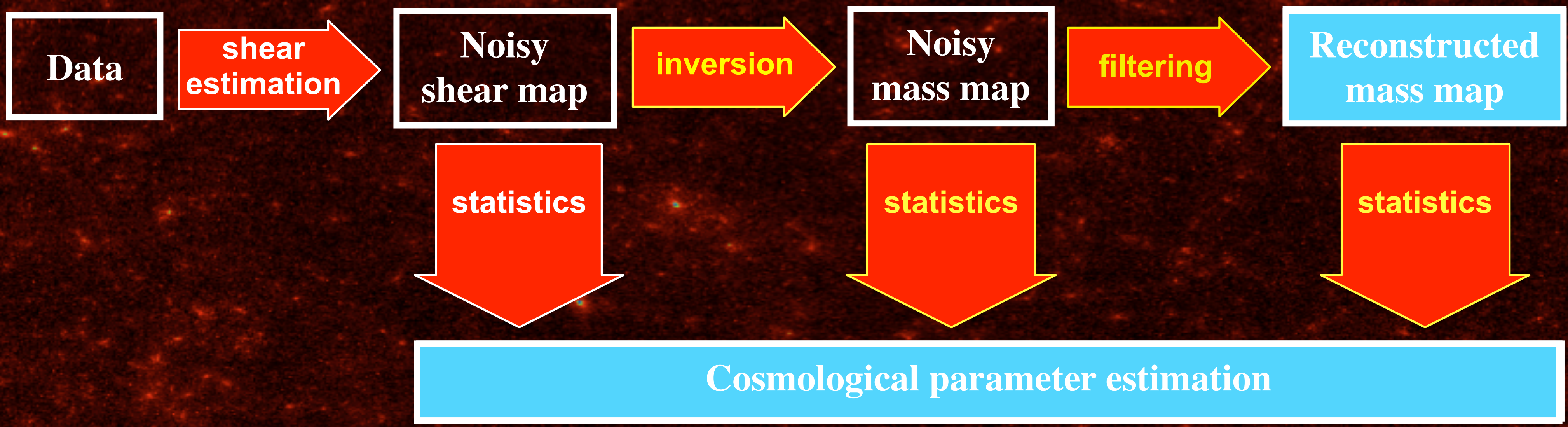
BACKGROUND GALAXIES

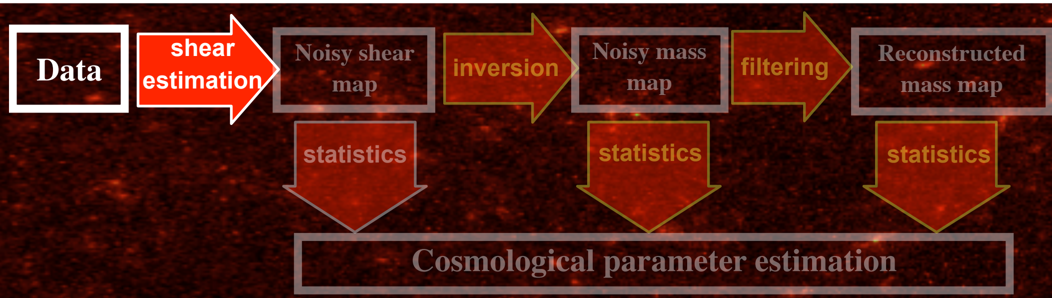


GRAVITATIONAL LENS

= GOAL =

Constrain cosmological parameters from weak lensing data

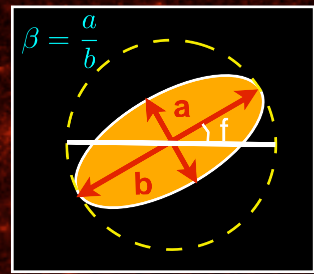




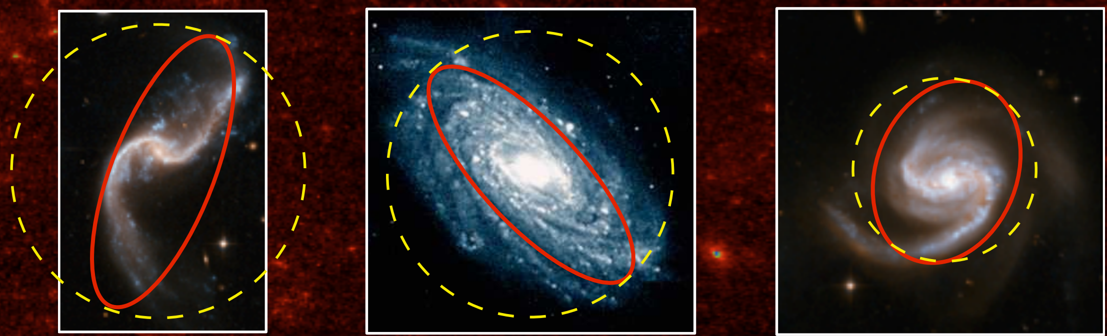
Shear estimation

✓ Shear estimation on each galaxy of the field

⇨ Ellipticity must be measured : $\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = \frac{1-\beta}{1+\beta} \begin{pmatrix} \cos 2\phi \\ \sin 2\phi \end{pmatrix}$



✓ Galaxies have an intrinsic ellipticipty

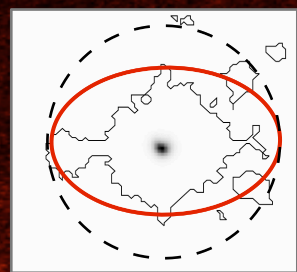


⇨ ellipticity must be averaged over several nearby galaxies :

$$< \epsilon_i > \approx \gamma_i$$

✓ Galaxies are convolved by an asymetric PSF

⇨ PSF have to be estimated and deconvolved



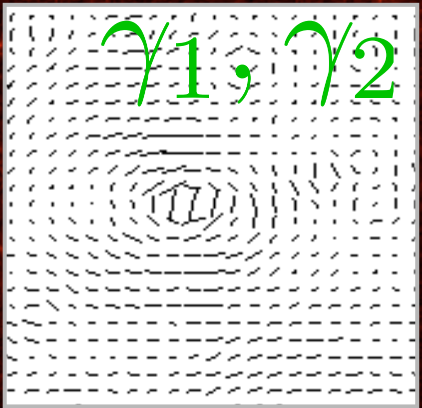
Data → shear estimation → Noisy shear map → inversion → Noisy mass map → filtering → Reconstructed mass map

statistics ↓ statistics ↓ statistics ↓

Cosmological parameter estimation

Inversion equations

SIMULATED SHEAR MAP



$$\gamma_1 = \frac{1}{2}\left(\partial_1^2 - \partial_2^2\right)\psi$$

$$\gamma_2 = \partial_1 \partial_2 \psi$$

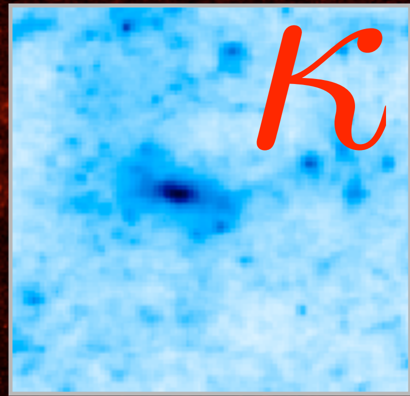
LENSING POTENTIAL

$$\psi$$

$$\frac{1}{2}\left(\partial_1^2 + \partial_2^2\right)\psi = \kappa$$

SIMULATED MASS MAP

(Vale & White, 2003)

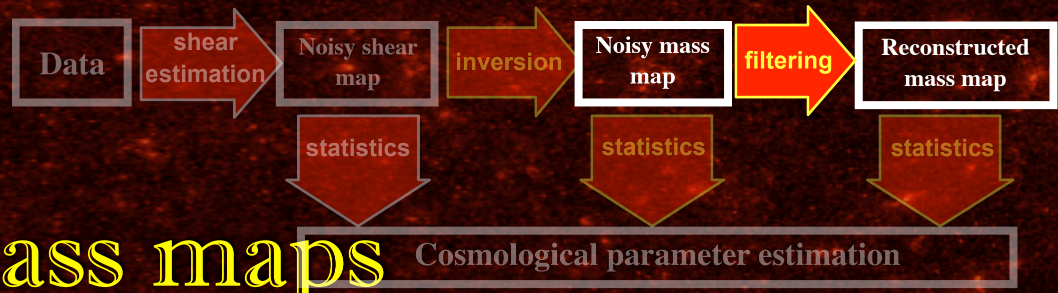


From mass to shear: $\gamma_i = \hat{P}_i \kappa$

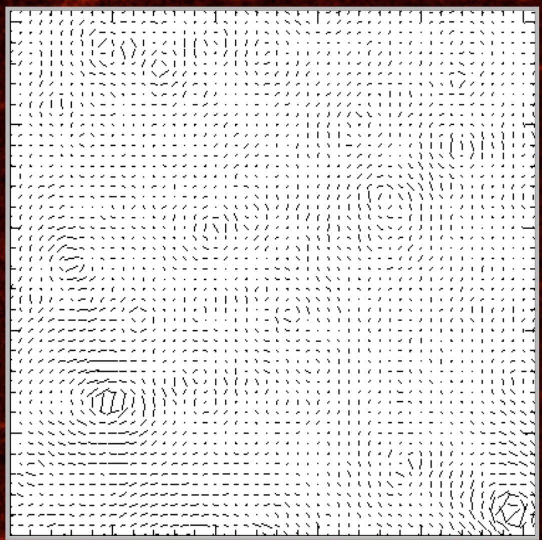
From shear to mass: $\kappa = \hat{P}_1 \gamma_1 + \hat{P}_2 \gamma_2$

$$\hat{P}_1(k) = \frac{k_1^2 - k_2^2}{k^2}$$

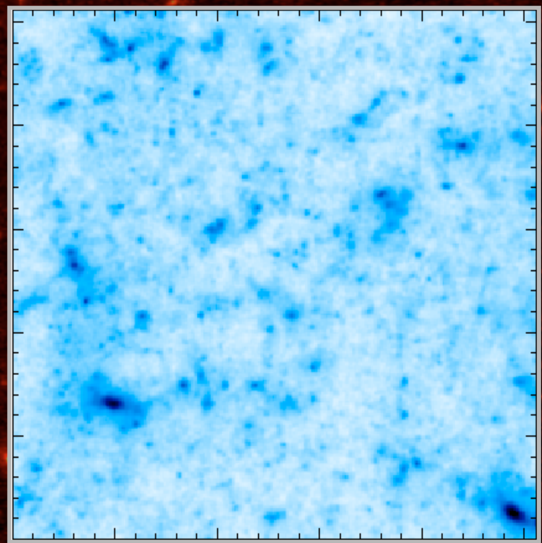
$$\hat{P}_2(k) = \frac{2 k_1 k_2}{k^2}$$



Noisy mass maps



SHEAR MAP



ORIGINAL MASS MAP
(Vale & White, 2003)

MASS MAP
(SPACE OBSERVATIONS)

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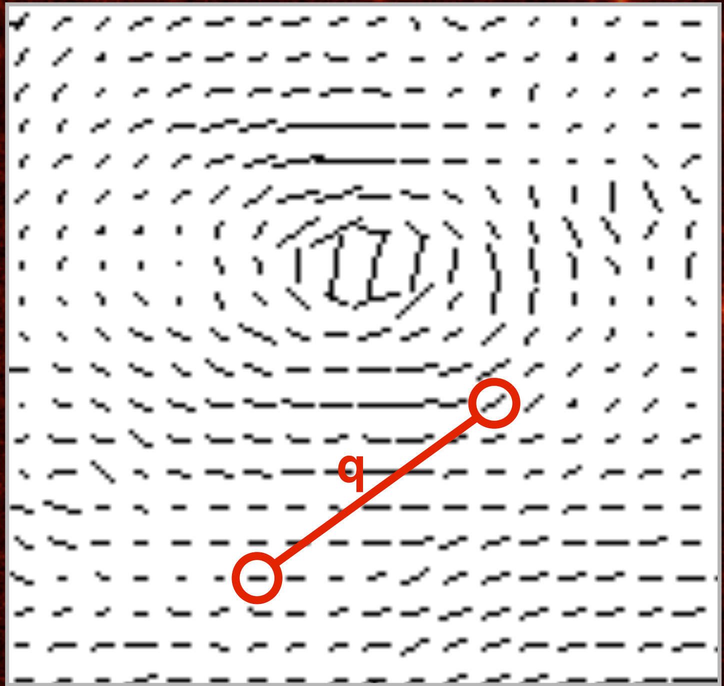
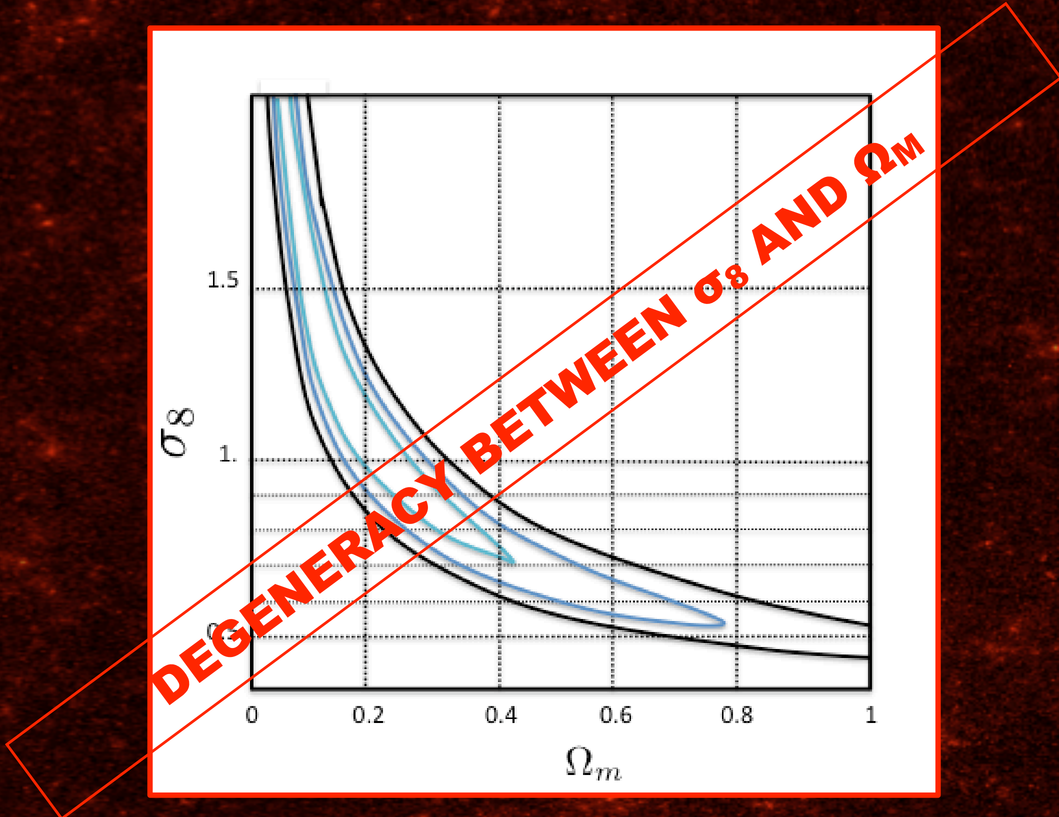
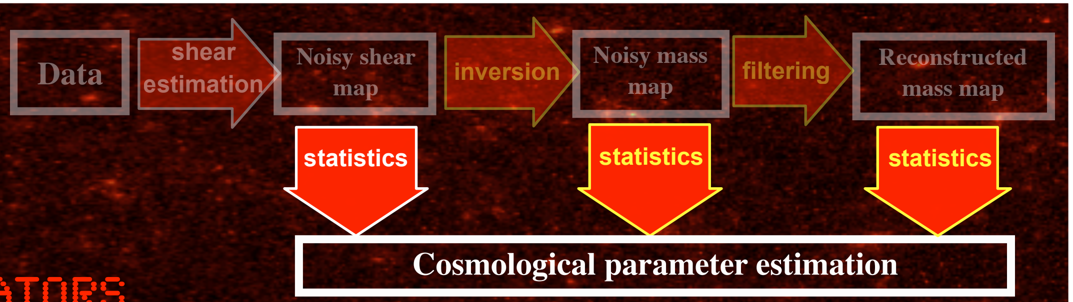
statistics / statistics / statistics → Cosmological parameter estimation

Statistic estimation



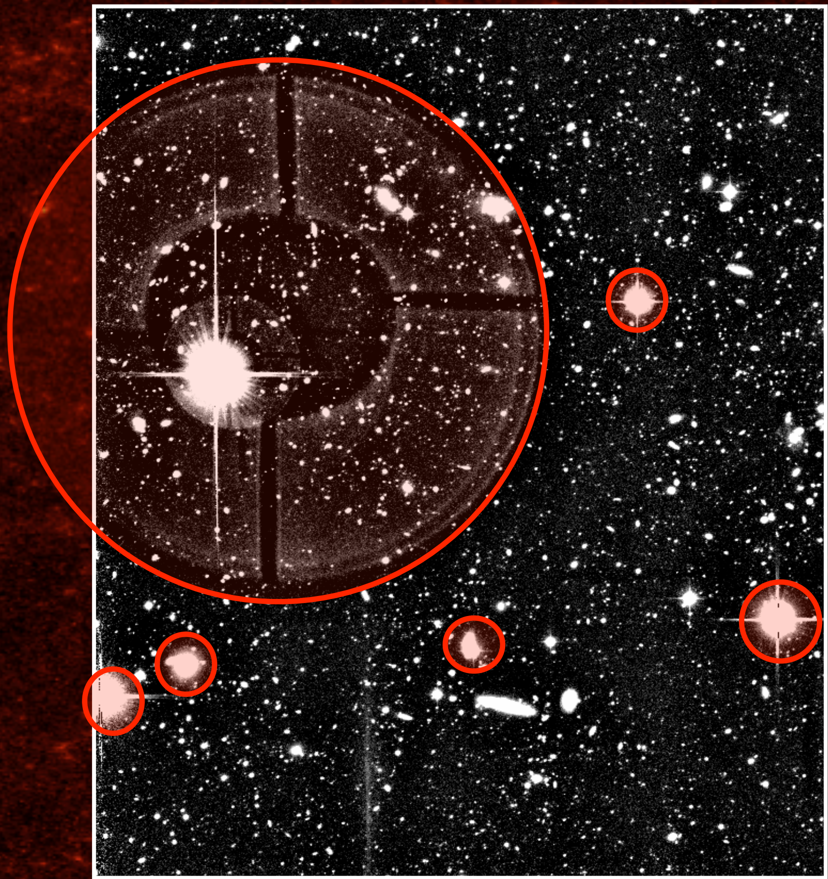


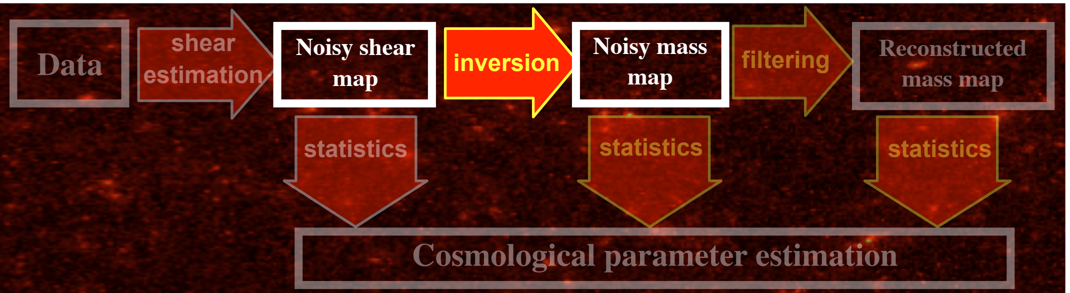


Statistic estimation

GAUSSIAN ESTIMATORS	NON-GAUSSIAN ESTIMATORS	
TWO-POINT STATISTICS	THREE-POINT STATISTICS	FOUR-POINT STATISTICS
$\sigma^2 = \sum_1^N (\kappa_i - \bar{\kappa})^2$ VARIANCE	$S = \frac{\sum_1^N (\kappa_i - \bar{\kappa})^3}{N\sigma^3}$ SKEWNESS	$K = \frac{\sum_1^N (\kappa_i - \bar{\kappa})^4}{N\sigma^4} - 3$ KURTOSIS
$\xi_{i,j} =< \kappa(\theta_i)\kappa(\theta_j) >$ TWO-POINT CORRELATION FUNCTION	$\xi_{i,j,k} =< \kappa(\theta_i)\kappa(\theta_j)\kappa(\theta_k) >$ THREE-POINT CORRELATION FUNCTION	$\xi_{i,j,k,l} =< \kappa(\theta_i)\kappa(\theta_j)\kappa(\theta_k)\kappa(\theta_l) >$ FOUR-POINT CORRELATION FUNCTION
$P_\kappa =< \hat{\kappa}(\theta_i)\hat{\kappa}(\theta_j) >$ POWER SPECTRUM	$B_\kappa =< \hat{\kappa}(\theta_i)\hat{\kappa}(\theta_j)\hat{\kappa}(\theta_k) >$ BISPECTRUM	$T_\kappa =< \hat{\kappa}(\theta_i)\hat{\kappa}(\theta_j)\hat{\kappa}(\theta_k)\hat{\kappa}(\theta_l) >$ TRISPECTRUM

Weak lensing missing data





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Statistical estimation with missing data

- ✓ Estimation of N-point correlation functions in direct space by avoiding the points falling in gaps
  - ✓ pros: unbiased by missing data
  - ✓ cons: time comsuming
- ✓ Estimation of the power spectrum in Fourier space by applying a mask correction
  - ✓ pros: fast estimation with FFT
  - ✓ cons:
    - ✓ stability depending on the shape of the mask
    - ✓ estimation of the mask correction can be long

Weak lensing inpainting algorithm

$\gamma_i^{obs} \rightarrow$ $\min_\kappa \|\Phi^t \kappa\|_{l_0}$ subject to $\sum_i \|\gamma_i^{obs} - M(P_i * \kappa)\|_{l_2}^2 \le \varepsilon$ $\rightarrow \kappa$

↑ Physical priors

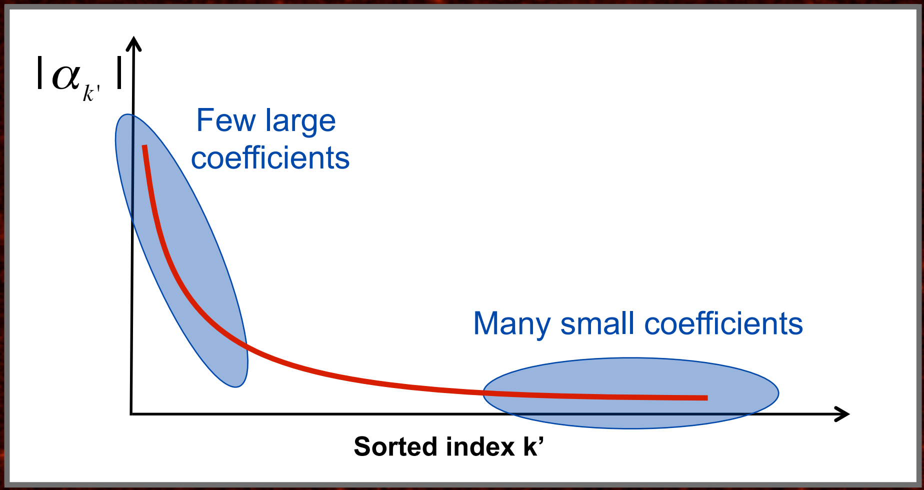
$$\gamma_i^{obs} = M.\gamma_i$$

$$\kappa = P_1 * \gamma_1 + P_2 * \gamma_2$$

Φ^t is the DCT

What is sparsity ?

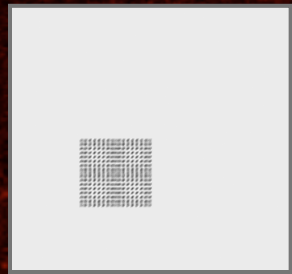
A signal S is sparse in a basis Φ if most of the coefficients α are equal to zero or closed to zero: $\min_S ||\phi^t S||_0^2$



Looking for Adapted representations

✓ Local DCT:

- Stationary textures
- Locally oscillatory



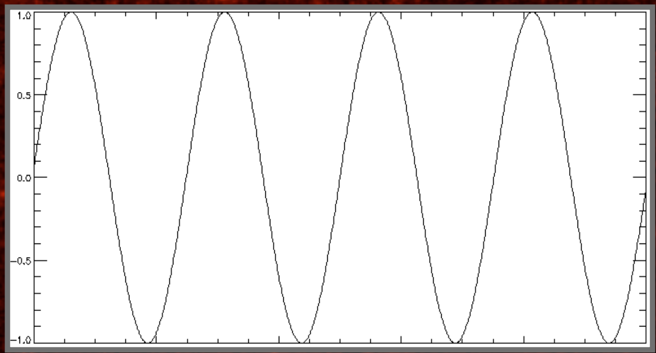
✓ Wavelet transform:

- Piecewise smooth
- Isotropic structures

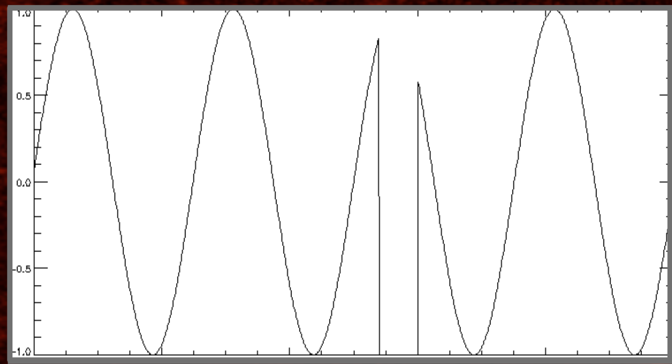
✓ Curvelet transform:

- Piecewise smooth
- Edge structures

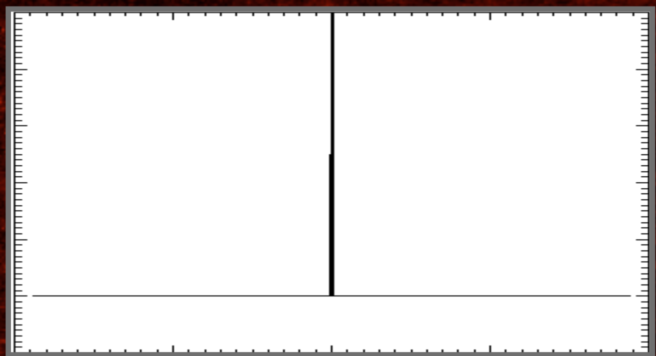
Inpainting based on sparse representation of data



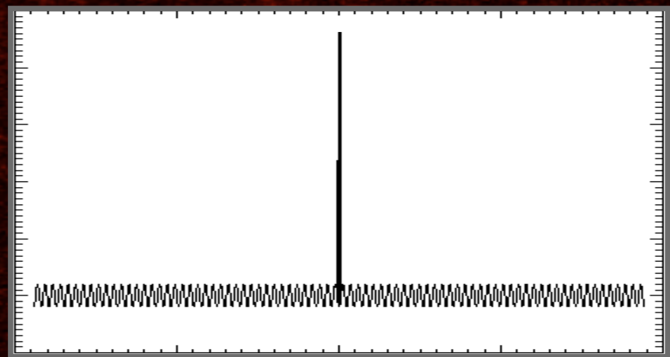
SINE CURVE



TRUNCATED SINE CURVE

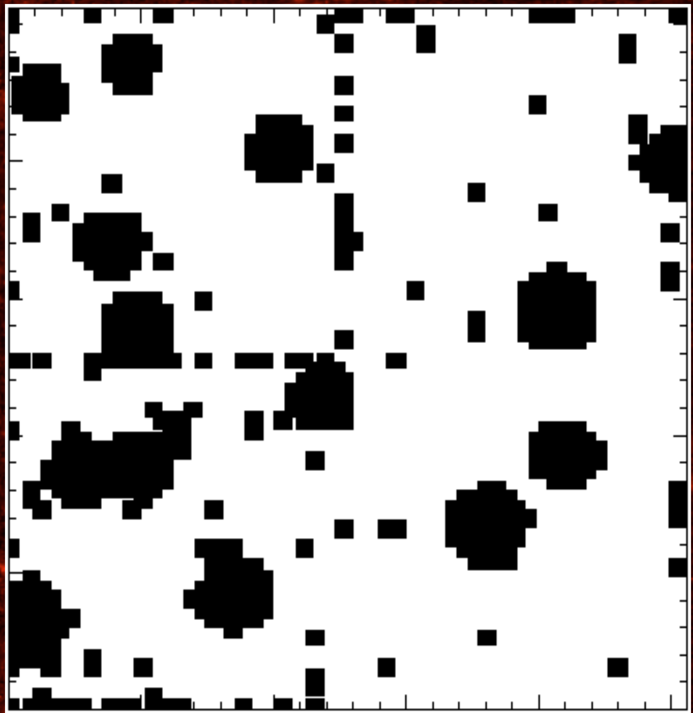


TF OF A SINE CURVE

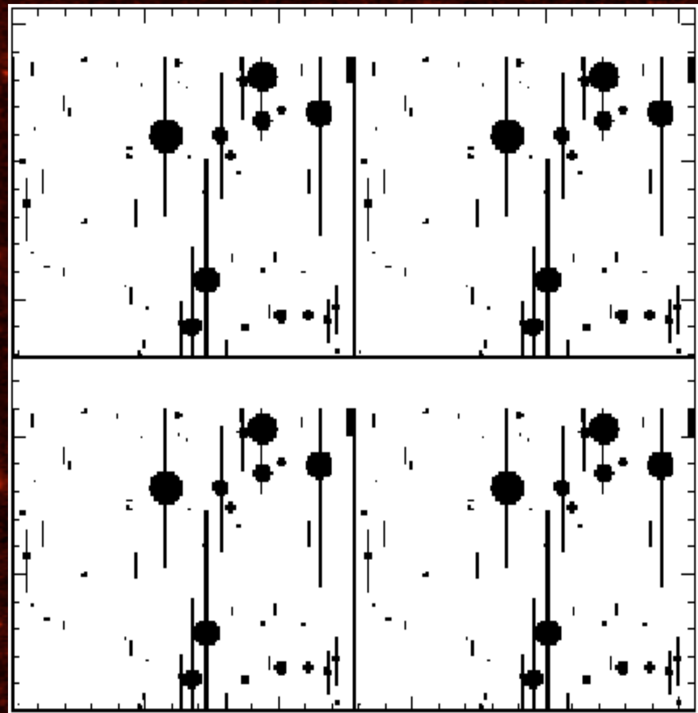


TF OF A TRUNCATED SINE CURVE

Masked masks



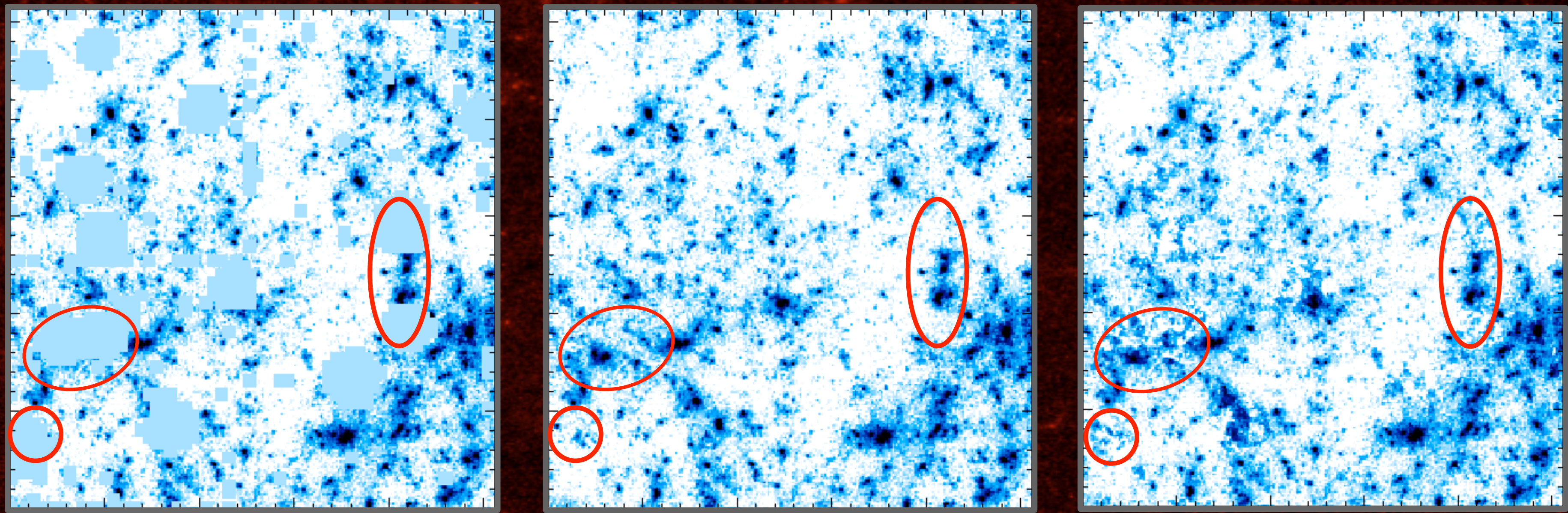
Mask pattern of CFHTLS survey on 1° x 1° field (courtesy J. Berge)



Mask pattern of Subaru survey on 1° x 1° field

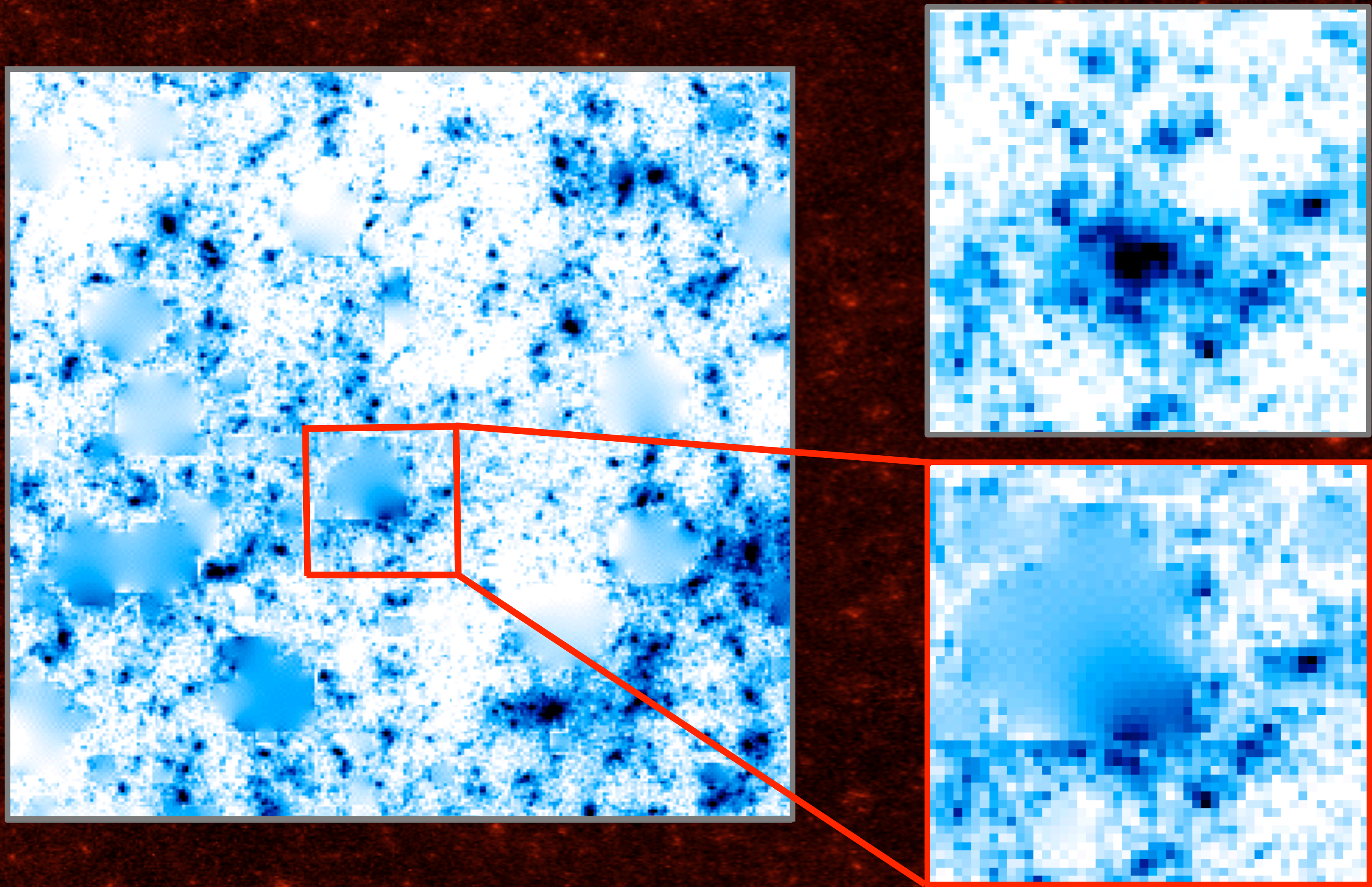
Inpainting on simulated weak lensing data

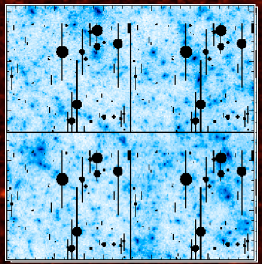
Pires et al 2008a submitted to MNRAS



WHICH IMAGE IS THE ORIGINAL ONE ?

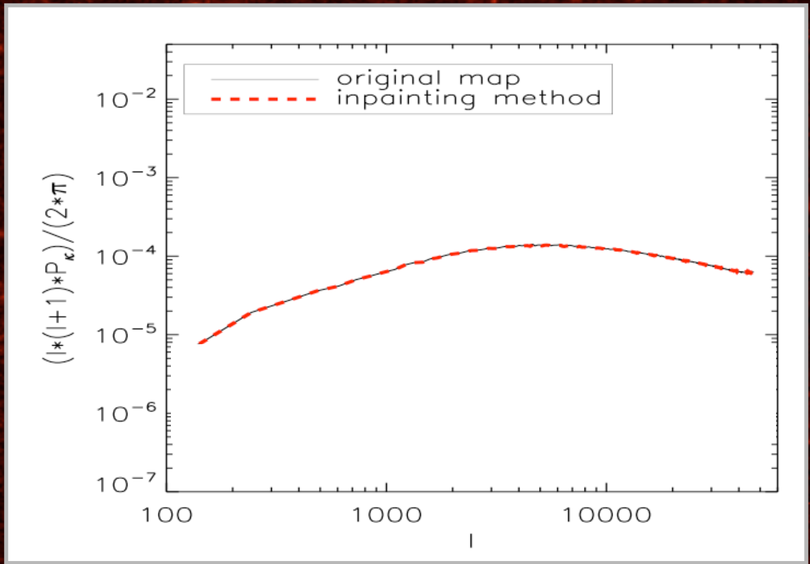
Inpainting on simulated weak lensing data





Power spectrum estimation

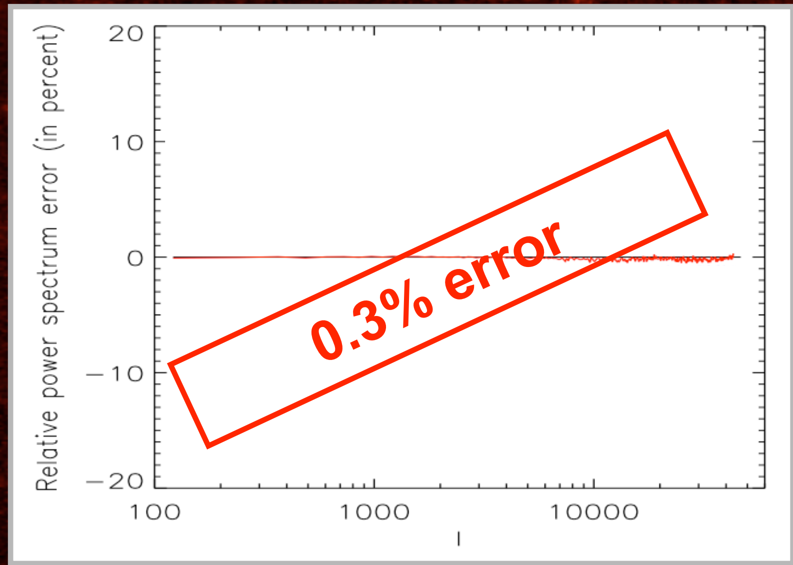
Pires et al 2008a

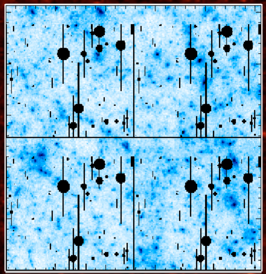


MEAN POWER SPECTRUM COMPUTED FROM

- 100 COMPLETE MASS MAPS (BLACK)
- 100 INPAINTED MAPS (RED).

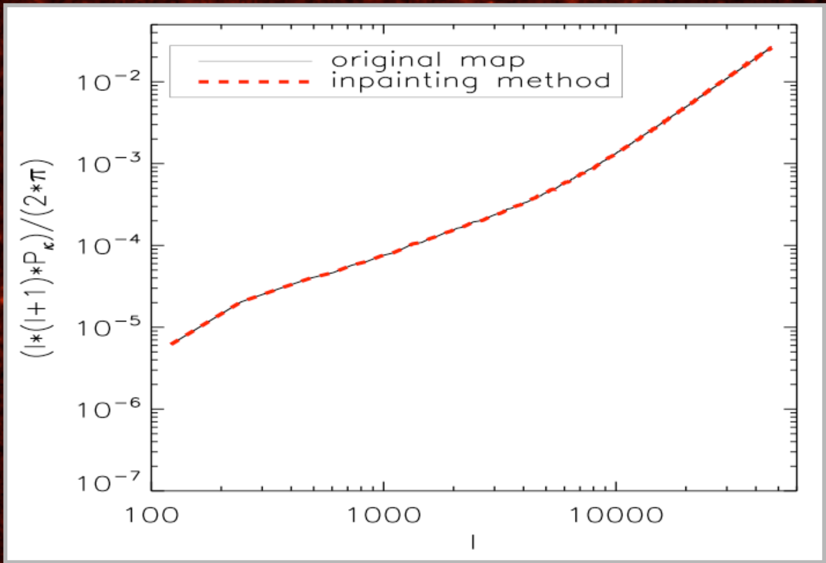
RELATIVE POWER SPECTRUM ERROR, I.E. THE NORMALIZED DIFFERENCE BETWEEN THE TWO UPPER CURVES OF THE LEFT PANEL.





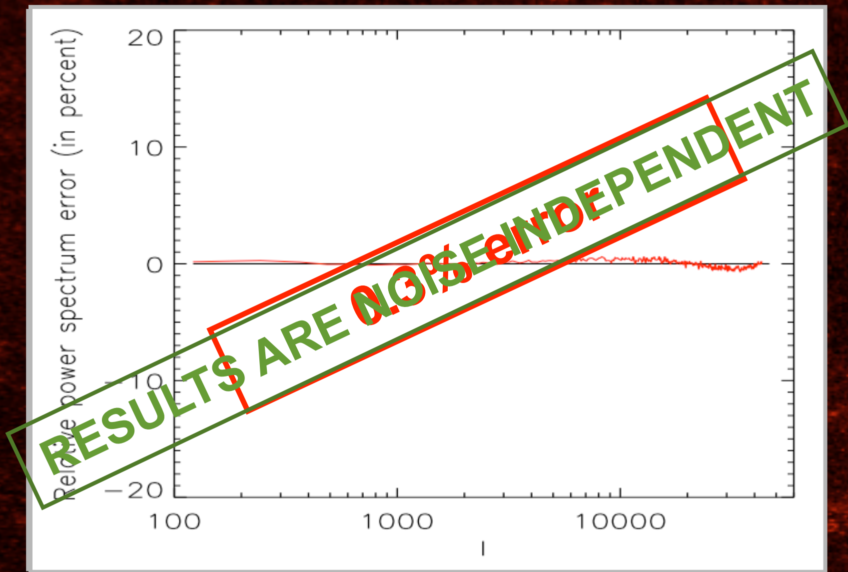
Noisy power spectrum estimation

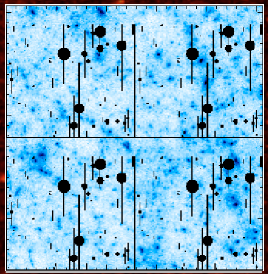
Pires et al 2008a



MEAN NOISY POWER SPECTRUM COMPUTED FROM
- 100 COMPLETE MASS MAPS (BLACK)
- 100 INPAINTED MAPS FROM INCOMPLETE SHEAR MAPS (RED)..

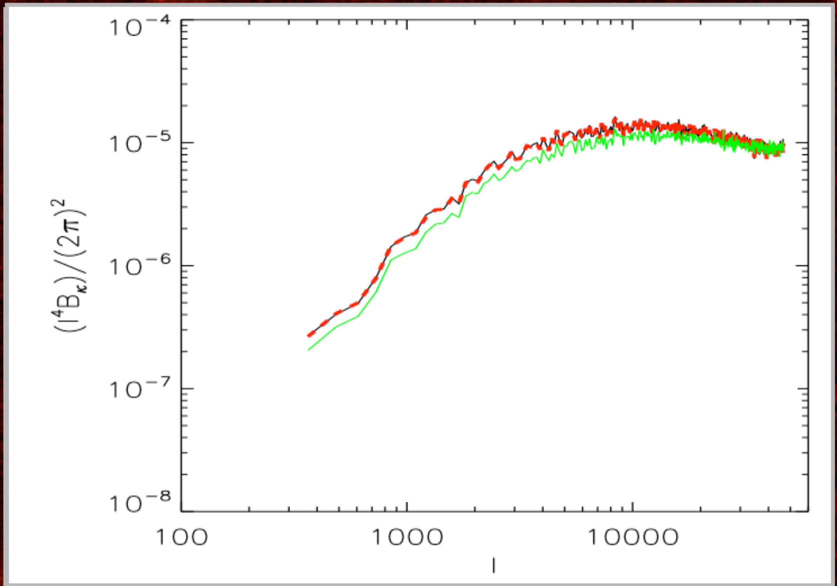
RELATIVE NOISY POWER SPECTRUM ERROR, I.E. THE NORMALIZED DIFFERENCE BETWEEN THE TWO CURVES OF THE LEFT PANEL.





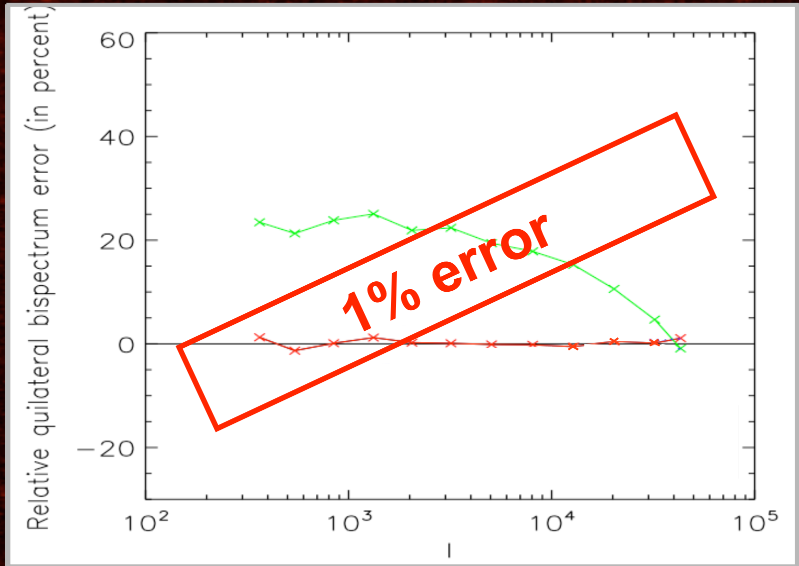
Equilateral bispectrum estimation

FASTLens, Pires et al 2008a



MEAN BISPECTRUM COMPUTED FROM
- 100 COMPLETE MASS MAPS (BLACK)
- 100 INPAINTED RECONSTRUCTED MAPS (RED)
- 100 INCOMPLETE MASS MAPS (GREEN) .

RELATIVE BISPECTRUM ERROR, I.E. THE NORMALIZED DIFFERENCE BETWEEN THE TWO UPPER CURVES OF THE LEFT PANEL.



FASTLens

(Pires et al 2008a)

- ✓ Make faster the estimation of statistics:
  - ✓ The maximum error on power spectrum estimation is 1%
  - ✓ The maximum error on bispectrum estimation is 3%
- ✓ Enables estimation of many statistics:
  - ✓ Power spectrum, Bispectrum, Trispectrum...
  - ✓ Dark matter statistics (cluster abundance, cluster correlations...)
- ✓ Enables filtering

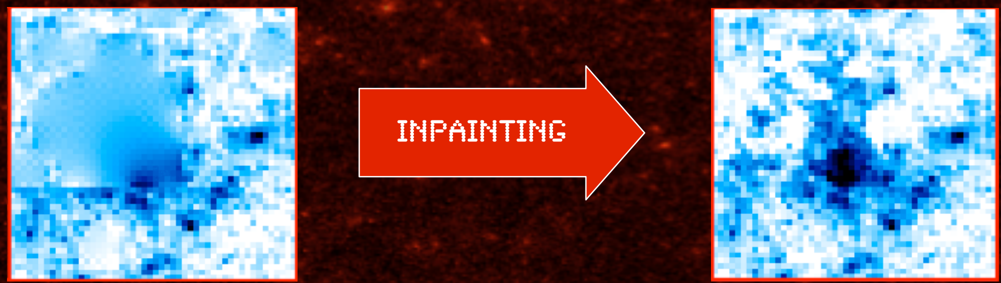
FASTLens

(FAst STatistics for weak Lensing)

http://www-irfu.cea.fr/Ast/fastlens_software.php

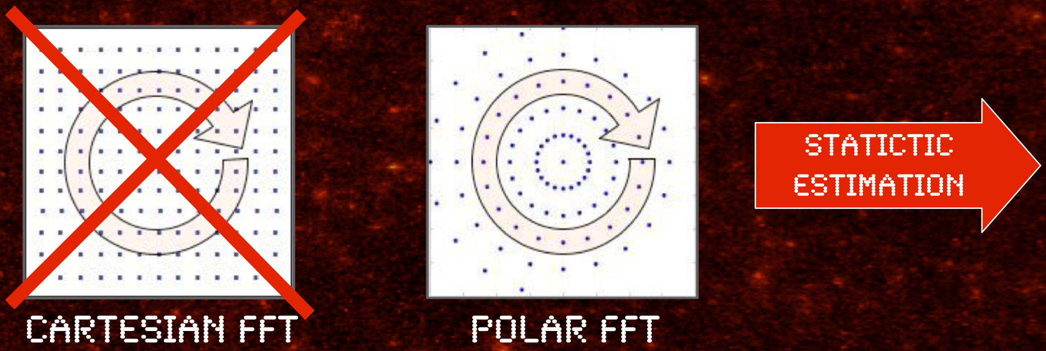
✓ Inpainting method:

- ✓ Estimation of a complete dark matter mass map from incomplete shear maps



✓ Polar FFT code:

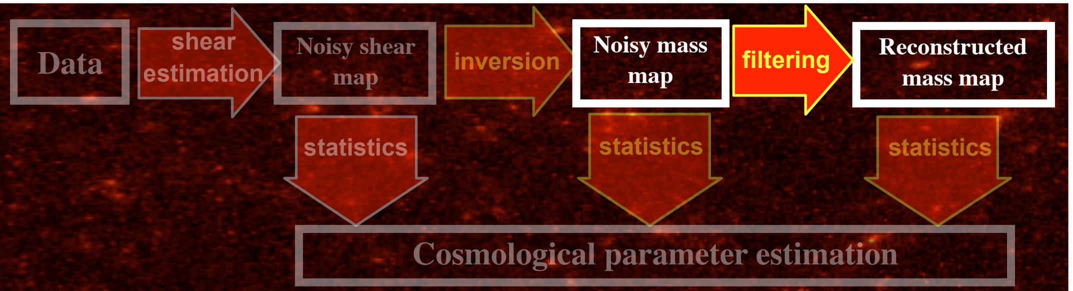
- ✓ Fast and Exact estimation of the power spectrum and the bispetrum



$$P_\kappa =< \hat{\kappa}(\theta_i)\hat{\kappa}(\theta_j) >$$

$$B_\kappa =< \hat{\kappa}(\theta_i)\hat{\kappa}(\theta_j)\hat{\kappa}(\theta_k) >$$

$$\ldots$$



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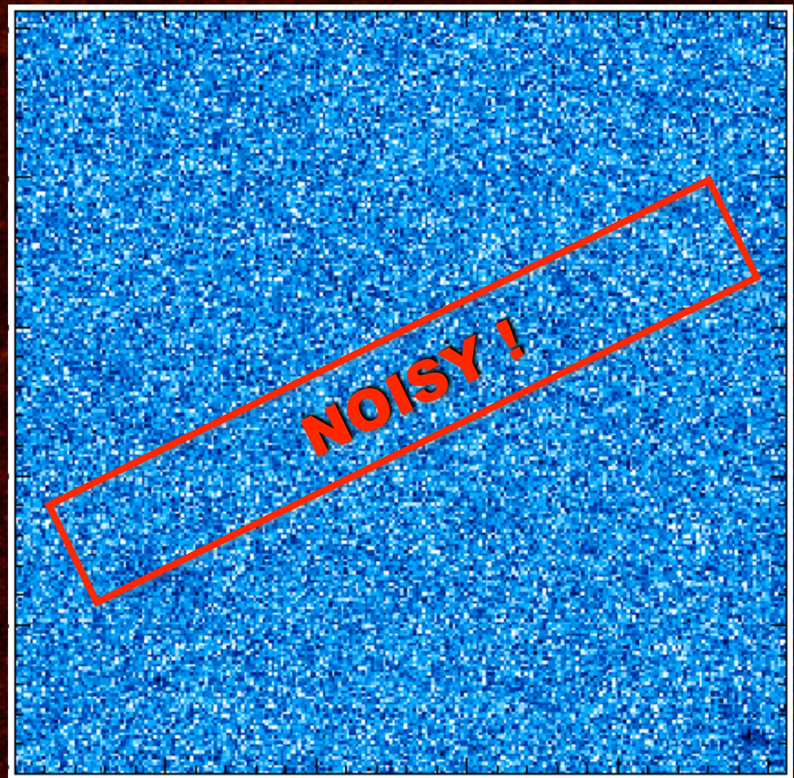
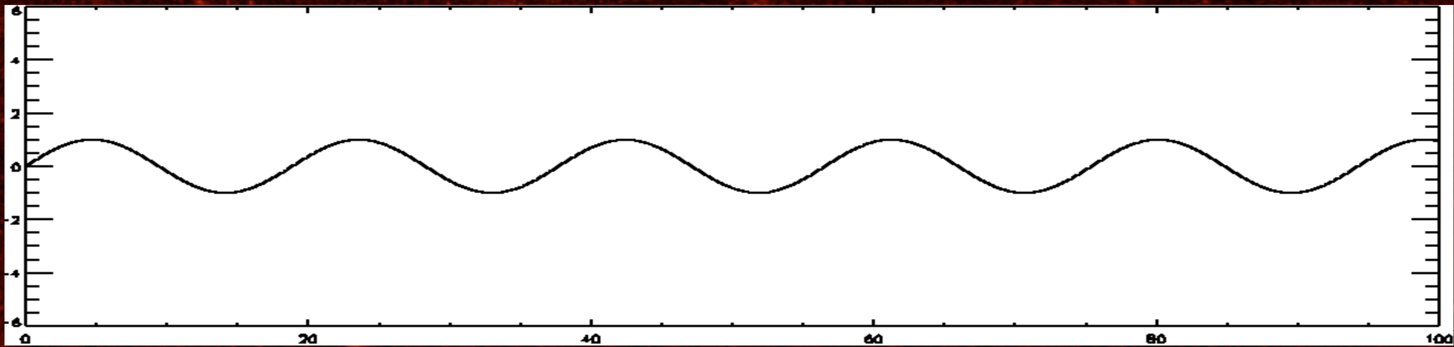
Noisy mass maps



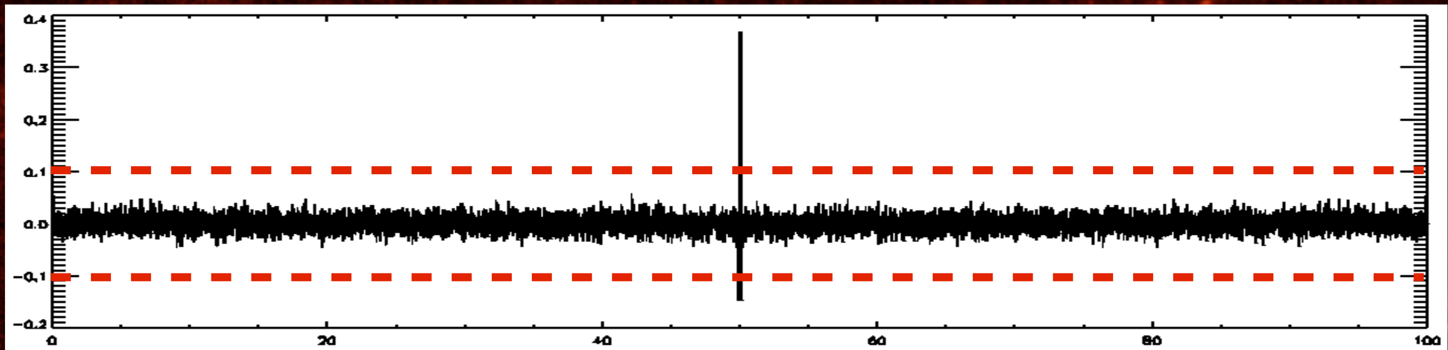
SIMULATED MASS MAP (SPACE OBSERVATIONS)

Noise filtering

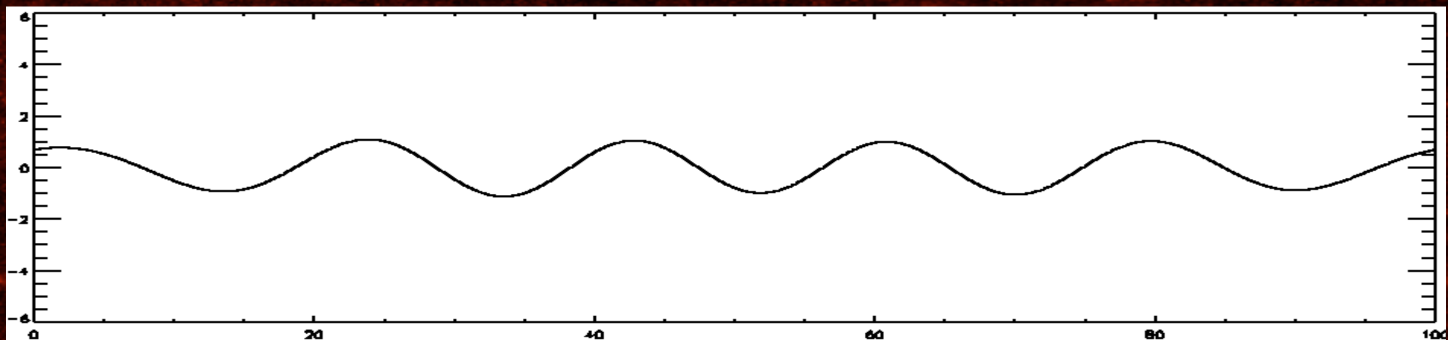


Original Signal

Noisy Signal



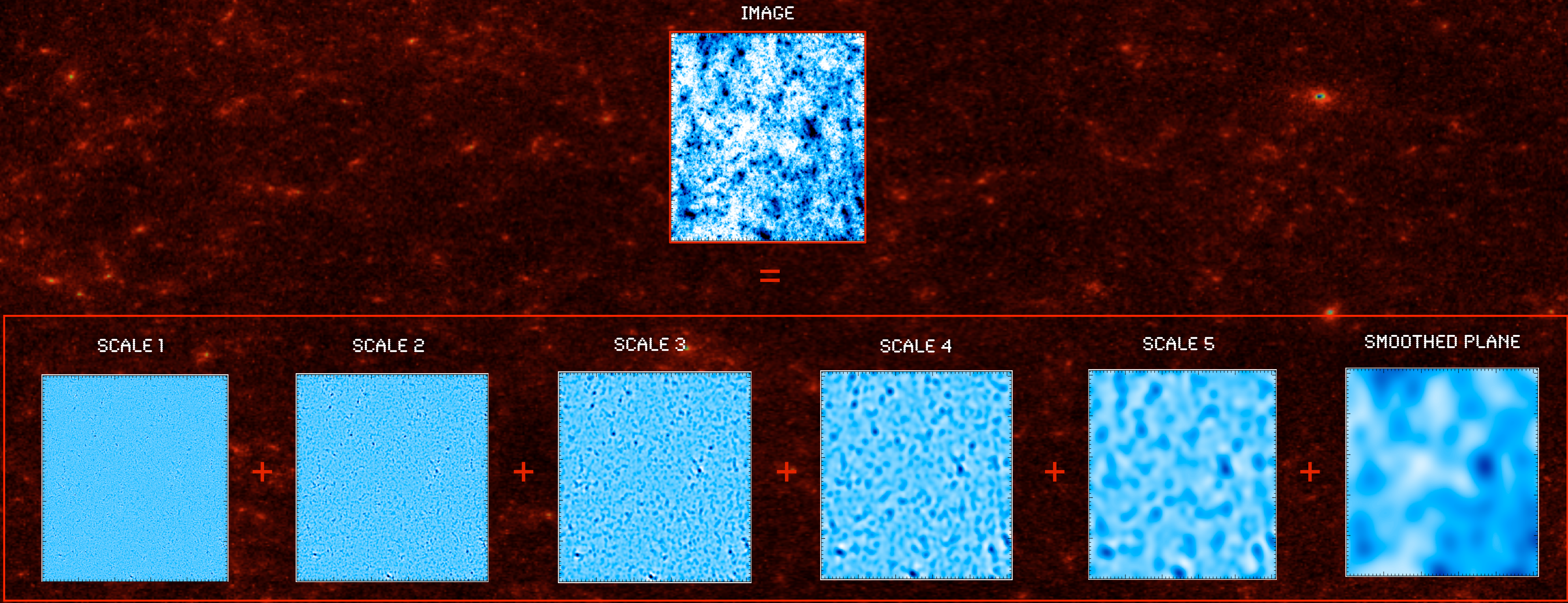
Noisy Signal
Fourier Transform



Filtered Signal

MRLENS : Multi-Resolution for weak LENSing

J.L. Starck, S. Pires and A. Réfrégier, A&A, 2006

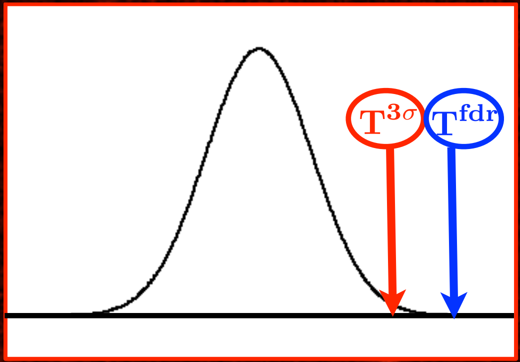


MRLENS :
False Discovery Rate method (FDR)

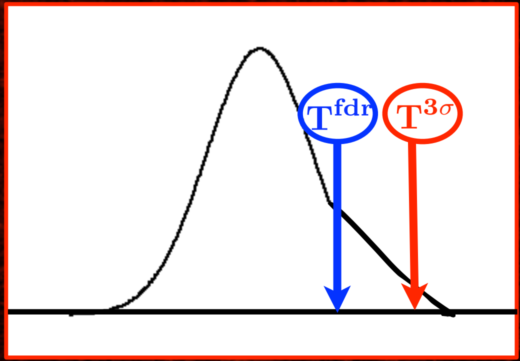
(Benjamini et al, 1995)

A Threshold is applied at each wavelet plane:

- ✓ kσ-Threshold: the number of false detections is depending on the number of sample
- ✓ FDR-Threshold: the number of false detections is depending on the number of true detections. The value of the threshold is then function of the level of the noise



ONLY NOISE



SIGNAL + NOISE

MRLENS : Maximum Multiresolution Entropy

BAYES' THEOREM: $$P(\kappa|\kappa_n) = \frac{P(\kappa_n|\kappa)P(\kappa)}{P(\kappa_n)}$$

$$\mathcal{Q} = -\log(P(\kappa|\kappa_n)) = -log(P(\kappa_n|\kappa)) - log(P(\kappa)) + Cte$$

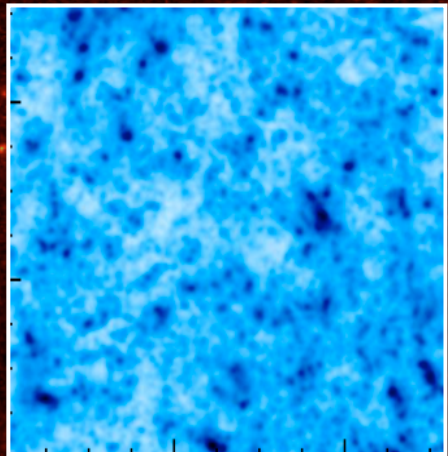
$$\mathcal{Q} = \frac{1}{2}\chi^2 - \alpha H$$

MULTISCALE ENTROPY

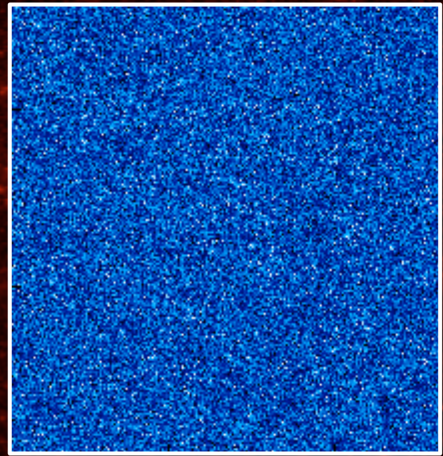
PENALISATION TERM

LIKELIHOOD TERM

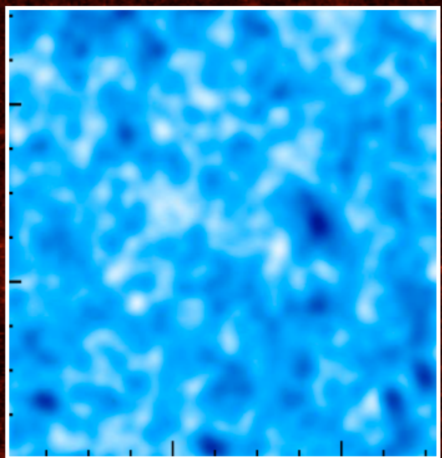
Comparison: Gaussian/Wiener/MRLens filter



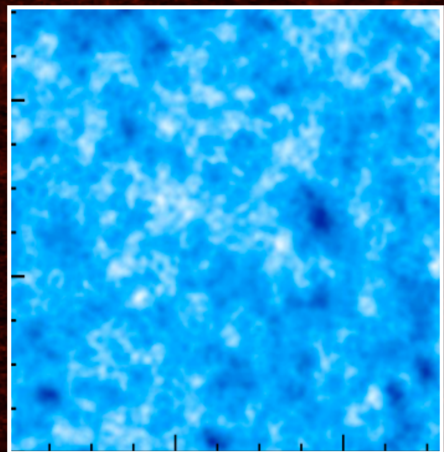
ORIGINAL MASS MAP



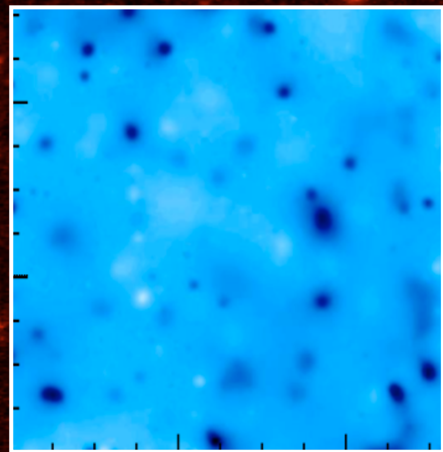
NOISY MASS MAP



GAUSSIAN FILTER



WIENER FILTER



MRLENS FILTER

MRLENS software

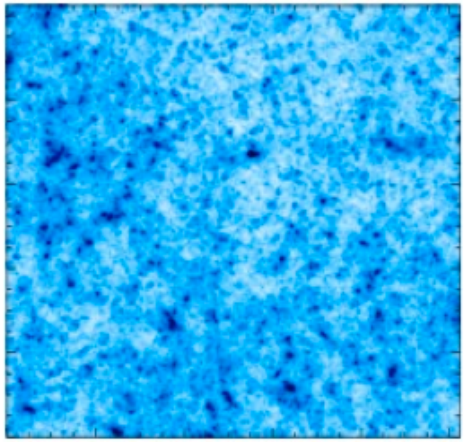
Multi-Resolution methods for gravitational LENSing

http://www-irfu.cea.fr/Ast/mrlens_software.php

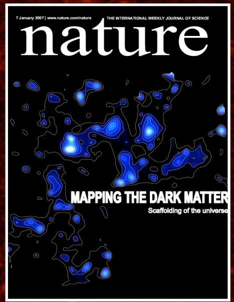
Software MRLENS : Multi-Resolution methods for gravitational LENSing

S. Pires, J.L. Starck and A. Réfrégier

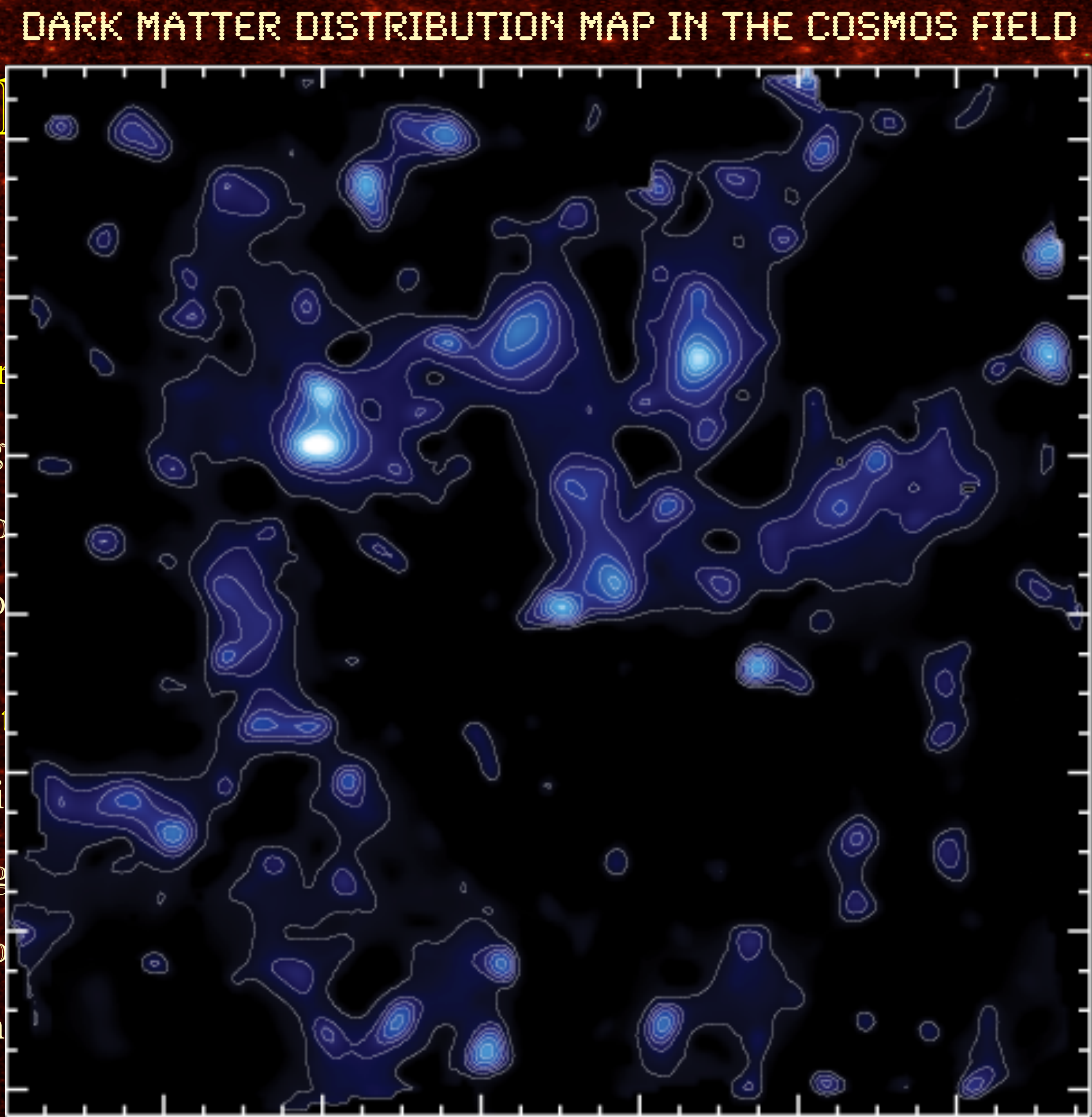
Welcome to the MRLENS web page.
This page introduce the MRLENS software (Version 1.0), contains links to our papers and allow you to download a copy of the MRLENS software and its user manual.



Simulated mass map from Vale and White (2003).



Appl… data

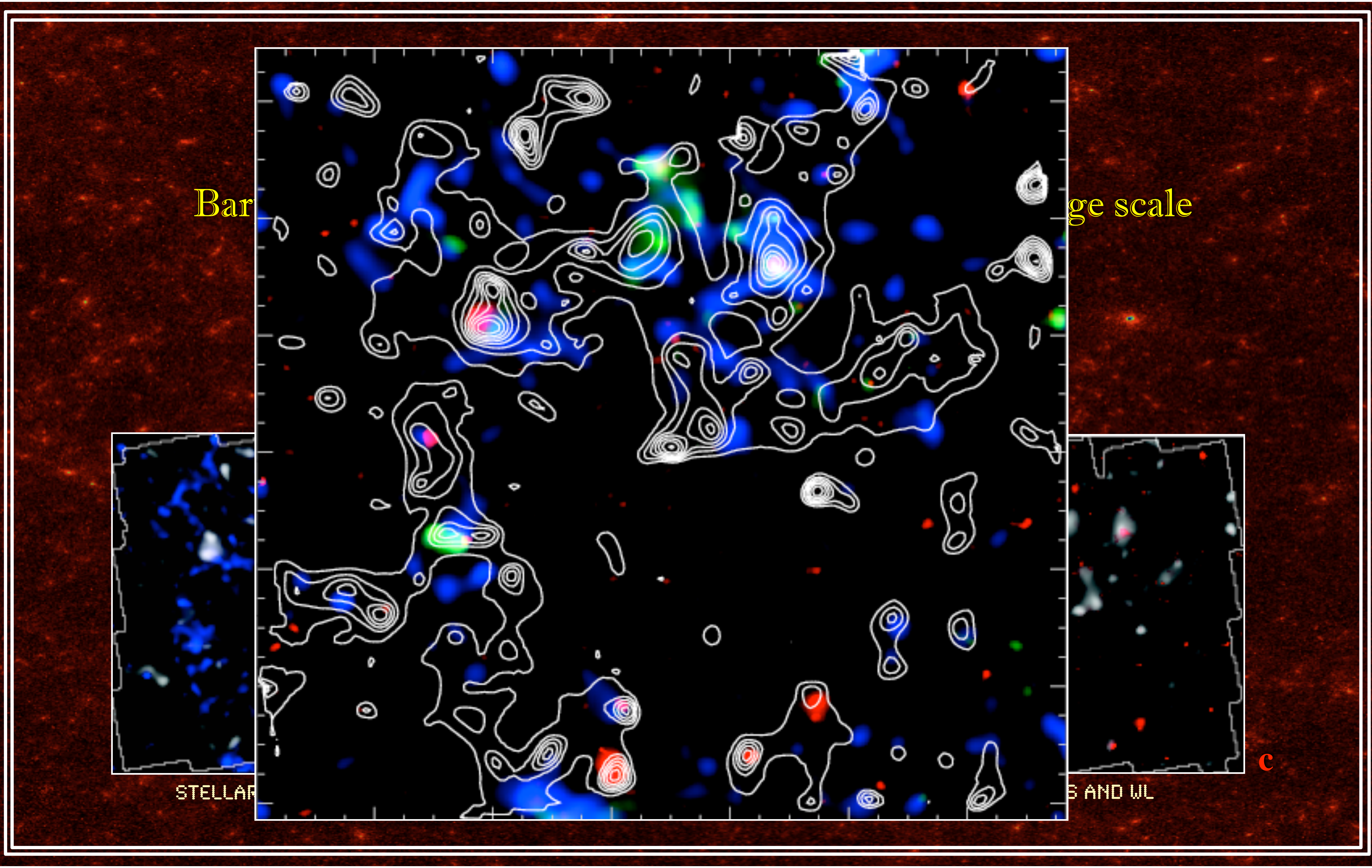


Data character…

- 575 pointing…
- Cover a regio…
- 500 000 shap…

Main steps on t…

- Raw processi…
- Making the g…
- Production o…
- Developmen…


Bar
ge scale
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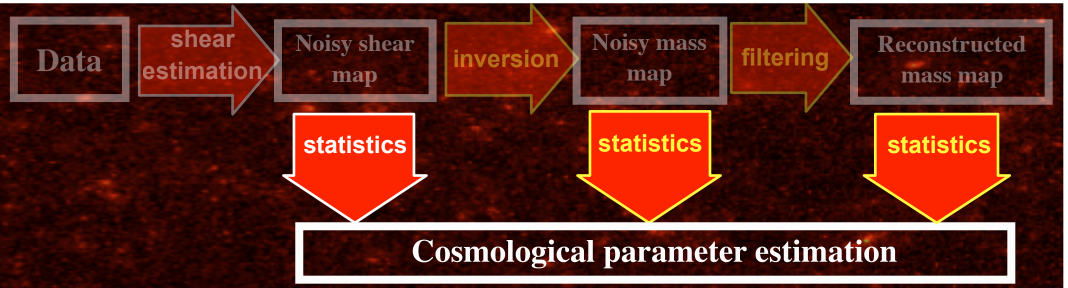
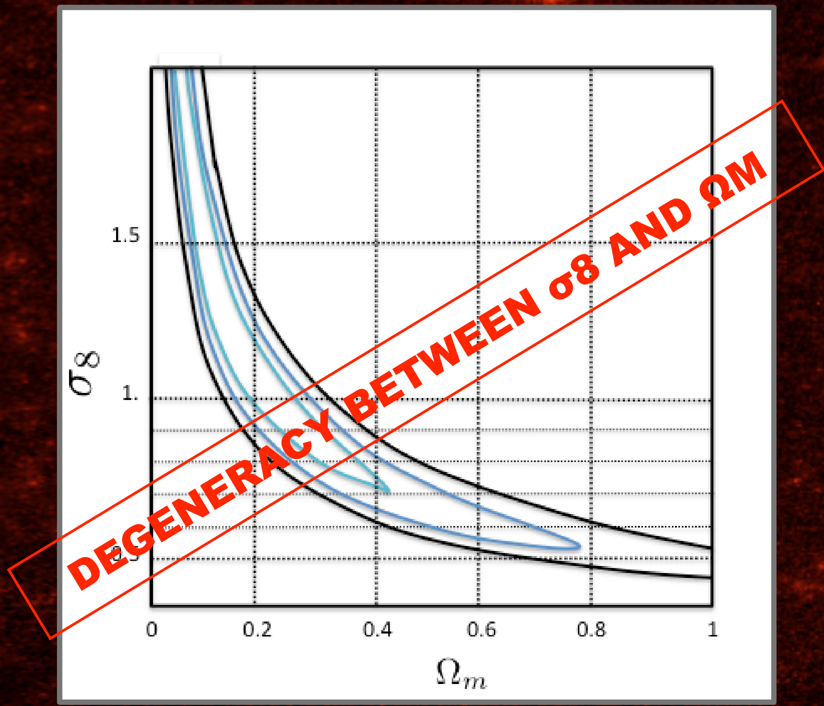
STELLAR
S AND WL


Data
shear estimation
Noisy shear map
inversion
Noisy mass map
filtering
Reconstructed mass map
statistics
statistics
statistics
Cosmological parameter estimation

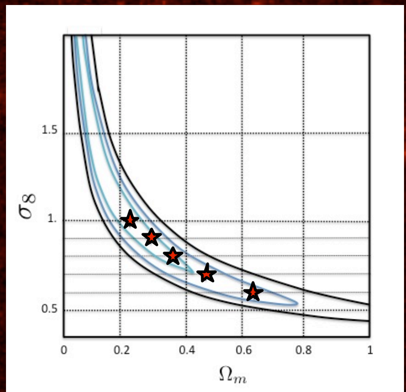
Outline

Statistical estimation to constrain cosmological parameters

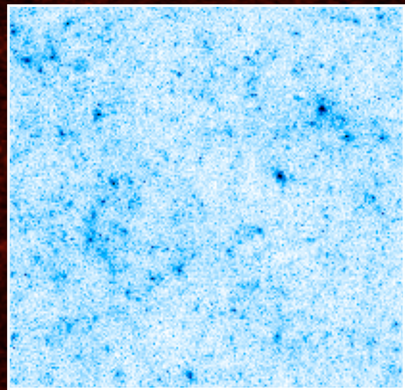
- ✓ Statistical estimation on shear maps:
  - ✓ Variance, skewness, kurtosis...
  - ✓ N-point correlation functions
  - ✓ Power spectrum, bispectrum, trispectrum...
- ✓ Statistical estimation on mass maps:
  - ✓ Variance, skewness, kurtosis...
  - ✓ N-point correlation functions
  - ✓ Power spectrum, bispectrum, trispectrum...
  - ✓ Cluster abundance, cluster correlations...



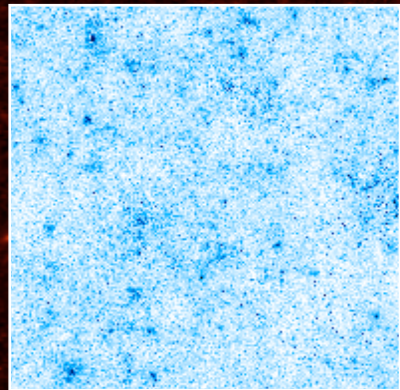
Cosmological model simulations



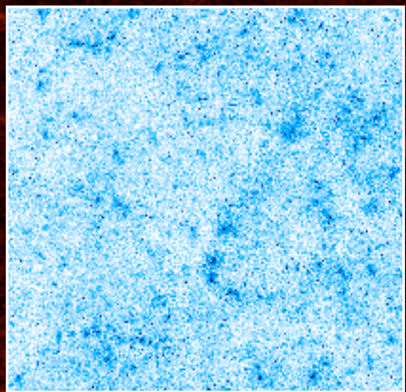
MODEL1 (σ_8=1, Ω_M=0.23)



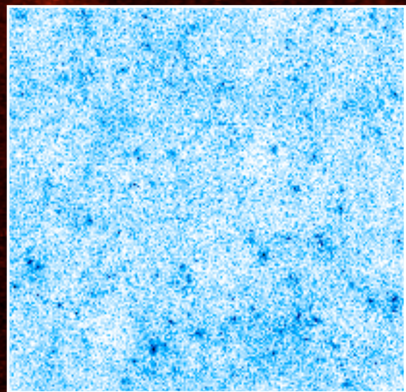
MODEL2 (σ_8=0.9, Ω_M=0.3)



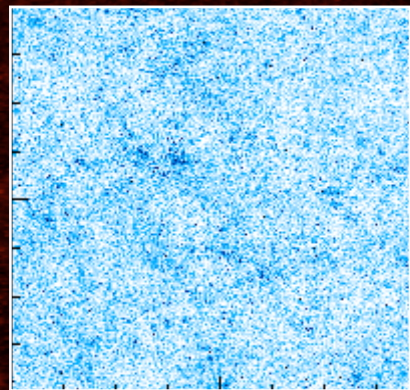
MODEL3 (σ_8=0.8, Ω_M=0.36)



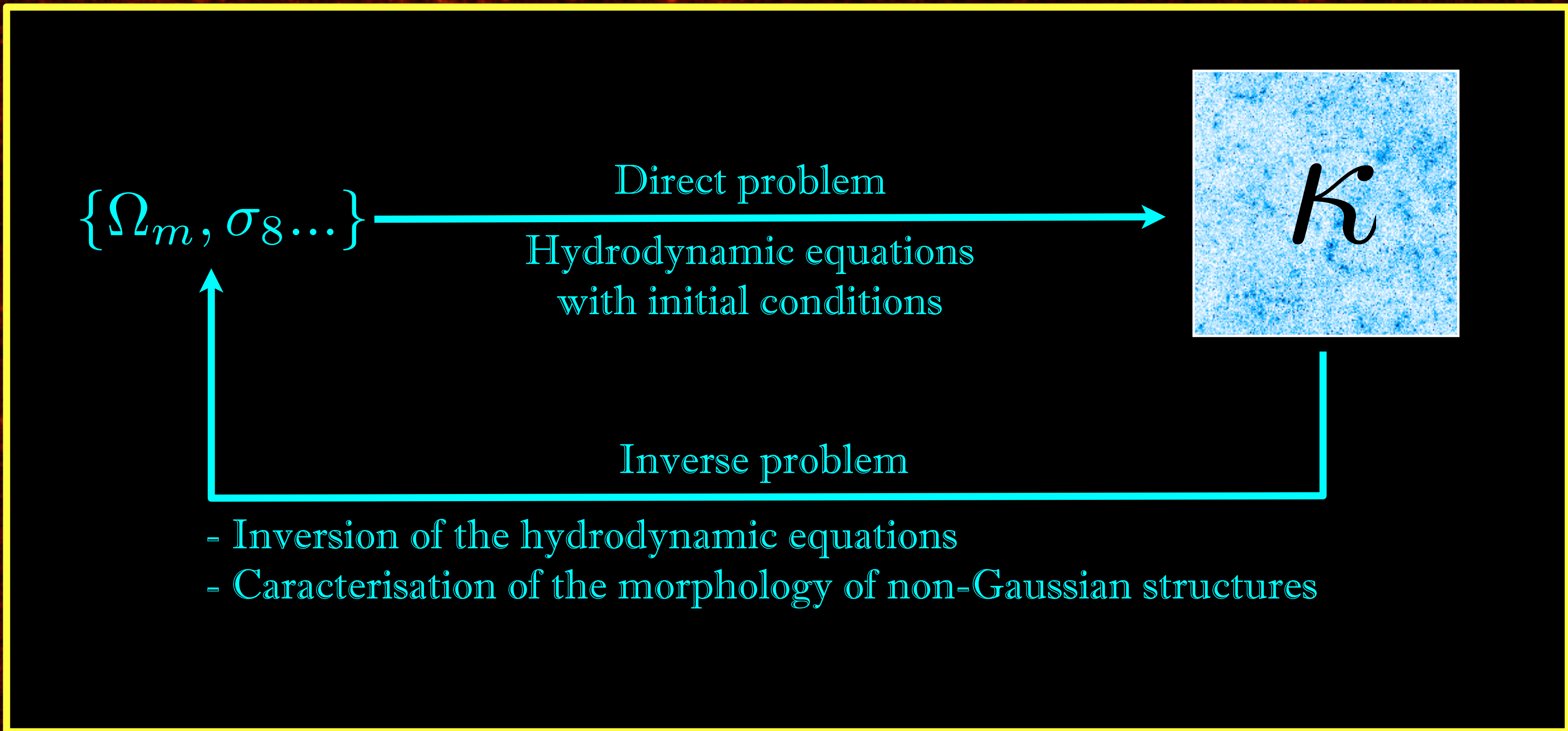
MODEL4 (σ_8=0.7, Ω_M=0.47)



MODEL5 (σ_8=0.6, Ω_M=0.64)



Inverse problem

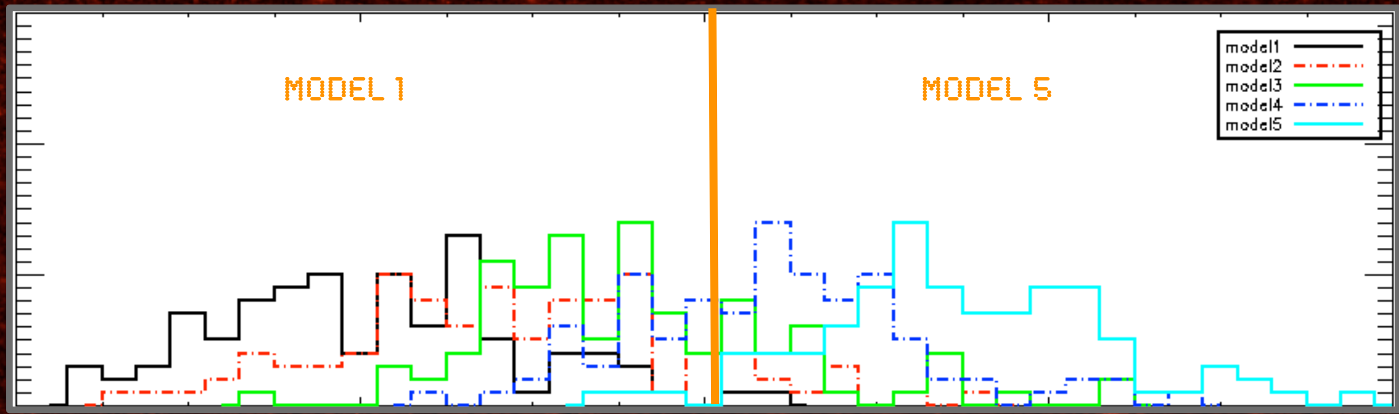


Candidate statistics

1. Skewness (third-order moment) estimated on a Direct, Fourier, Wavelet, Ridgelet and Curvelet representation
2. Kurtosis (fourth-order moment) estimated on a Direct, Fourier, Wavelet, Ridgelet and Curvelet representation
3. Higher Criticism (Donoho & Jin, 2004) estimated on a Direct, Fourier, Wavelet, Ridgelet and Curvelet representation
4. Bispectrum (Fourier space analog of the three-point correlation function)
5. Peak counting (cluster abundance)
6. WPC - Wavelet Peak Counting (Pires et al, 2008b)

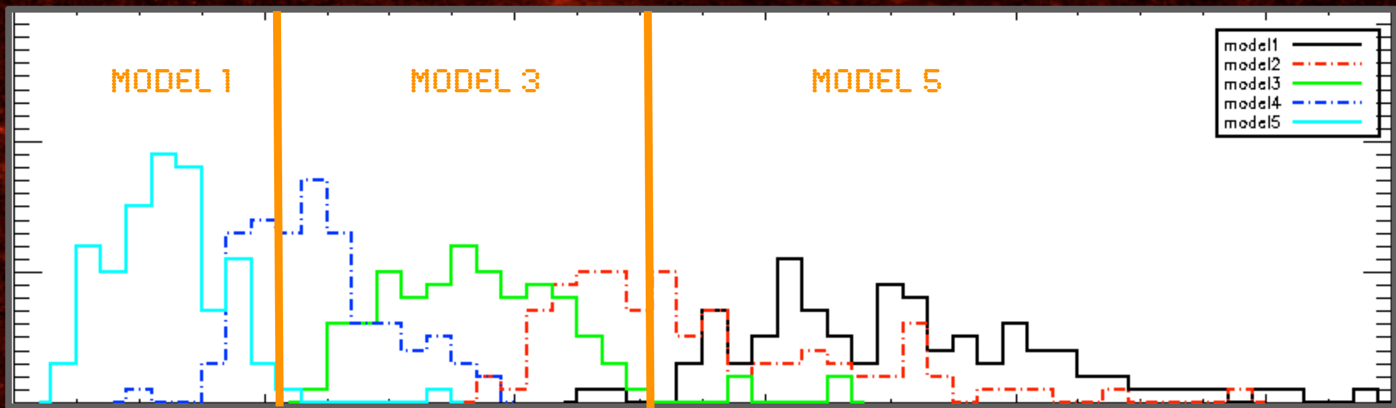
Discrimination Power

Pires et al, 2008b, submitted to A&A



SKEWNESS ON NOISY MAPS (AT SCALE OF ABOUT 1 ARCMIN)

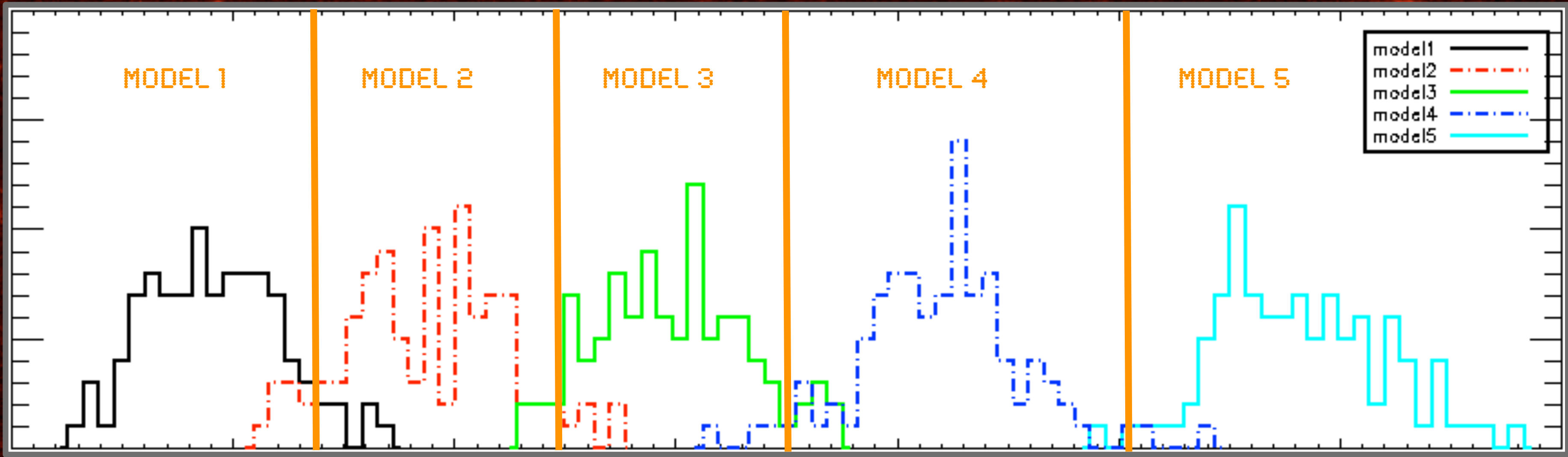
(kilbinger, 2005)



SKEWNESS ON MRLENS FILTRED MAPS (AT SCALE OF ABOUT 1 ARCMIN)

Best discrimination power : WPC

Pires et al, 2008b, submitted to A&A



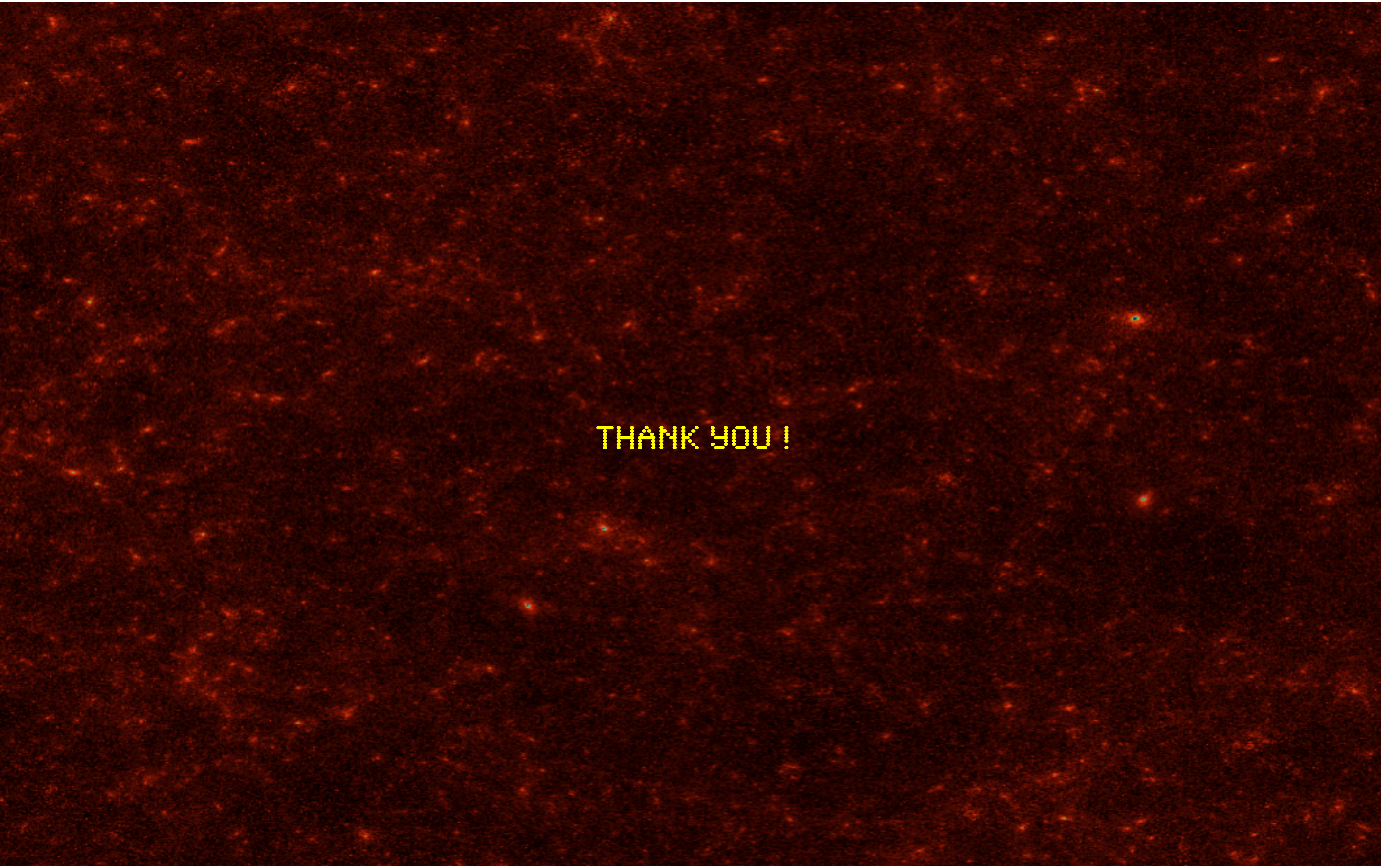
WAVELET PEAK COUNTING ON MRLENS FILTRED MAPS (AT SCALE OF ABOUT 1 ARCMIN)

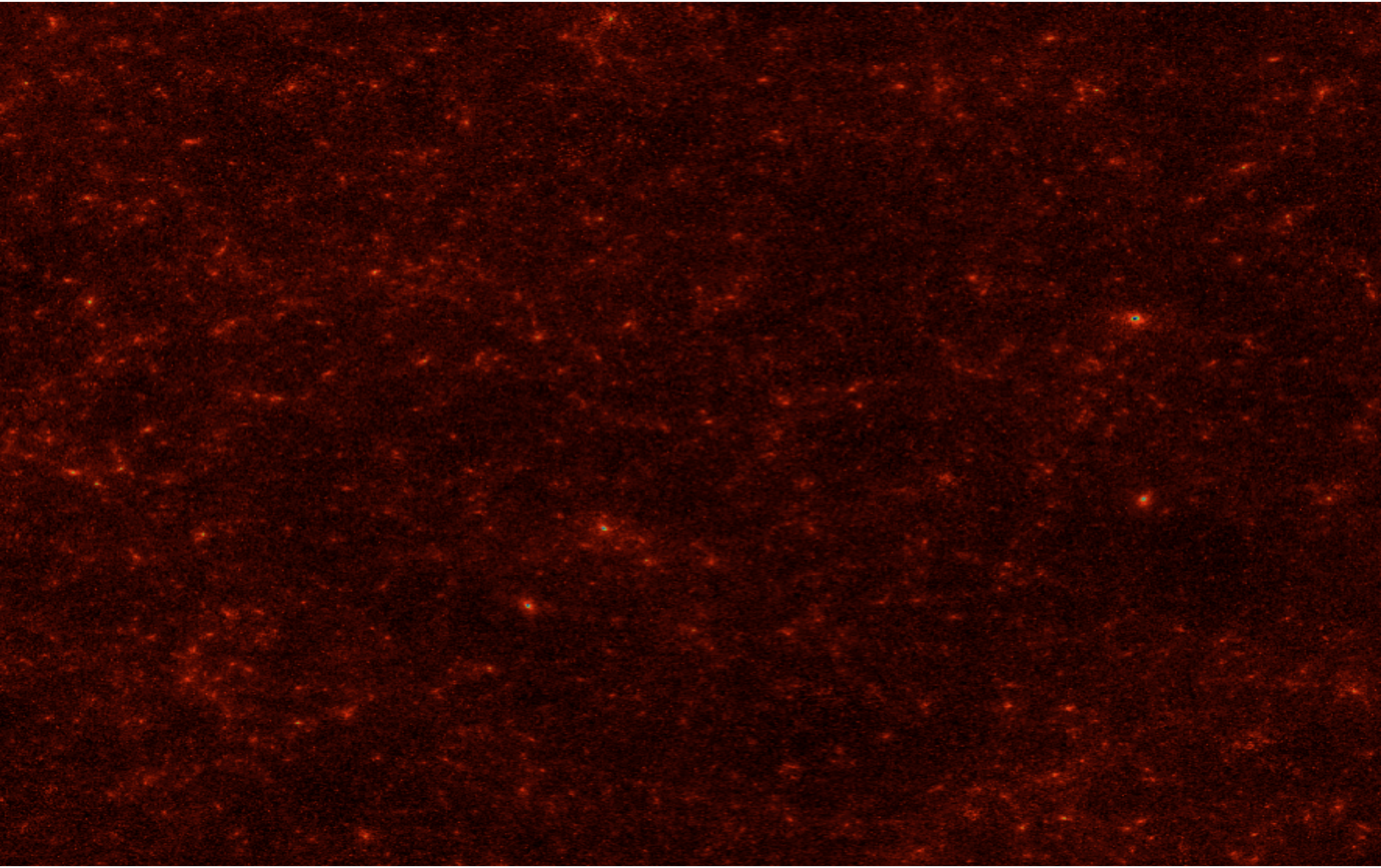
Conclusions

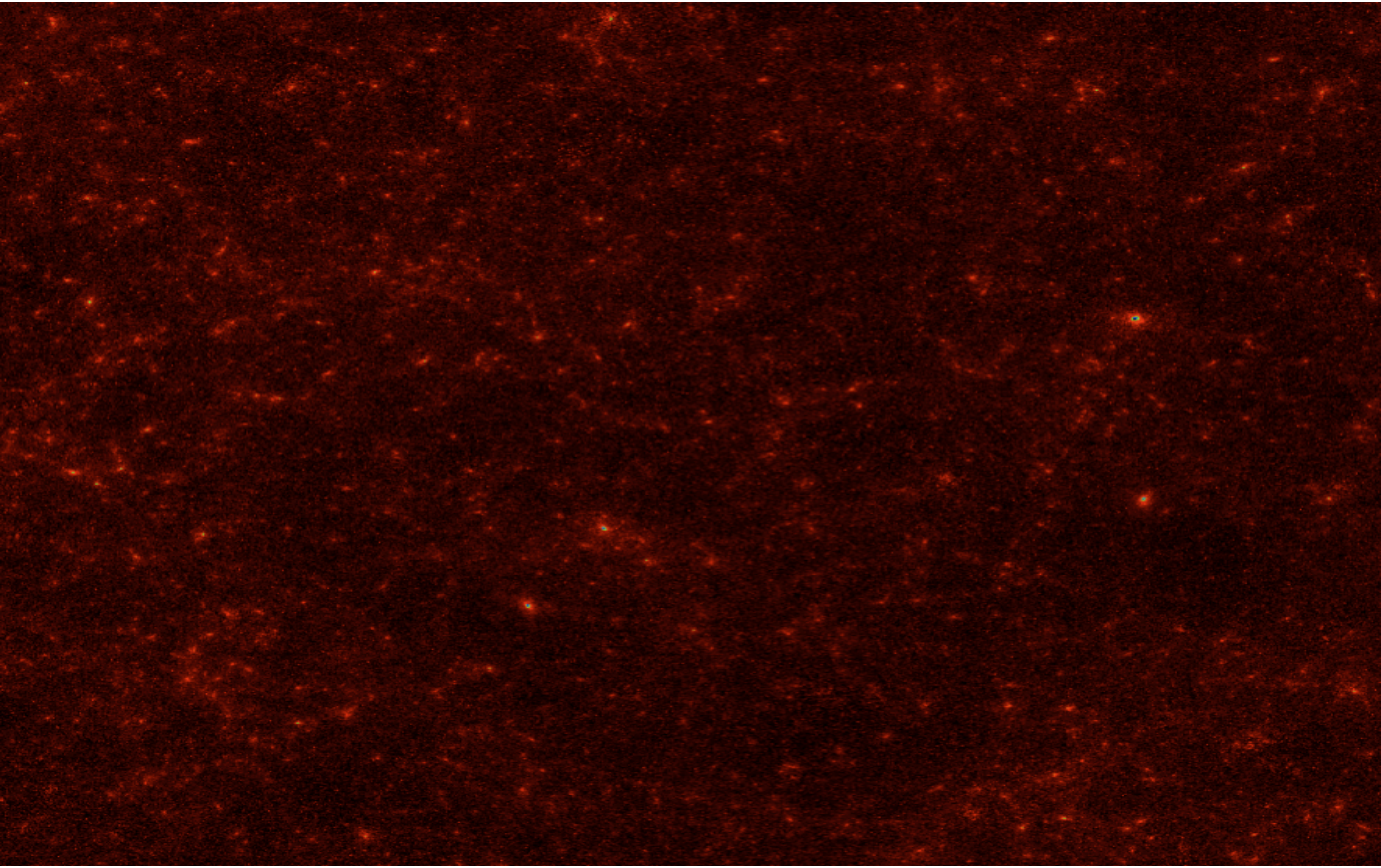
- A method to reconstruct full Weak Lensing mass map from incomplete shear maps has been developed (FASTLens)
  - The maximum error in the estimation of the power spectrum is 1%
  - The maximum error in the estimation of the bispectrum is 3%
  - FASTLens will be soon available including a method to estimate the equilateral bispectrum
- A method for filtering the noise of Weak Lensing dark matter mass map has been developed (MRLens)
  - Outperforms existing methods
  - Applied to real data (COSMOS field)
  - MRLens is freely available on the web (google mrlens)
- We have studied the best way to constrain the cosmological model
  - The better statistic is the Wavelet Peak Counting

Perspectives

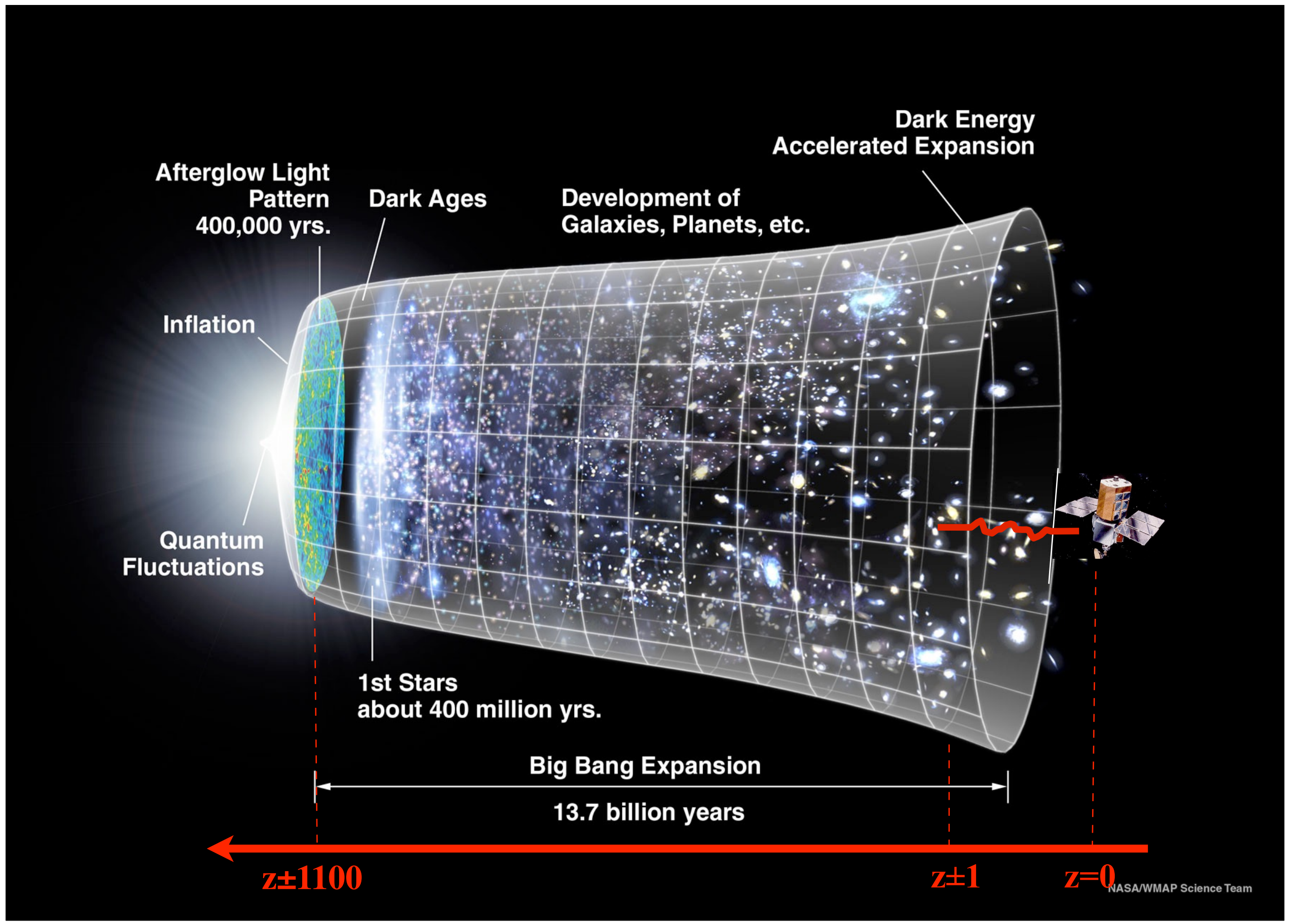
- ✓ Application of the method MRLens and FASTLens to CFHTLS data
- ✓ Extension of the MRLens filter to the processing of data on the sphere (Euclid project)
- ✓ Extension of the MRLens filter to the processing of 3D Weak Lensing data.
- ✓ Developement of a new method to estimate the shear using sparsity in the GREAT08 projet (Bridle, 2008)


THANK YOU !

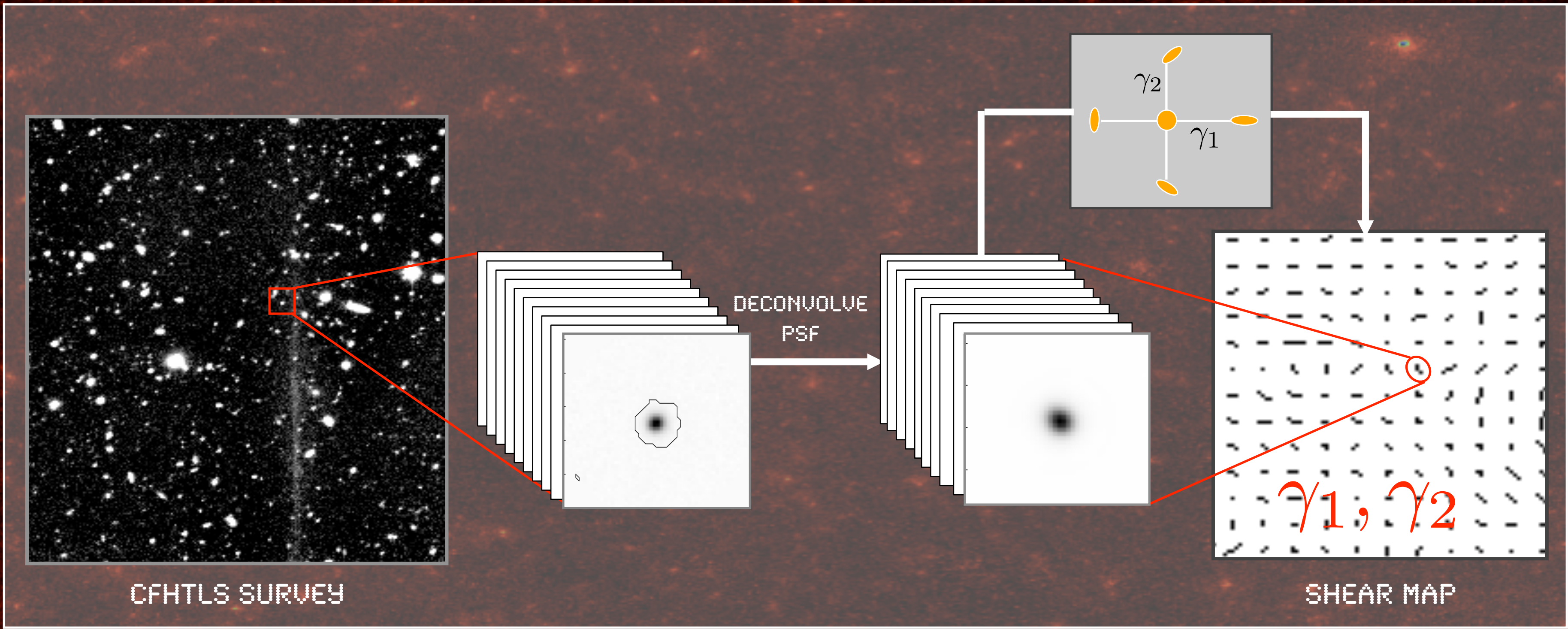




Weak Lensing observations



From shear measurements to shear map



Cosmological model

✓ Einstein metric represents a Universe static with matter:

$$ds^2 = (\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2) - cdt^2$$

Courbure spatiale: $k = \sqrt{\Lambda}$

✓ De sitter metric represents a Universe in expansion without matter:

$$ds^2 = a(t)^2(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2) - cdt^2$$

✓ Friedmann-Lemaître-Robertson-Walker metric represents a Universe (with matter) isotropic and homogeneous in expansion:

$$ds^2 = a(t)^2(\frac{dr^2}{1 - Rr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2) - cdt^2$$

Courbure spatiale: $R = -1, 0, +1$

From ellipticities to the shear

(Kitching et al, 2007)

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = \frac{1-\beta}{1+\beta} \begin{pmatrix} cos[2\phi] \\ sin[2\phi] \end{pmatrix}$$

$$\epsilon = \frac{\epsilon^I + g}{1 + g^* \epsilon^I}$$ where: $$g \equiv \frac{\gamma}{1-\kappa}$$ with: $$|g| < 1$$

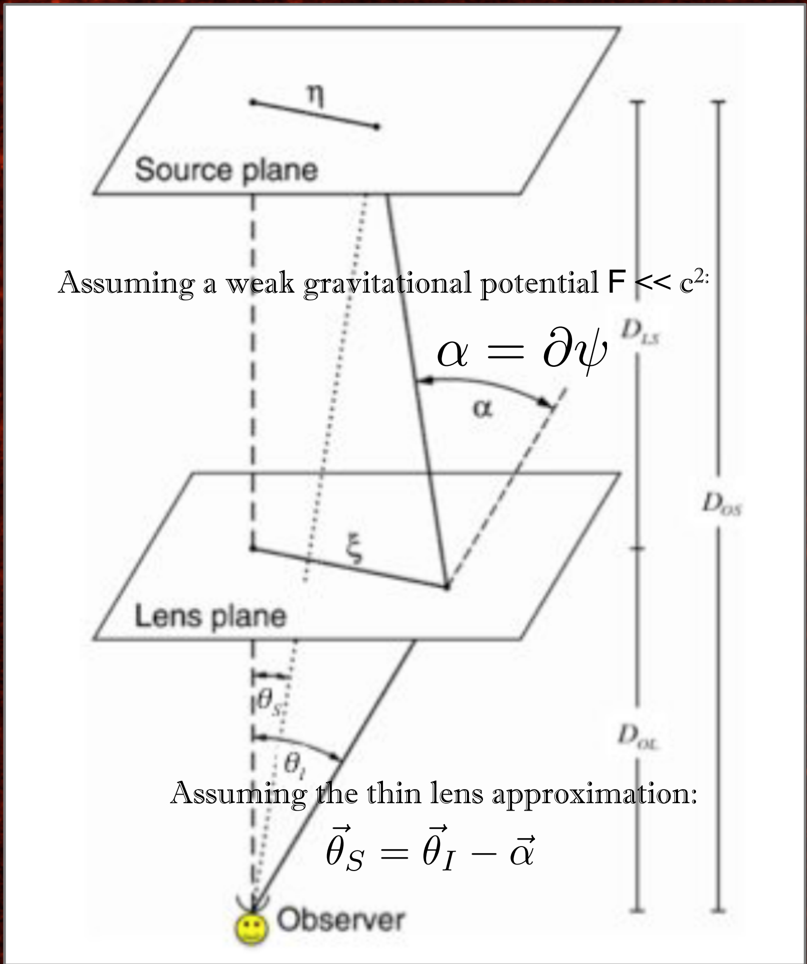
(Seitz & Schneider, 97)

(Mellier, 99; Bartelmann & Schneider, 2001)

$$< \epsilon > \approx g$$ where: $$g \approx \gamma$$ with: $$\kappa \ll 1$$

$$< \epsilon > = \frac{\gamma}{P^\gamma}$$ with: $$P^\gamma = < \epsilon^I >$$

Lensing and shear equations



$$\theta_{S,I} = A_{i,j}\theta_{I,j} \qquad A_{i,j} = \delta_{i,j} - \frac{\partial \alpha_i}{\partial \theta_I}$$

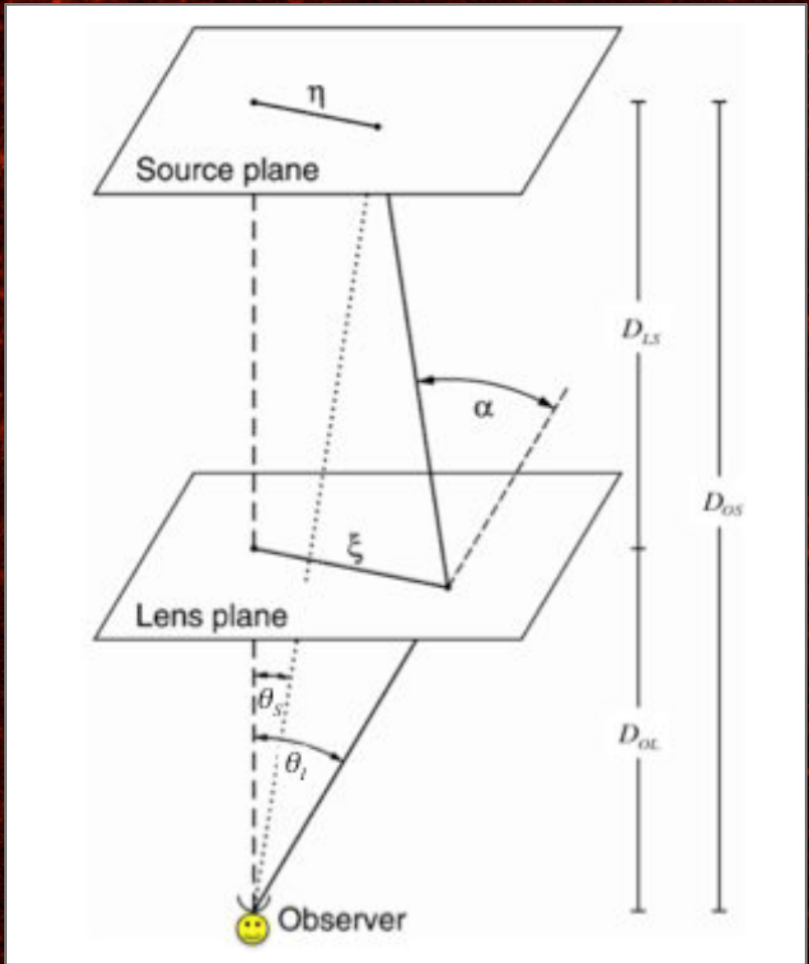
$$A_{i,j} = \begin{pmatrix} 1-\kappa-\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

$$\kappa = \frac{1}{2}\left(\partial_1^2 + \partial_2^2\right)\psi$$

$$\gamma_1 = \frac{1}{2}\left(\partial_1^2 - \partial_2^2\right)\psi$$

$$\gamma_2 = \partial_1\partial_2\psi$$

Lensing and flexion equations



$$\theta_{S,I} = A_{i,j}\theta_{I,j} + \frac{1}{2}D_{i,j,k}\theta_{I,j}\theta_{I,k}$$

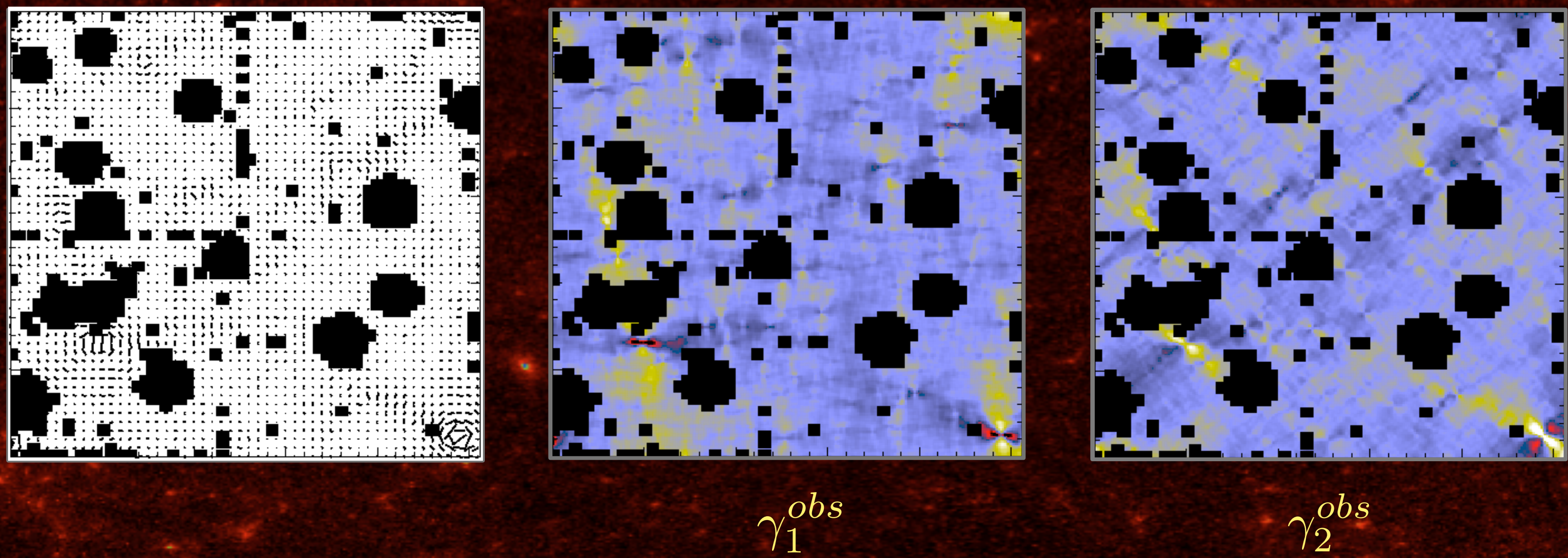
$$D_{i,j,1} = \begin{pmatrix} -2\frac{\partial\gamma_1}{\partial\theta_{I,1}} - \frac{\partial\gamma_2}{\partial\theta_{I,2}} & \frac{\partial\gamma_2}{\partial\theta_{I,1}} \\ -\frac{\partial\gamma_2}{\partial\theta_{I,1}} & \frac{\partial\gamma_2}{\partial\theta_{I,2}} \end{pmatrix}$$

$$D_{i,j,2} = \begin{pmatrix} -\frac{\partial\gamma_2}{\partial\theta_{I,1}} & \frac{\partial\gamma_2}{\partial\theta_{I,2}} \\ -\frac{\partial\gamma_2}{\partial\theta_{I,2}} & 2\frac{\partial\gamma_2}{\partial\theta_{I,2}} - \frac{\partial\gamma_2}{\partial\theta_{I,1}} \end{pmatrix}$$

$$\mathcal{F} = (\partial_1\gamma_1 + \partial_2\gamma_2) + i(\partial_1\gamma_2 - \partial_2\gamma_1)$$

$$\mathcal{G} = (\partial_1\gamma_1 - \partial_2\gamma_2) + i(\partial_1\gamma_2 + \partial_2\gamma_1)$$

Inpainting from Shear maps



γ_1^{obs} γ_2^{obs}

$$
\begin{aligned}
Y &= P_1 * \gamma_1^{obs} + P_2 * \gamma_2^{obs} \\
I_{max} &= 100 \\
\kappa^0 &= 0 \\
R^0 &= Y \\
\lambda_{max} &= max(|\alpha = \Phi^T Y|) \\
\lambda_{min} &= 0
\end{aligned}
$$

Inpainting algorithm from shear maps

for $n = 0$ *to* I_{max} *do begin*

$U = \kappa^n + MR^n(\gamma^{obs})$ *et*

$R^n(\gamma^{obs}) = P_1 * (\gamma_1^{obs} - P_1 * \kappa_n) + P_2 * (\gamma_2^{obs} - P_2 * \kappa_n)$

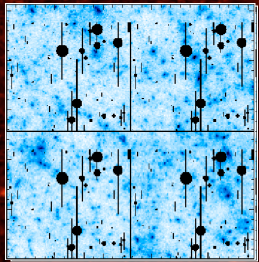
Digital Cosine Transform (DCT) of U: $\alpha = \Phi^T U$

Threshold determination: λ_n

Hard-thresholding of α *with* $\alpha_n : \tilde{\alpha} = S_{\lambda_n} \alpha$

$\kappa^{n+1} = \Phi \tilde{\alpha}$

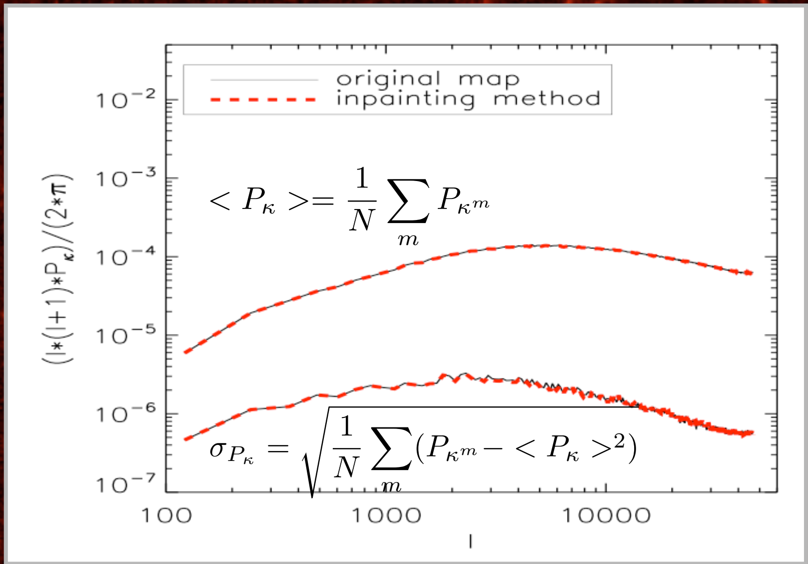
$n = n + 1 \text{ if } n < I_{max}(2)$

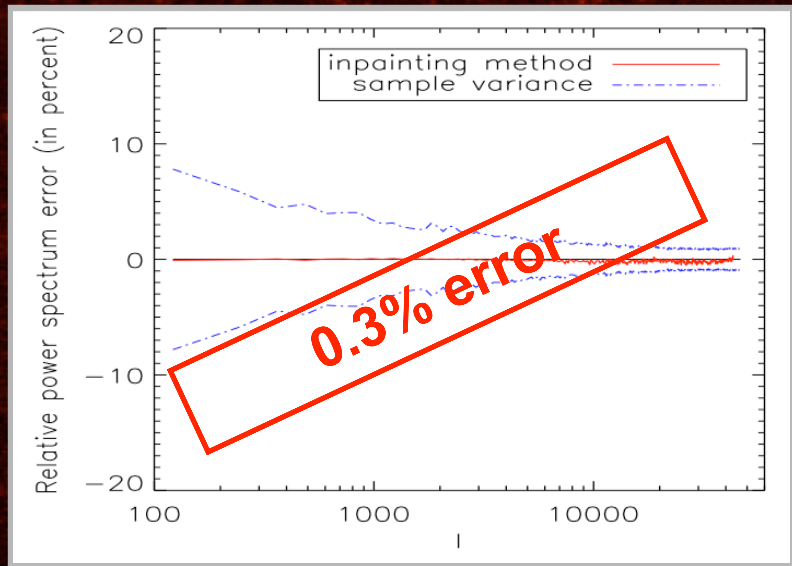


Power spectrum estimation

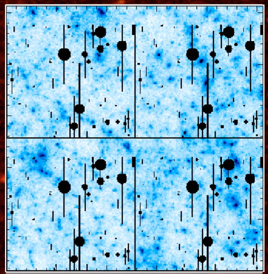
Pires et al 2008a

RELATIVE POWER SPECTRUM ERROR, I.E. THE NORMALIZED DIFFERENCE BETWEEN THE TWO UPPER CURVES OF THE LEFT PANEL.



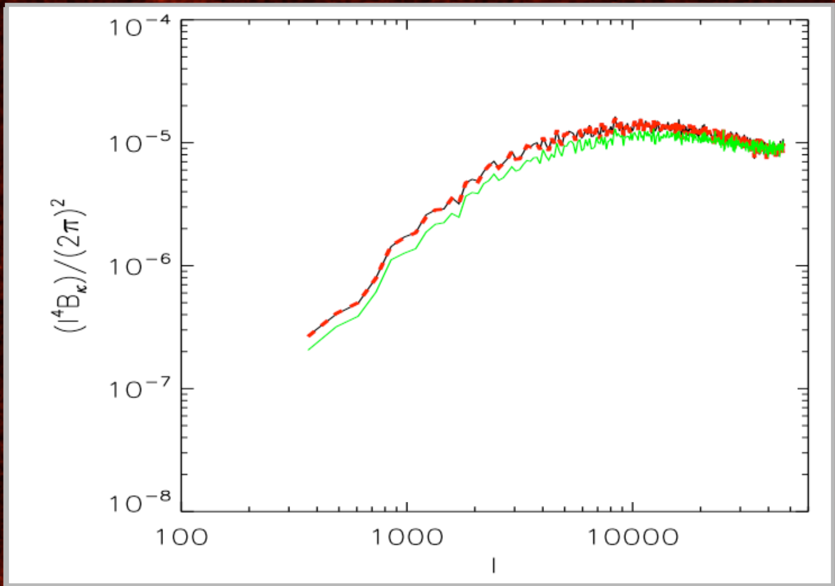


MEAN POWER SPECTRUM COMPUTED FROM
- 100 COMPLETE MASS MAPS (BLACK)
- 100 INPAINTED MAPS (RED).



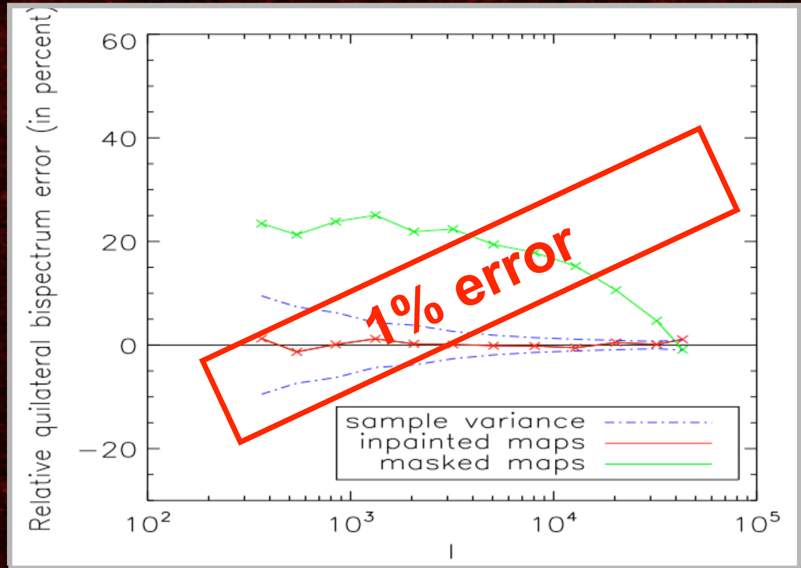
Equilateral bispectrum estimation

FASTLens, Pires et al 2008a

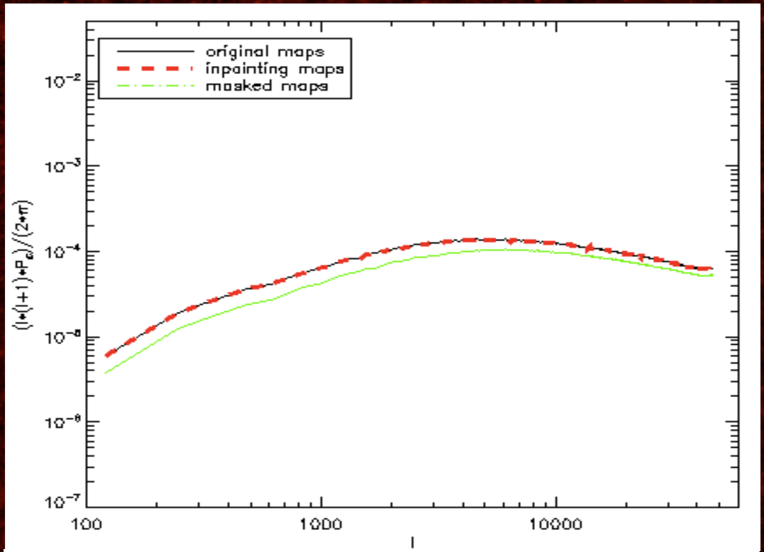


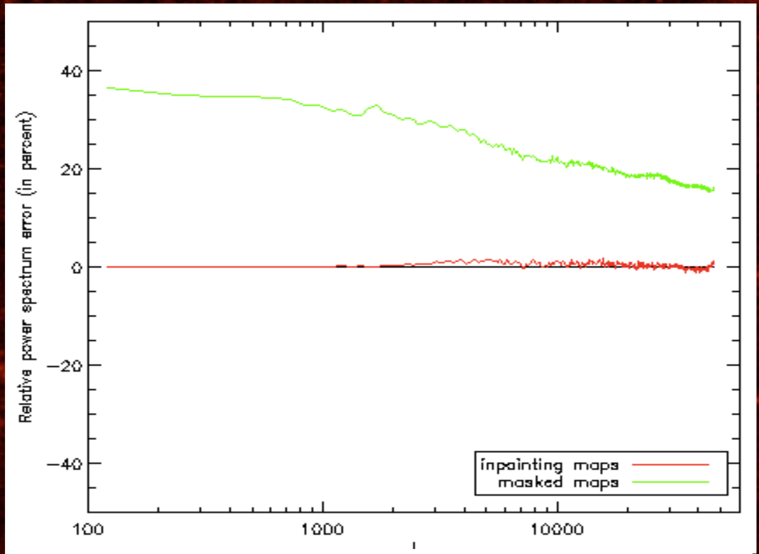
MEAN BISPECTRUM COMPUTED FROM
- 100 COMPLETE MASS MAPS (BLACK),
- 100 INPAINTED RECONSTRUCTED MAPS (RED)
- 100 INCOMPLETE MASS MAPS (GREEN) .

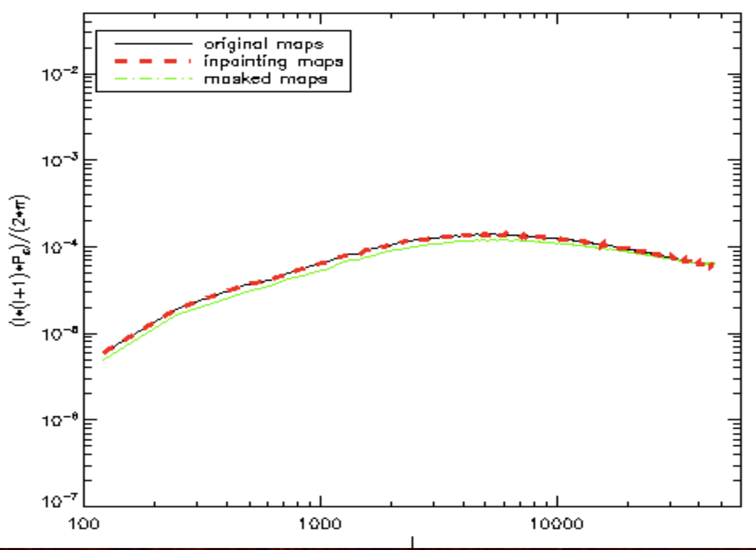
RELATIVE BISPECTRUM ERROR, I.E. THE NORMALIZED DIFFERENCE BETWEEN THE TWO UPPER CURVES OF THE LEFT PANEL.

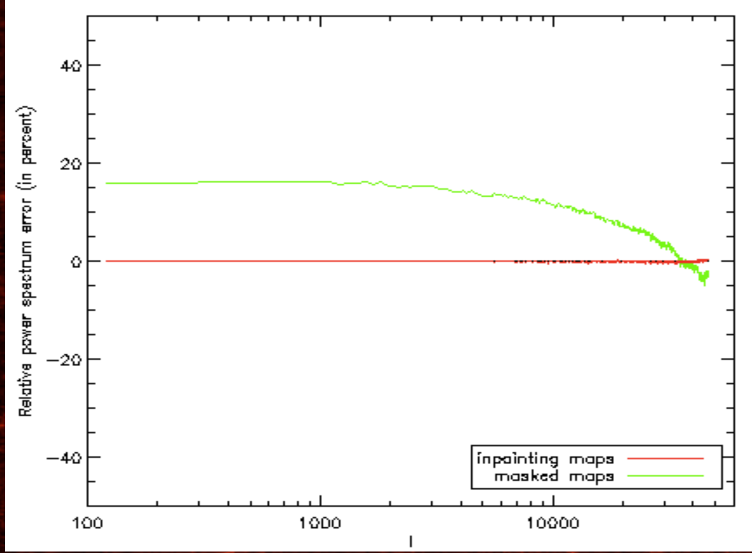


Power spectrum estimation









MRLens : Multiscale Entropy definition

$$Q = \frac{1}{2}\chi^2 - \alpha h_n$$

$$h_n(w_j(x,y)) = \bar{M}_j(x,y)h(w_j(x,y))$$

$$FDR = \frac{V_{ia}}{D_a}$$

$$E(FDR) \leq \alpha$$

NOISE MSE

- ✓ Close to a quadratic function on small coefficients
- ✓ Close to l1-norme on large coefficients

$$h(w_j(x,y)) = \frac{1}{\sigma_{n_j}} \int_0^{|w_j(x,y)|} u.erfc(\frac{|w_j(x,y)| - u}{\sqrt{2}\sigma_j})du$$

ITERATIVE ALGORITHM

$$\tilde{\kappa}^i = \tilde{\kappa}^{i-1} + \mathcal{W}^{-1}(\bar{M}(\tilde{w} - \mathcal{W}\tilde{\kappa}^{i-1}))$$

Penalisation term different from an Entropy : Wavelet regularisation

Penalisation term on the dependency of a pixel with their neighbor pixels

$$C(\kappa) = \beta \sum_x \sum_y (\phi(\kappa(x,y) - \kappa(x,y+1))^2 + \phi(\kappa(x,y) - \kappa(x+1,y))^2)^{\frac{1}{2}}$$

$\phi(x) = x^2$: Quadratic function

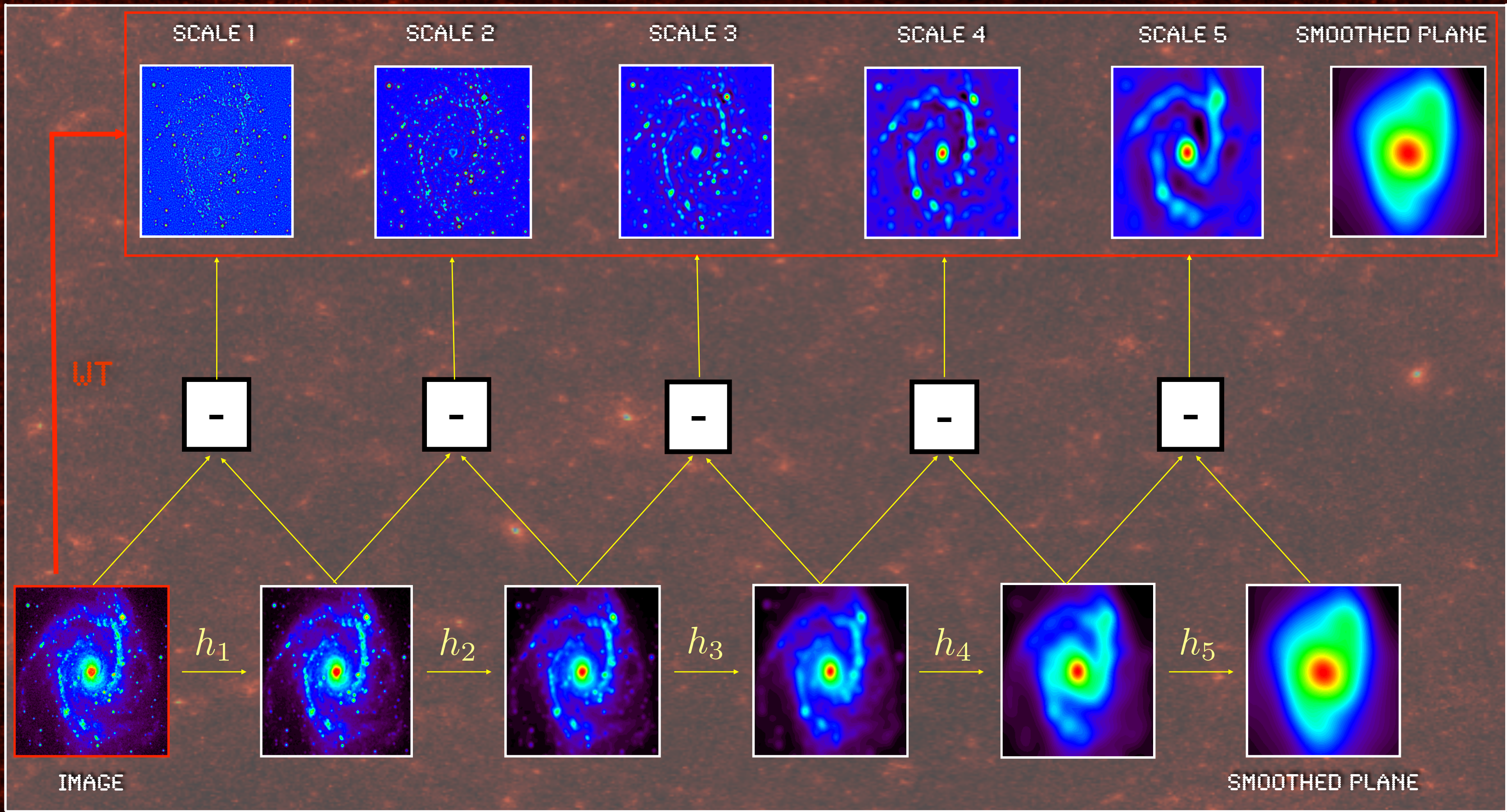
$\phi(x) = |x|$: Total variation

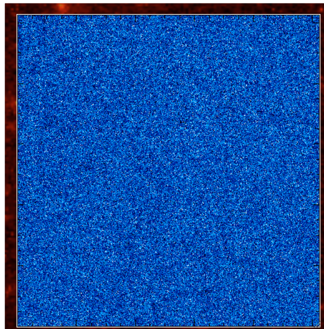
Multiscale penalisation term on the dependency of a pixel with their neighbor pixels

$$C_w(\kappa) = \beta \sum_{j,k,l} (\phi(||(\mathcal{W}_\kappa)_{j,k,l}||_p)$$

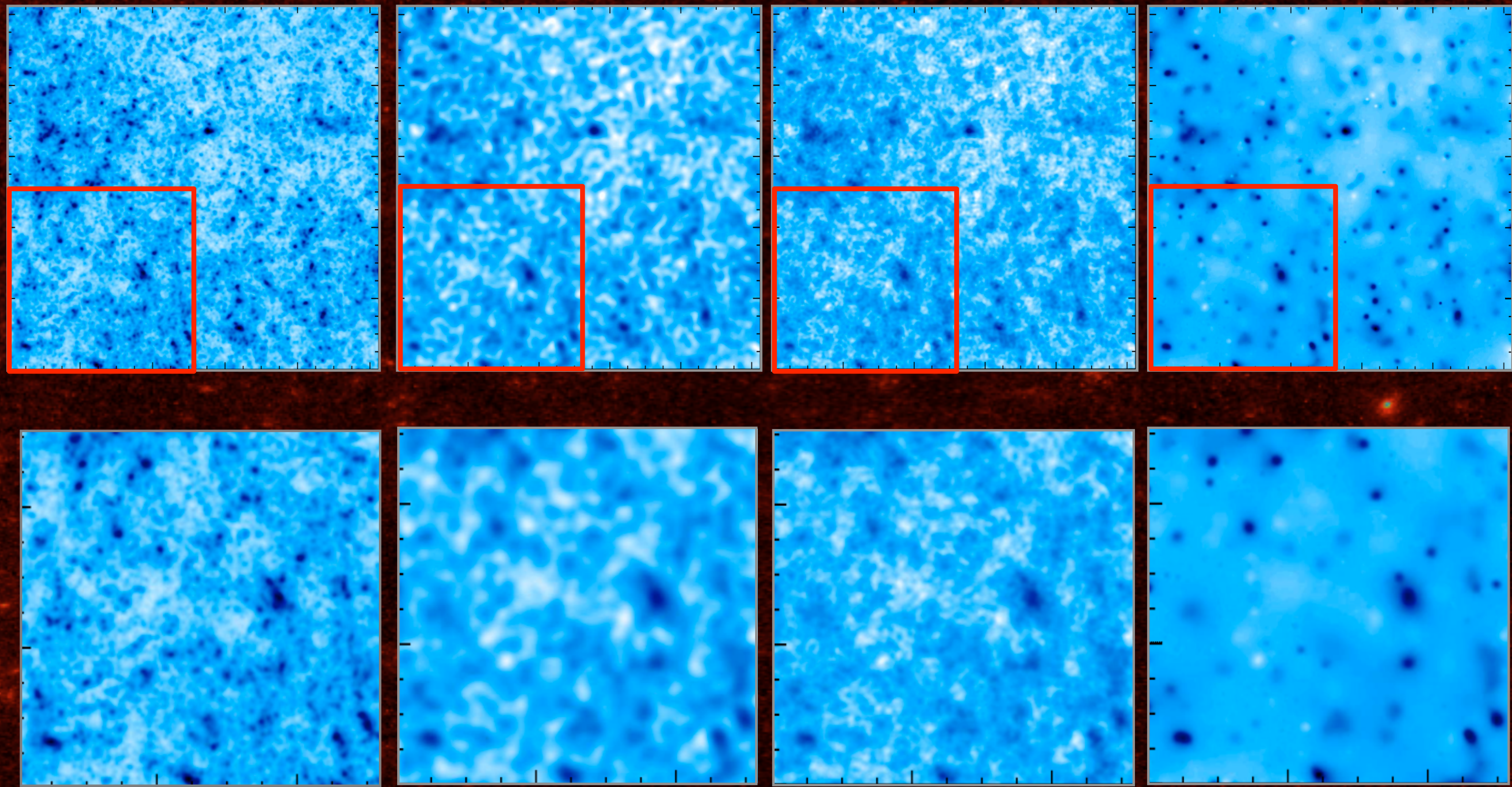
MRLENS : Multi-Resolution for weak LENSing

J.L. Starck, S. Pires and A. Réfrégier, A&A, 2006



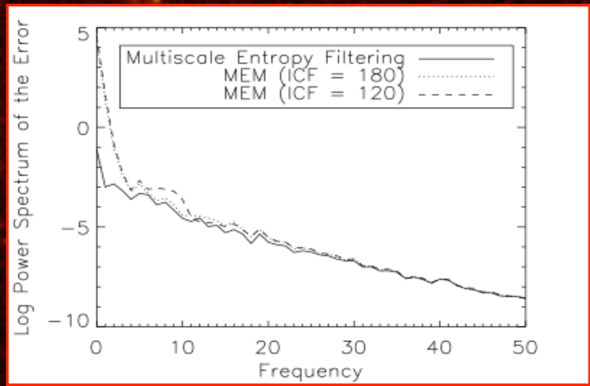


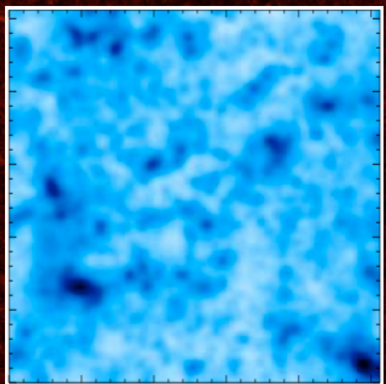
Comparison between Gaussian, Wiener and MRLens filter

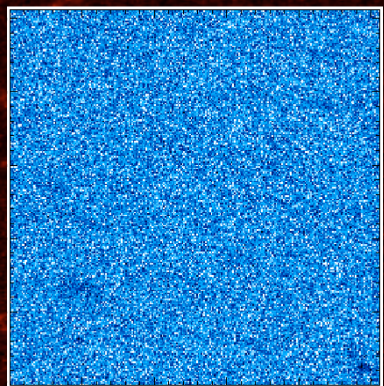


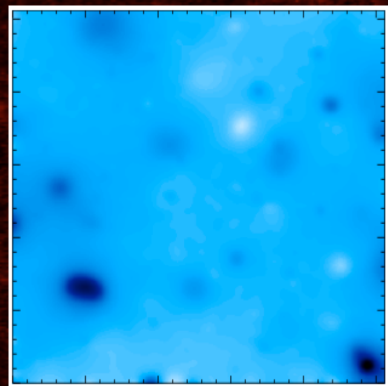
ORIGINAL MAP | GAUSSIAN FILTER | WIENER FILTER | MRLENS FILTER

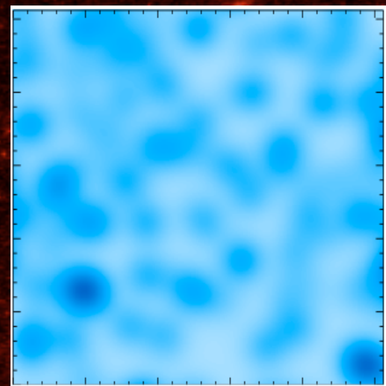
Comparison: MEM / MRLens filter



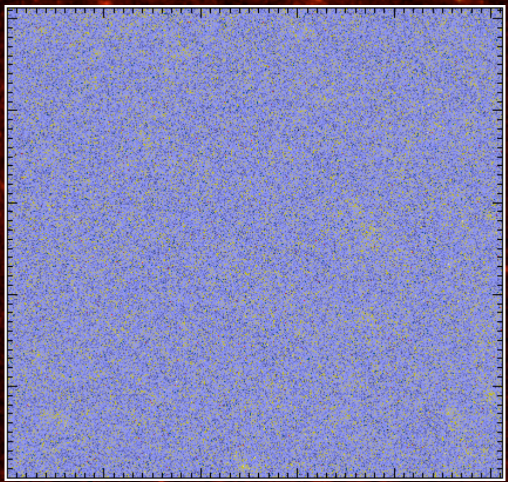

SIMULATED MASS MAP

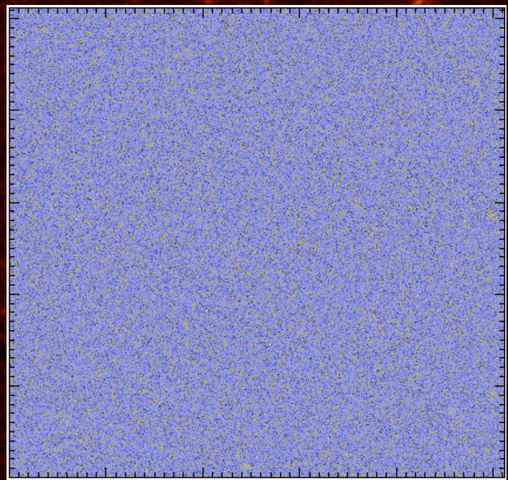

SIMULATED MASS MAP
(SPACE OBSERVATIONS)

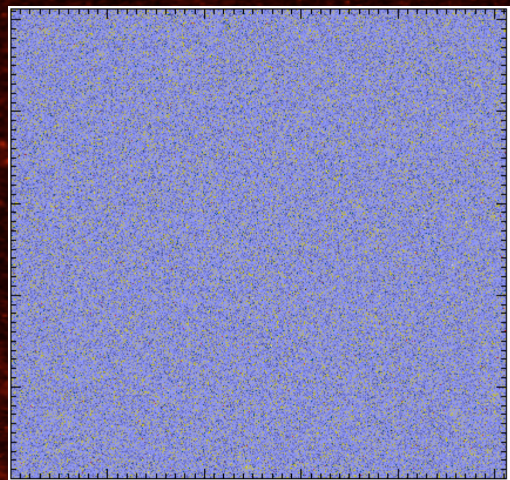

MASS MAP FILTERED
BY MRLENS

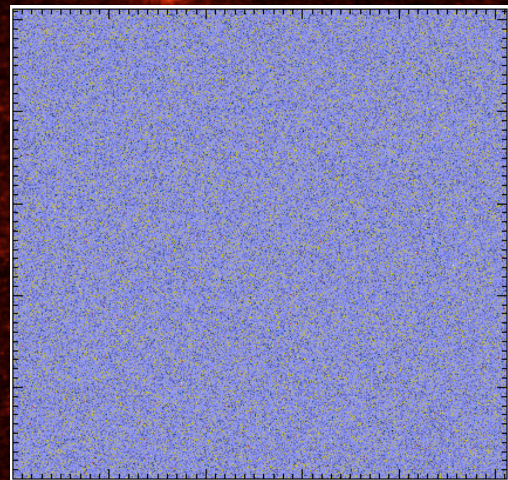

MASS MAP FILTERED BY MEM
(MAXIMUM ENTROPY METHOD)

Filtering Residual


NOISY IMAGE


GAUSSIAN RESIDUAL


WIENER RESIDUAL

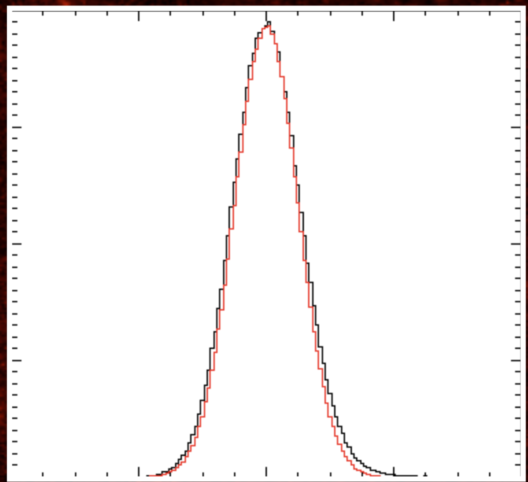

MRLENS RESIDUAL

Higher Criticism
(Donoho and Jin, 2004)

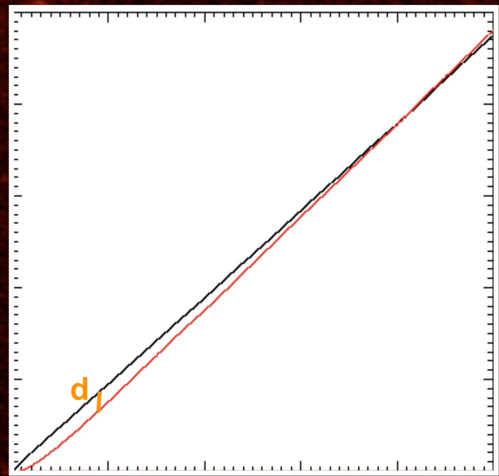
$$HC = \max_i \frac{\sqrt{n}\left(i/n - p(i)\right)}{\sqrt{p(i)(1-p(i))}}$$

d = $\sqrt{n}\left(i/n - p(i)\right)$

Higher Criticism is a measure of non-gaussianity. Values of 2 or greater indicate non-gaussianity.



histogram of a map



Sorted pvalues

Simulations numériques

- ✓ 3D N-body simulation by solving the hydrodynamic equations on a AMR grid (Ramses code)
- ✓ Dark matter mass maps simulation by projeting the density along the line of sight (using the Born approximation):

$$\kappa_e \approx \frac{3H_0^2{}_mL}{2c^2} \sum_i \frac{\chi_i(\chi_0 - \chi_i)}{\chi_0 a(\chi_i)} \left(\frac{n_p R^2}{N_t s^2} - \Delta r_{f_i} \right)$$